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STATISTICAL PROPERTIES OF TRAPEZIUM SYSTEMS

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RESUMEN

Hemos obtenido recientemente un catálogo de más de 900 sistemas múltiples de tipo trapecio. Al construir el catálogo se intentó excluir a los sistemas ópticos. Se estudia la distribución galáctica de los trapecios de distintos tipos espectrales y se compara ésta con la de sistemas múltiples ordinarios y binarias visuales del IDS. Se estima el número esperado de pseudotrapecios en cada tipo espectral. Este estudio muestra que la mayor parte de los trapecios son trapecios verdaderos, y por lo tanto, deben ser jóvenes. Esta conclusión es válida para trapecios de todos los tipos espectrales.

ABSTRACT

A catalogue of more than 900 multiple systems of trapezium type has recently been completed. In constructing the catalogue an attempt was made to exclude optical systems. We study the galactic distribution of trapezia of different spectral types, and we compare it with that of ordinary multiple systems and of visual binaries from the IDS. An estimate is made of the expected number of pseudo-trapezia among each spectral type. This study shows that most of our trapezia are true trapezia and should therefore be young. This conclusion holds for trapezia of all spectral types.

I. INTRODUCTION

It has been repeatedly pointed out that multiple :ar systems of the trapezium type are very young Sharpless 1954; Ambartsumian 1954). Although ıe "expansion ages" for trapezia have turned out › be spurious (Poveda 1973; Allen, Poveda and Vorley 1974), dynamical studies have confirmed the outh of these systems (Allen and Poveda 1974, 975).

Following Ambartsumian we divide multiple star ystems in two categories: trapezium-type and orinary systems. For the sake of clarity, we will refer › the latter as hierarchical systems. If among the omponents of a multiple system (of three or more tars) three distances of the same order of magitude (i.e., with ratios larger than 1/3 but less than 3) can be found, then the system is of trapezium type; if no three distances of the same order of magnitude can be found, then the system is of hierarchical type. Although this classification is purely observational in nature, since it applies to the projected separations of the components, a very similar criterion has been shown to be significant dynamically, when applied to the physical separations (Harrington 1972).

Two lists of trapezia are known in the literature: a list given by Sharpless (1954) of 25 objects associated with emission nebulae, and Ambartsumian's (1954) catalogue, containing 108 systems.

A new catalogue of trapezia containing 915 systems is presently being completed (Allen *et al.* 1977). These trapezia were obtained from the IDS catalogue; only about 100 of them have appeared in previous lists of trapezia. The new sample is thus considerably larger, and it seems desirable to subject it to a statistical study. If trapezia are, on the whole, young objects, as dynamical studies have led us to believe, then their statistical properties should be different from those of hierarchical multiple systems or of visual binaries. The purpose of this paper is to study the statistical properties of trapezium systems and to compare them with the properties of hierarchical multiple systems and of visual binaries.

II. THE "FILTERING" PROCEDURE

When dealing with statistical properties of multiple systems it is of crucial importance to make sure that most of the systems of the sample are, in fact, true physical multiples; that is, we have to try to sieve out, to "filter" out, the optical companions; these can be very numerous in the sample, and can completely mask the properties of the true systems. It is only after filtering out the optical systems that we can be reasonably certain that the derived statistical properties of the sample of trapezia will be relatively free of the spurious correlations (e.g., with the galactic plane) that have appeared in previous studies of multiple systems (Wallenquist 1944, Wierzbinsky 1964).

In his study of trapezium systems, Ambartsumian (1954) used a combined separation-apparent magnitude of secondary criterion, defined as follows:

m	d''
11.5 − 12.5	<10
10.5 − 11.5	<30
9.5 − 10.5	<50
8.5 − 9.5	<80
7.5 − 8.5	<140

(Components fainter than magnitude 12.5 were rejected by Ambartsumian). Here d is the separation of the secondary from the primary, and m is the visual magnitude of the secondary.

For our study we have chosen a "1% filter", defined as follows:

$$\pi d^2 N(m)_{l,b} < 0.01$$

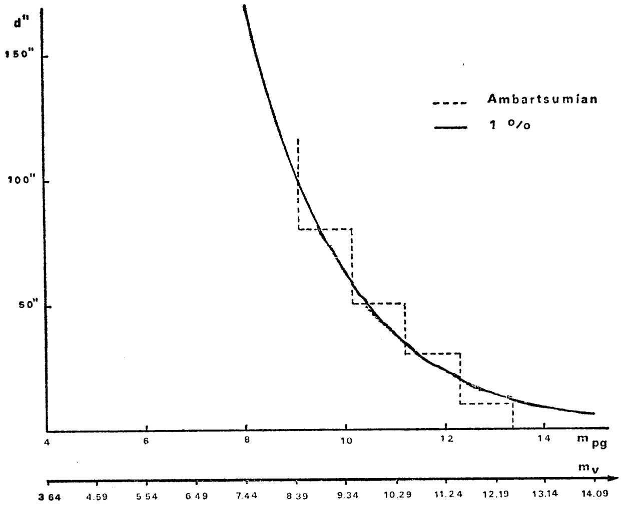

FIG. 1. Ambartsumian's filter (broken line) and 1% filter (solid line) for the direction $l^{\mathrm{I}} = 210°$, $b^{\mathrm{I}} = 5°$.

where $N(m)_{l,b}$ is the number of stars per unit area brighter than visual magnitude m in a given direction, determined by the galactic coordinates l and b. Numerical values for $N(m)_{l,b}$ were obtained from the tables of Seares and Joyner (1928). If we have a population of P stars, and if a *systematic* search for duplicity among them has yielded a complete sample of M doubles, then the application of the "1% filter" to either population leads to the result that the expected number of optical systems among the M doubles is 0.01 P.

Figure 1 illustrates comparatively the effect of both filters. We have plotted as a dashed line Ambartsumian's filter; the full line corresponds to the 1% filter for a rather dense galactic region ($l^{\mathrm{I}} = 210°$, $b^{\mathrm{I}} = 5°$). All companions falling above the limiting lines were rejected; companions falling below were left in the sample.

Of the 915 trapezia in the catalogue, 900 are systems that passed the 1% filter. The remaining 15 systems passed Ambartsumian's filter, but were marginally rejected by the 1% filter.

All comparisons of the properties of trapezia and the properties of hierarchical multiples or binaries were made after "filtering" each sample with the same filter (either Ambartsumian's or 1%).

We note in passing that if we apply Ambartsumian's filter to his list of 108 trapezia, taking the magnitudes and separations from the IDS, we find that more than 40 systems should have been rejected. This could be due to the fact that the magnitudes available to Ambartsumian are not necessarily those listed in the IDS; it is well known that the latter can be inaccurate.

If we know the population out of which the binaries and multiples were picked out, and if we allow for incompleteness of discovery of these systems, we can, in principle, estimate the number of optical systems that we expect to be present in the filtered samples. Full details of the estimates for all the examined samples will be given elsewhere (Allen *et al.* 1977). We will quote here only the result for trapezia: among the 915 systems of the catalogue we expect to find about 30 optical systems.

III. RESULTS AND DISCUSSION

The basic material for our study was obtained by computer-scanning the IDS tape with both Ambartsumian's and the 1% filter. In Table 1 we summarize the main results; a total of 10589 pairs by Smart, Stein, Pourteau and Baillaud was removed from the counts listed in the table. Most —though not all— of these pairs are strongly suspected of being spurious (Deutsch 1971).

The first line in Table 1 gives the total number of pairs found in the IDS tape, unfiltered and filtered with both filters. Here, most triple systems were counted as two pairs, most quadruples as three pairs, and so on. The second line gives the number of pairs after removing all multiple systems.

TABLE 1

SUMMARY OF RESULTS

Population*	Filter: None	Filter: Ambartsumian	Filter: 1%
IDS Pairs	53836	41013	48015
IDS Pairs (without multiples)	41533	34839	39431
IDS Multiples	5141	1627	2975
IDS Trapezia	2595	402	900

Total number of trapezia: 915

* A total of 10589 pairs by Smart, Stein, Pourteau and Baillaud was removed from the above counts.

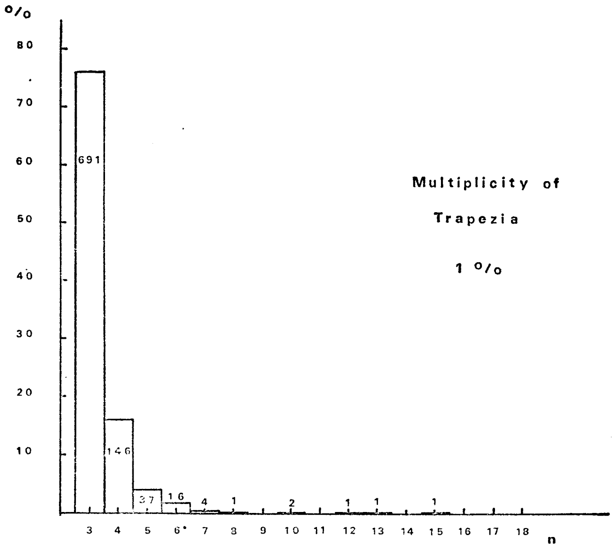

FIG. 2. Percentage distribution of trapezia having n components (1% filter).

The third line lists the number of multiples, the fourth the number of trapezia. In the last two lines the effect of the filters is most dramatically illustrated: the majority of the unfiltered multiples and trapezia turned out to be optical systems.

Since Ambartsumian's filter is the more stringent of the two, especially at the faint end, we expect the samples of column 2 to be contained in those of column 3. In the case of trapezia this is true for all but 15 systems; as previously mentioned, these 15 trapezia, having passed Ambartsumian's filter were marginally rejected by the 1% filter.

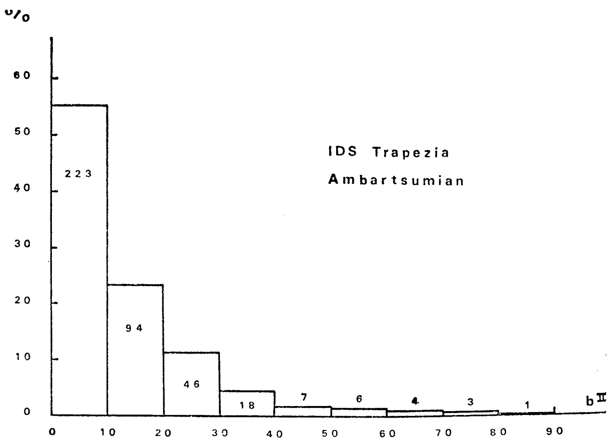

FIG. 3. Distribution in galactic latitude of trapezia from the IDS (Ambartsumian's filter). In this and all successive figures the number of systems in each bin is indicated.

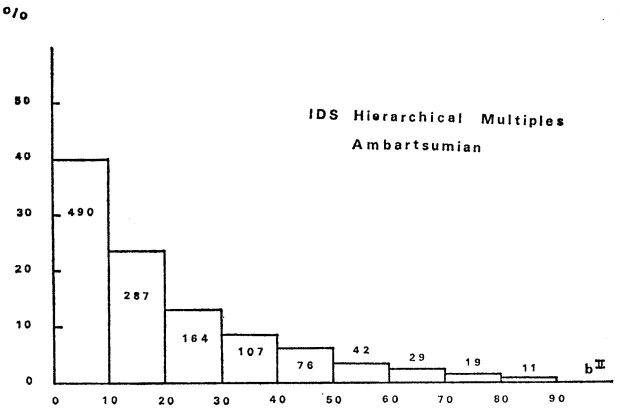

FIG. 4. Distribution in galactic latitude of hierarchical multiple systems from the IDS (Ambartsumian's filter).

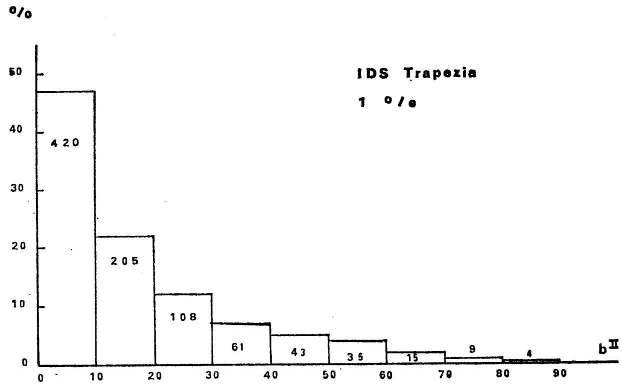

FIG. 6. Distribution in galactic latitude of trapezia from the IDS (1% filter).

In Figure 2 we have plotted the distribution of the number of components of each system for the 900 trapezia that passed the 1% filter. We notice that 77% of them are triple systems.

We next turn our attention to the galactic concentration of trapezium systems. Figures 3 and 6 show histograms of the percentage of trapezia from two samples per interval of galactic latitude; we also indicate the number of systems in each bin. Figures 4, 5, 7 and 8 show analogous histograms for the hierarchical multiple systems and for the IDS pairs (after removing multiples). In all cases trapezia display a higher degree of galactic concentration than do hierarchical multiples or pairs.

A similar type of analysis has been carried out for each spectral type separately. As examples, we

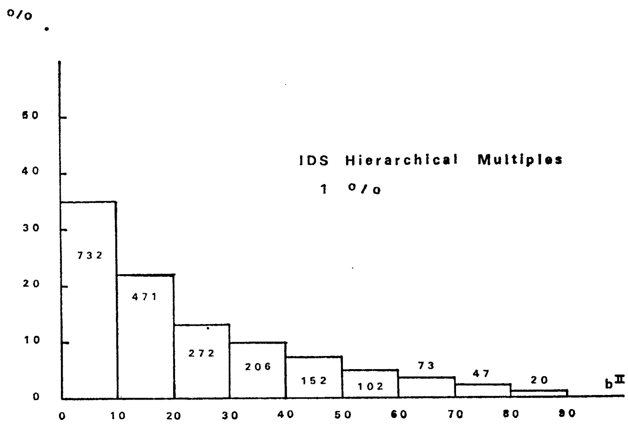

FIG. 7. Distribution in galactic latitude of hierarchical multiple systems from the IDS (1% filter).

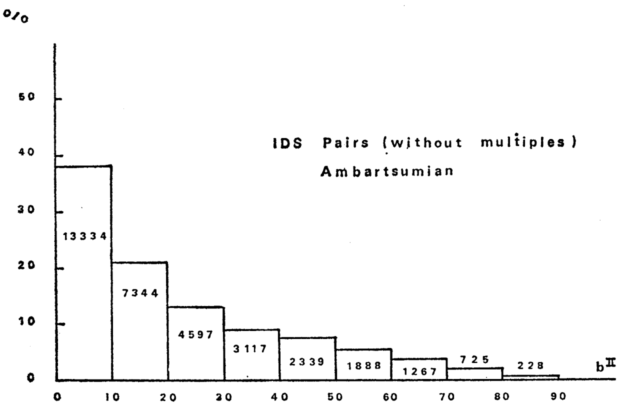

FIG. 5. Distribution in galactic latitude of pairs from the IDS after removing multiples (Ambartsumian's filter).

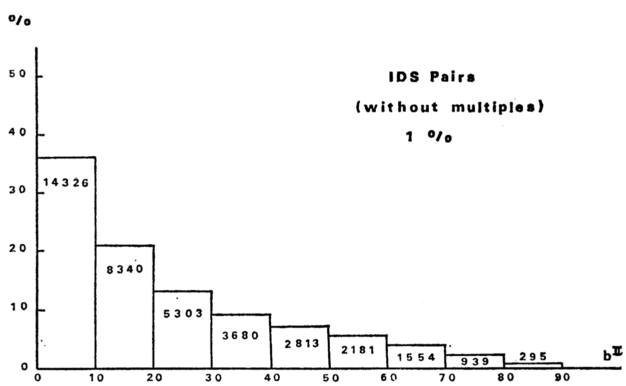

FIG. 8. Distribution in galactic latitude of pairs from the IDS after removing multiples (1% filter).

TABLE 2

GALACTIC CONCENTRATION INDICES

(Ambartsumian's Filter)

Spectral Type	All Pairs	Pairs without Multiples	Hierarchical Multiples	Trapezia
O	∞	∞	∞	∞
B	47.7	50.8	27.5	69.0
A	8.0	8.3	6.6	48.0
F	1.4	1.4	2.1	4.0
G	1.1	1.0	1.5	3.2
K	1.7	1.6	2.1	9.3
M	3.0	2.7	5.0	5.0
P	∞	∞	—	∞
Unknown	4.8	4.6	10.7	20.3
Total	3.4	3.2	4.4	15.0

show in Figures 9 to 11, respectively, the histograms obtained for trapezia, hierarchical multiples and pairs of spectral types O + B, and in Figures 12 to 14 those for spectral type K. We note again the higher galactic concentration of trapezia.

The results of the above analysis are summarized in Tables 2 and 3, where we show "galactic concentration indices" for several groups of objects. The "galactic concentration index" is defined, in a simple way, as the number of objects having galactic latitude between 0° and ±20° divided by the number of objects having galactic latitude between ±40° and ±90°. Since the corresponding areas in the sky are nearly equal, this means that a spherically uniform distribution of objects would yield a galactic concentration index of nearly 1.0. The tables show that the most marked differences in galactic concentration between trapezia and hierarchical multiples exist for objects of spectral types B, K, M and of unknown spectral type.

TABLE 3

GALACTIC CONCENTRATION INDICES

(1% Filter)

Spectral Type	All Pairs	Pairs without Multiples	Hierarchical Multiples	Trapezia
O	∞	∞	∞	∞
B	37.4	38.0	29.0	114.0
A	7.2	7.4	6.3	7.8
F	1.3	1.3	1.5	1.5
G	1.0	1.0	1.1	1.7
K	1.6	1.6	1.3	3.0
M	2.4	2.2	3.0	3.5
P	∞	∞	—	∞
Unknown	4.3	4.1	5.8	9.0
Total	3.1	2.9	3.1	5.9

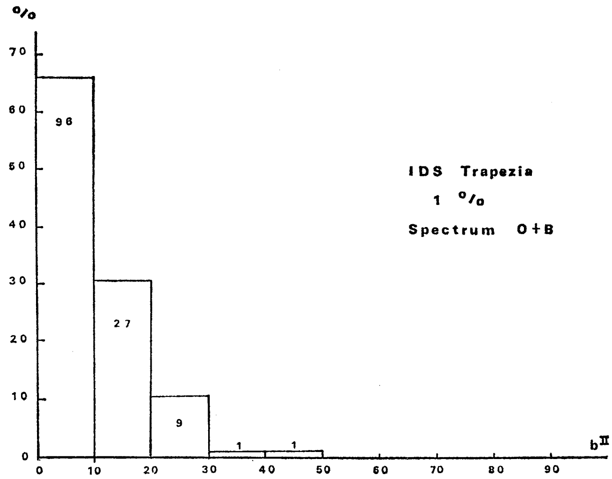

FIG. 9. Distribution in galactic latitude of trapezia from the IDS of spectral types 0 and B (1% filter).

Since all comparisons have been made with samples that were obtained using the same filter, we feel reasonably confident that the higher galactic concentration exhibited by trapezium systems of nearly all spectral types is a real effect, and not due to the overwhelming presence of optical com-

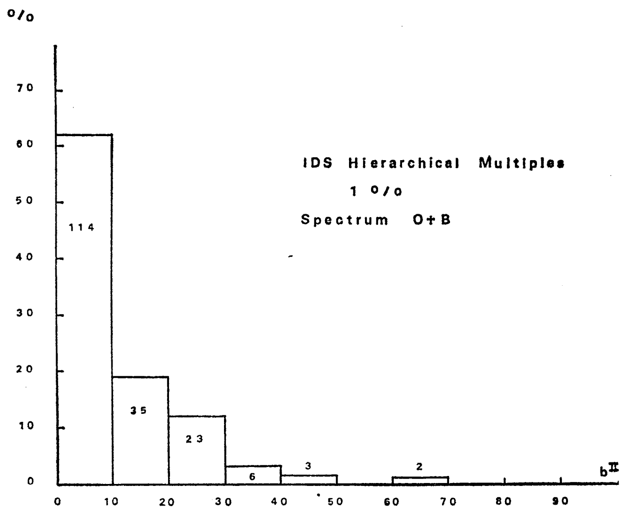

FIG. 10. Distribution in galactic latitude of hierarchical multiple systems from the IDS of spectral types O and B (1% filter).

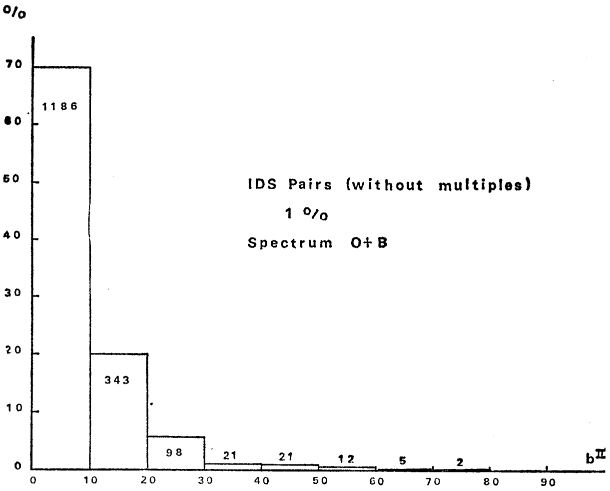

FIG. 11. Distribution in galactic latitude of pairs from the IDS of spectral types O and B, after removing multiples (1% filter).

TABLE 4

TRAPEZIA FROM THE IDS
(Ambartsumian's Filter)

Spectral Type	Multiples (all)	Expected Pseudo-trapezia	Observed Trapezia
O	12	0.3	9
B	216	13.3	76
A	330	26.9	58
F	268	23.3	33
G	198	16.3	30
K	129	9.1	37
M	14	0.7	7
P	4	0.2	2
Unknown	462	30.9	150
Total	1627	121.4	402

TABLE 5

TRAPEZIA FROM THE IDS
(1% Filter)

Spectral Type	Multiples (all)	Expected Pseudo-trapezia	Observed Trapezia
O	14	0.4	10
B	303	17.8	124
A	565	42.4	137
F	485	39.3	88
G	384	30.0	81
K	272	20.0	70
M	24	1.0	14
P	4	0.1	3
Unknown	924	54.8	373
Total	2975	205.7	900

panions in the sample. This, in turn, suggests the presence of young objects among trapezia even of late spectral types.

An important point remains to be cleared up; as a result of projection effects, it is possible for a hierarchical multiple to appear as a trapezium; such systems were called pseudo-trapezia by Ambartsumian. Since the number of hierarchical multiples is larger than the number of trapezia, even a small fraction of pseudo-trapezia can heavily contaminate the sample of observed trapezia. By means of an elaborate statistical analysis, Ambartsumian computed the probability of a hierarchical multiple appearing as trapezium due to projection; this probability turns out to be 9%. The computation takes into account the different eccentricities of the orbits, as well as the different inclinations of the orbital planes. Applying this result to his counts of trapezia and hierarchical multiples of different spectral types, Ambartsumian found a rather sharp line of demarcation; for spectral types O, B and M, the expected number of pseudo-trapezia was significantly smaller than the observed number of trapezia, whereas for other spectral types the expected number of pseudo-trapezia was nearly equal to the observed number of trapezia, the differences being of the order of the sampling errors. Consequently, Ambartsumian concluded that true trapezia exist only among systems of spectral types O, B and M.

We have carried out a similar analysis with our material. The expected numbers of pseudo-trapezia for each spectral type are shown in Tables 4 and 5.

TABLE 6

ADS TRAPEZIA FROM THE IDS
(Ambartsumian's Filter)

Spectral Type	Multiples (all)	Expected Pseudo-trapezia	Observed Trapezia
O	5	0.0	5
B	91	4.7	44
A	179	14.1	37
F	138	11.3	24
G	80	6.4	15
K	55	3.8	17
M	5	0.1	4
P	0	0.0	0
Unknown	204	14.2	61
Total	757	54.5	207

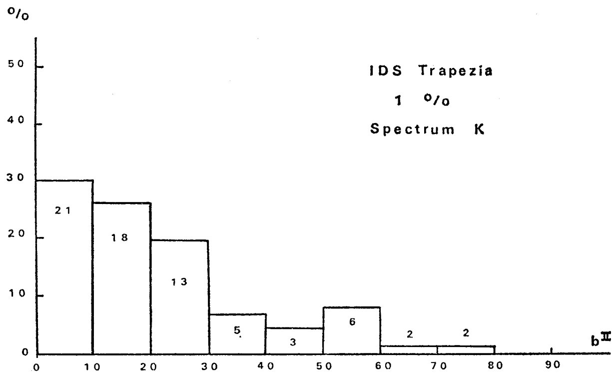

Fig. 12. Distribution in galactic latitude of trapezia from the IDS of spectral type K (1% filter).

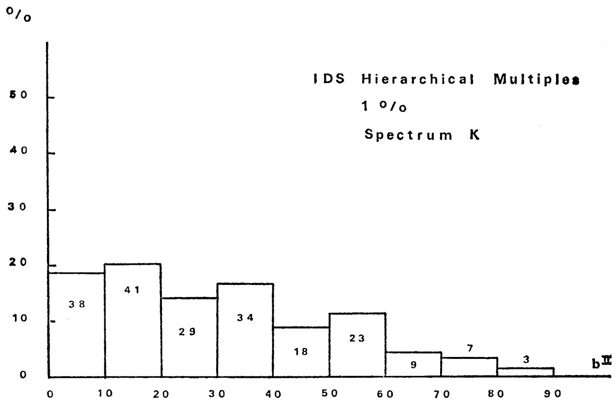

Fig. 13. Distribution in galactic latitude of hierarchical multiple systems from the IDS of spectral type K (1% filter).

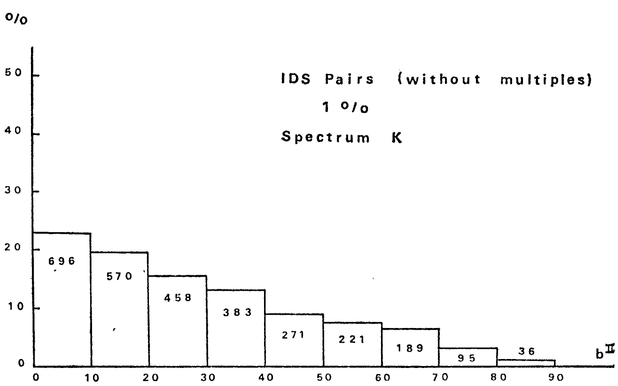

Fig. 14. Distribution in galactic latitude of pairs from the IDS of spectral type K, after removing multiples (1% filter).

It is easily seen that no sharp line of demarcation exists: the expected number of pseudo-trapezia is always considerably smaller than the observed number of trapezia (in the worst case —spectrum F— it is smaller by a factor of 2). The differences are much larger than the sampling errors.

Ambartsumian obtained his material mostly from the ADS; in an attempt to duplicate his sample and check his results, we picked out the ADS systems from the IDS tape, and performed on them the same type of analysis. The results are shown in Table 6; again, no sharp line of demarcation was found. Rather, the results of Table 6 substantially agree with those of Tables 4 and 5. Whether our disagreement with Ambartsumian stems from his having used magnitudes and/or spectral types different from those of the IDS, or from other causes, is a question that will be left for future study.

Taking at face value our results —which were all obtained from the IDS— we are led to conclude that the great majority of the trapezia contained in our catalogue are true trapezia, and not optical systems or hierarchical multiples seen in projection. This conclusion seems to hold for all spectral types, and not only for the early types, as found by Ambartsumian.

On the other hand, it has been shown that the dynamical ages for trapezia are rather short; the half-lives are of the order of 10^6 years for OB trapezia and only a few million years for less massive systems (Allen and Poveda 1974, 1975). This implies that many of the trapezia in our catalogue should be very young systems. Observational studies of these systems are likely to prove rewarding.

The authors are indebted to Dr. A. Poveda for inspiring discussions and constructive criticism. It it a pleasure to thank Dr. C. Worley, for kindly providing a copy of the IDS type. All computations were done at the Centro de Servicios de Cómputo of the National University, Mexico; we gratefully acknowledge the generous facilities given to us. Last but not least, our thanks go to Mr. J. Luna for drawing all the graphs.

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DISCUSSION

Pişmiş: It is clear from your data shown in the tables that the maximum number of trapezia occurs for spectral type A of the primary. Do you have any comment on this point?
Allen: This point will be extensively dealt with in Dr. Poveda's contribution. Perhaps he would like to make a comment now.
Poveda: As one goes along the spectral sequence, toward later types, the number of trapezia increases. On the other hand, the *fraction* of multiple stars that are trapezia decreases along the same sequence because the dynamical half-life of a trapezium becomes much shorter than the nuclear age of the primary. These two effects combine to give a maximum around type A.
Abt: Are the K-type trapezium systems ones with supergiants, or giants, or possibly dwarfs?
Poveda: The galactic concentration of the K-type trapezia indicates that their primaries are high-luminosity stars.
Worley: The spectral types in the IDS are mostly from the HD catalog. I would advise anyone using the IDS for statistical purposes *not* to use either the spetral types or magnitudes as their primary source for these quantities.
Allen: We are indeed aware of the fact that *both* magnitudes *and* spectral types in the IDS can be inaccurate. We feel, however, that the material contained in the IDS is so rich that statistical studies are worthwhile in spite of some uncertaintes, provided that these are clearly pointed out and kept in mind when evaluating the results.