Revista Mexicana de Astronomía y Astrofísica, Volumen 3, Mayo 1977

STABLE ORBITS IN THE CIRCULAR PLANE RESTRICTED THREE-BODY PROBLEM*

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RESUMEN

Se investigan por exploración numérica las regiones extendidas de estabilidad para satélites retrógrados cuando la masa del cuerpo poco masivo crece hasta $\mu = 0.5$. Se hace un estudio preliminar de las características y propiedades de estabilidad de las familias de órbitas periódicas. En seguida se considera el caso general de órbitas no periódicas. Dichas exploraciones numéricas muestran que para grandes órbitas aperiódicas el movimiento se puede descomponer en un "movimiento de referencia" rápido y una libración lenta. Estudiamos esta libración teóricamente en el caso de Hill, por el cual el "movimiento de referencia" es elíptico.

ABSTRACT

We investigate by numerical exploration the extended regions of stability for retrograde satellites when the mass of the less massive body increases up to $\mu = 0.5$. A preliminary study is made of the characteristic and stability properties of the families of periodic orbits; then the general case of non-periodic orbits is considered. Those numerical explorations show that for stable large non-periodic orbits, the motion can be decomposed into a fast "reference motion" and slow libration; we study theoretically this libration in Hill's case for which the "reference motion" is elliptic.

I. NUMERICAL EXPLORATION OF STABLE ORBITS†

a) *Introduction*

In the circular plane restricted three-body problem, we study by numerical simulation the satellites (B_3) of the less massive body (B_2). We use rotating axes with origin in B_2 and we denote by μ the mass ratio $m_2/(m_1 + m_2)$ with $m_2 \leq m_1$. B_3 starts perpendicularly from the right part of the X axis, so that an orbit can be represented by a point in the plane (X_0, V_0), where we can examine the limits of the subspace of initial conditions for stable orbits. Figure 1 shows this subspace in Hill's case. The "band" of stable retrograde orbits, extending to much larger distances, surrounds a family of symmetrical simple-periodic orbits where and only where this family is stable: the stability of a periodic orbit is defined through the value of an index of stability, denoted by a such that $|a| < 1$ is the mean stability, but the isolated unstable value $a = -0.5$.

To show the evolution of this "band" when μ increases from 0 to 0.5 we first study this simple-periodic family and possibly other periodic families.

b) *Periodic Families*

i) *Family f.*

Figure 2 shows how the characteristic of family f evolves when μ increases from 10^{-6} (curve 1) to 0.5 (curve 12).

* Figures 2, 3 and 6c have been reproduced from *Astron. and Astrophys., 1974,* **32**, 39. Figures 4a-h, 5, and 6d-n from *Astron. and Astrophys., 1975,* **45**, 353 published by Springer-Verlag Berlin-Heidelberg-New York on behalf of the Board of Directors, ESO.

† This section is the synthesis of two papers: Benest (1974) and Benest (1975*a*).

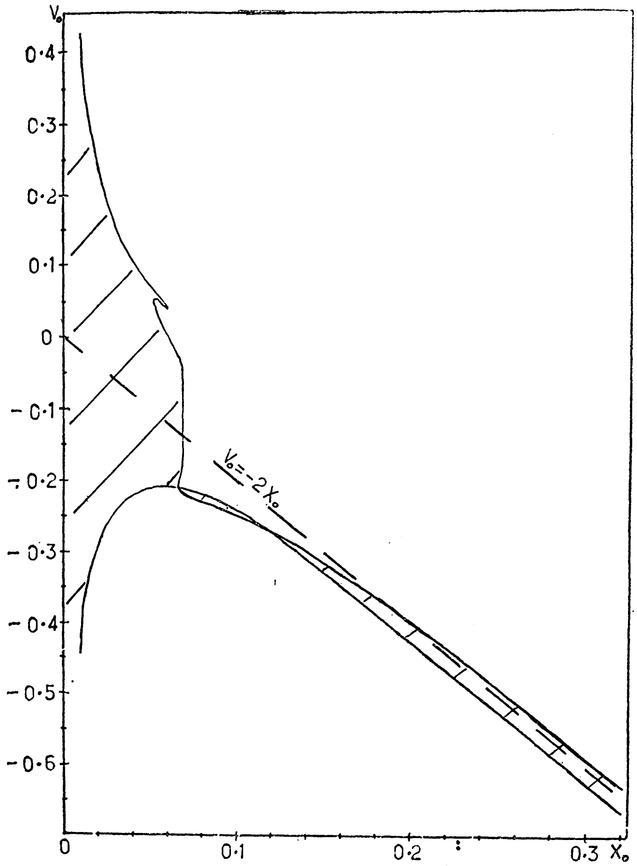

FIG. 1. Non-periodic stability zone (hatched region) in Hill's case.

As X_0 varies monotonically along the characteristic, we can represent the stability properties in the (μ, X_0) plane (Fig. 3): for $\mu < 0.0477\ldots$, the orbits are continuously stable until ejection.

ii) The Double-Periodic Family φ.

For this critical value of μ, an unstable interval appears on the characteristic between two points which are intersections with the characteristic of a symmetrical double periodic family (called φ). Figure 4 shows the evolution of this characteristic from $\mu = 0.0477\ldots$; we shall see later that for $\mu > 0.150$, the family φ has no influence on the "band" of stable retrograde non-periodic orbits.

The orbits of family φ have two perpendicular intersections at the right of the X axis (whose abscissas are noted X_{o_1} and X_{o_2}) and the numerical results show that, up to $\mu = 0.130$, te quantity $(X_{o_1} + X_{o_2})/2$ varies monotonically along the characteristic, so that we can, as for family *f*, represent the stability properties in a (μ, X_o) plane (Fig. 5).

iii) Other Periodic Families

Many N-periodic orbits were found, systematically or by chance, for various values of N, but they were mostly unstable and had no influence at all on the "band" of stable retrograde non-periodic orbits.

c) *Non-Periodic Orbits*

Figure 6 shows the evolution of the "band" of stable retrograde non-periodic orbits: up to $\mu = 0.0477$, the figure is topologically identical to Hill's case (cases *a, b, c*); from 0.0477 to 0.063, the family φ is always stable and can neutralize at least partly, the unstable interval of family *f* (case *d*); from 0.063 on the family φ has one or two unstable intervals and the "band" breaks up into several parts (cases *e, f, g*); from 0.097 on a new stable interval in family *f* gives rise to an extension of the "band", which grows when, at the same time, the effect of the family φ decreases, and disappears for 0.150 (cases *h, i, j*); then, the "band" evolves slowly to its final shape for 0.5 (cases *k* to *n*).

II. LIBRATION EFFECTS FOR LARGE ORBITS‡

Numerical explorations of the restricted problem has shown that for stable large non-periodic retrograde satellite orbits, the motion can be decomposed into a fast "reference motion" and a slow libration around B_2. We study here this libration in the circular plane Hill's case, for which the "reference motion" is elliptic. We establish the equations of motion for the coordinates of the center of this ellipse. We find two integrals of motion: the first is the semi-major axis of the ellipse; the second is essentially Jacobi's integral, translated into the new coordinates. We give a formula for the period of the libration and we find its limiting value for small libration amplitudes. A numerical experiment gives very good agreement for all these results.

‡ This section is detailed in an earlier paper (Benest 1975*b*).

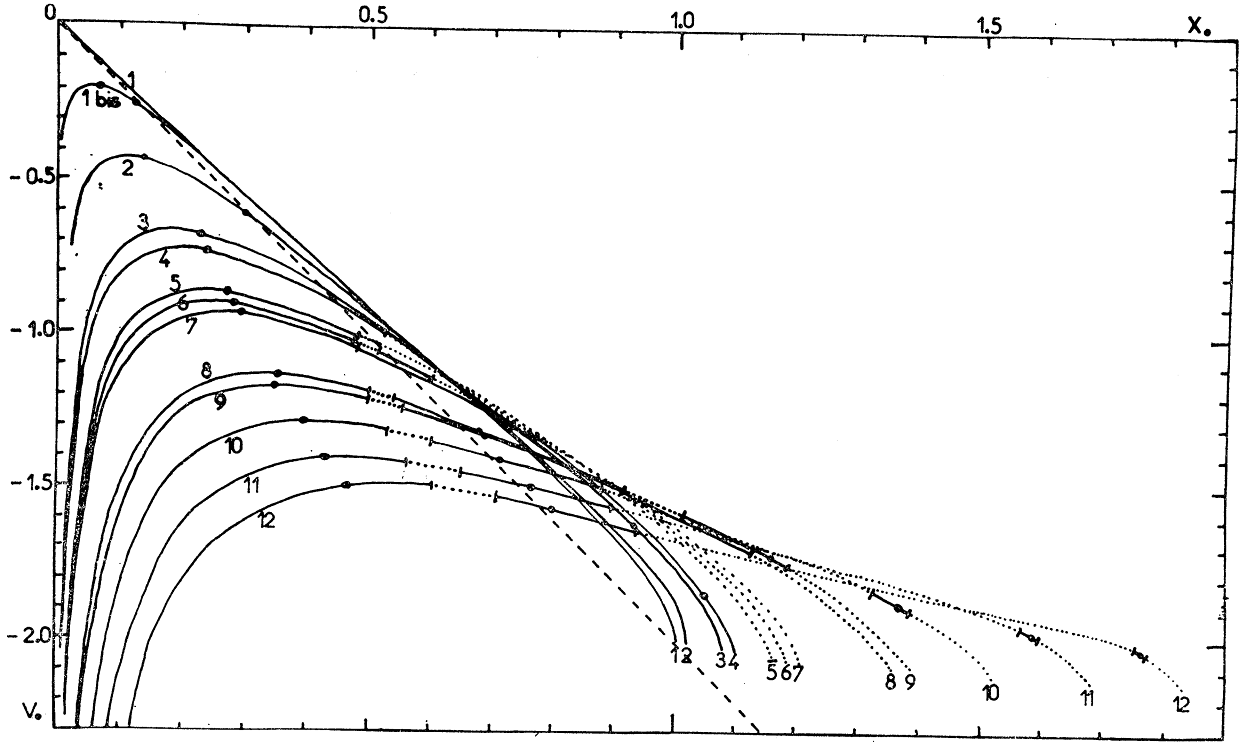

FIG. 2. Evolution of the characteristic of familyf. Full lines: stable parts ($|a| < 1$); perpendicular dashes: $|a| = 1$; dotted lines: unstable parts ($|a| > 1$); circles: $a = -0.5$. The value of μ for each curve is as follows:

Curve No.	1	1 bis	2	3	4	5	6	7	8	9	10	11	12
μ	$\leqq 10^{-6}$	0.001	0.01	0.04	0.05	0.085	0.095	0.108	0.2	0.22	0.3	0.4	0.5

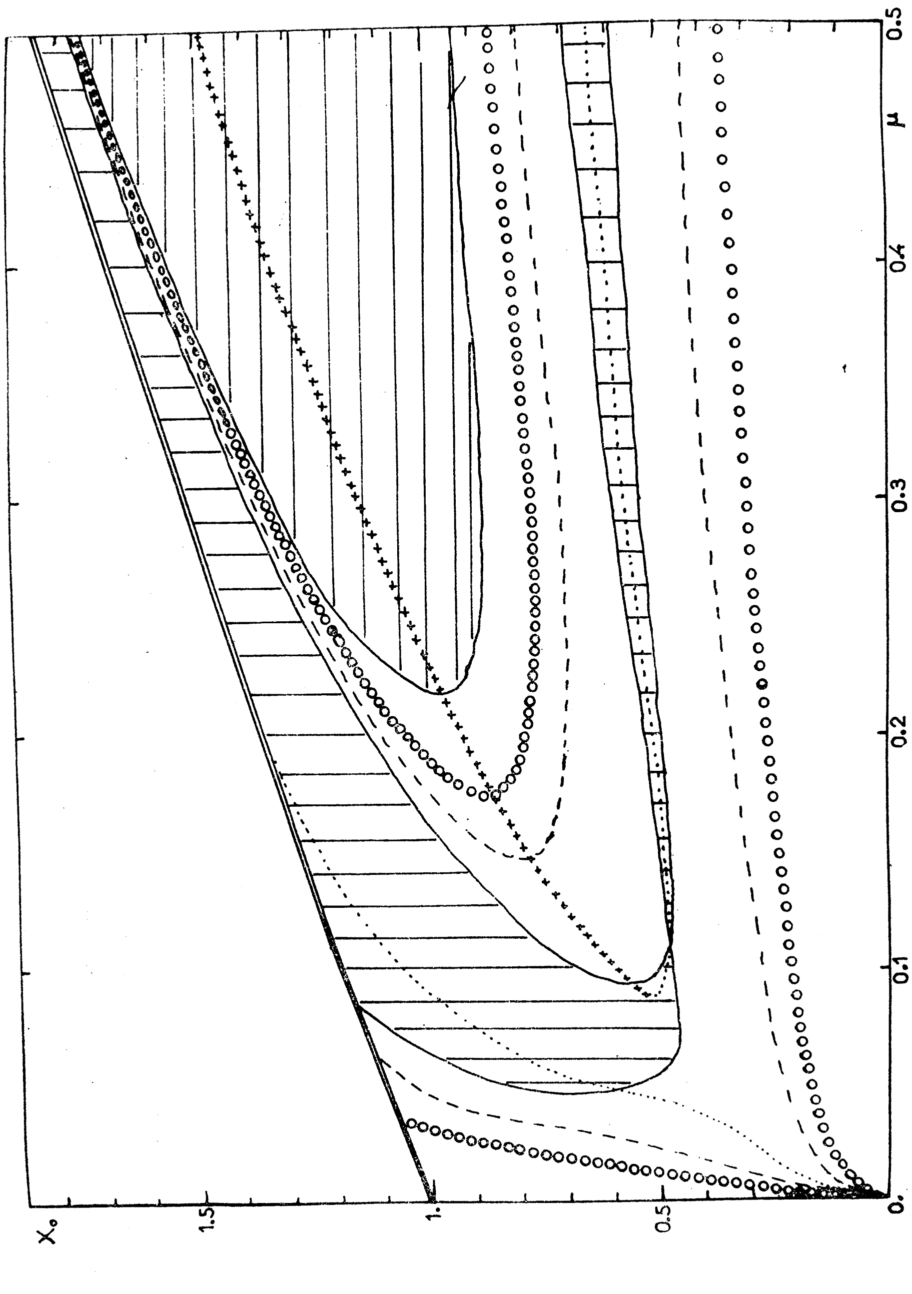

FIG. 3. Stability of family f: full lines: critical orbits ($|a| = 1$); dotted lines: a minimum; vertical hatchings: $a < -1$; crosses: a maximum; horizontal hatchings: $a > +1$; dashed lines: $a = -0.5$; circles: $a = 0$; double line: ejection orbits.

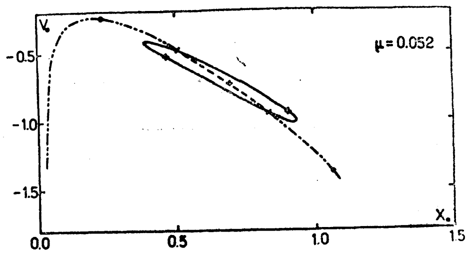

FIG. 4-a

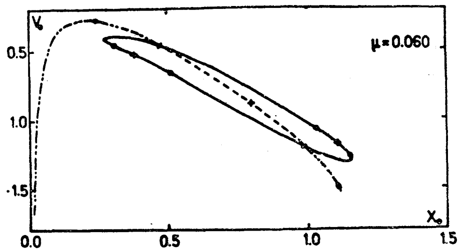

FIG. 4-b

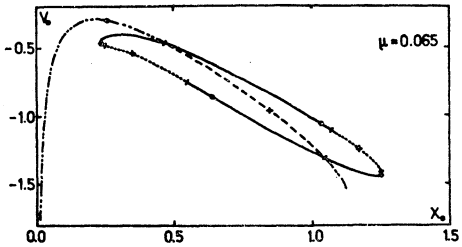

FIG. 4-c

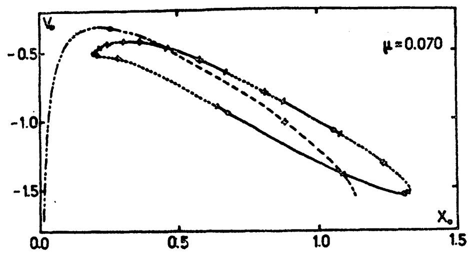

FIG. 4-d

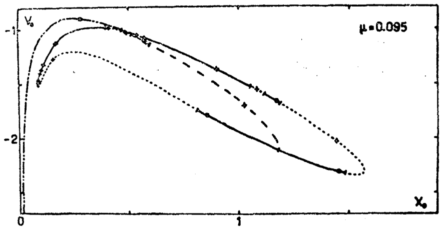

FIG. 4-e

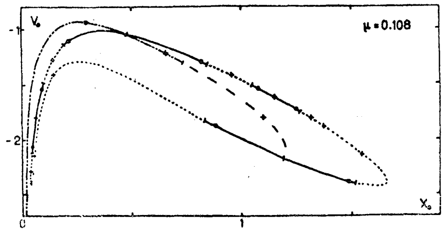

FIG. 4-f

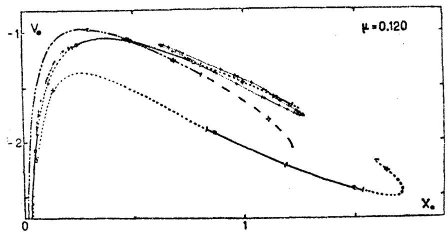

FIG. 4-g

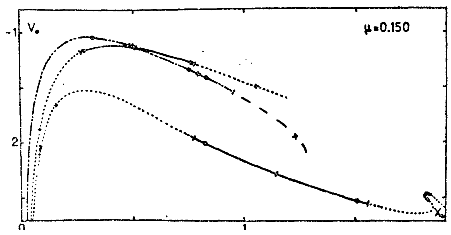

FIG. 4-h

FIG. 4. Evolution of the characteristic of family ϕ.

— . . — . . — : family f, $|a| < 1$;

— . . — . . — : family f, $|a| < 1$;

———————— : family ϕ $|a| < 1$;

- - - - - - - - : family ϕ $|a| > 1$;

crosses : a is extremum;

circles : $a = -0.5$.

FIG. 6. a to n. Evolution with μ of the non-periodic stability zone: horizontal hatchings; full and dotted lines: stable and unstable parts of periodic families; circles: $a = -0.5$; perpendicular dashes: $|a| = 1$; crosses: extrema of a.

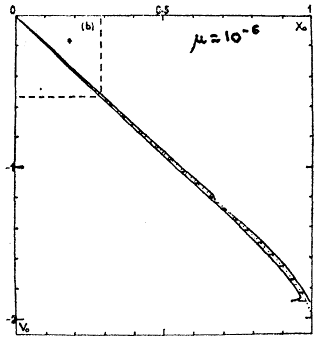

FIG. 6-a

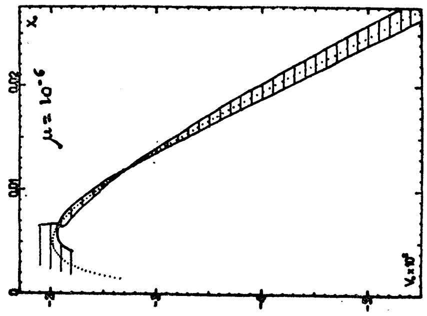

FIG. 6-b

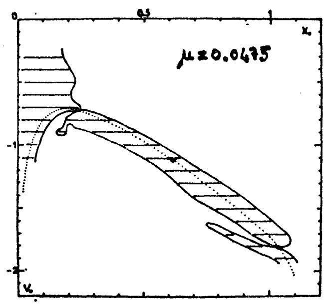

FIG. 6-c

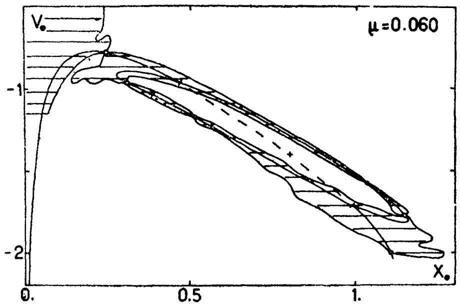

FIG. 6-d

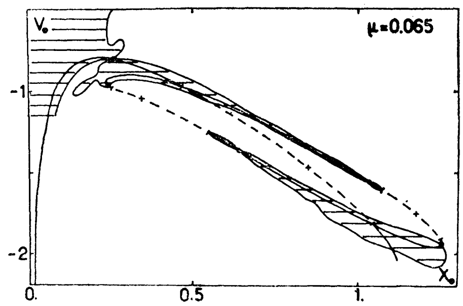

FIG. 6-e

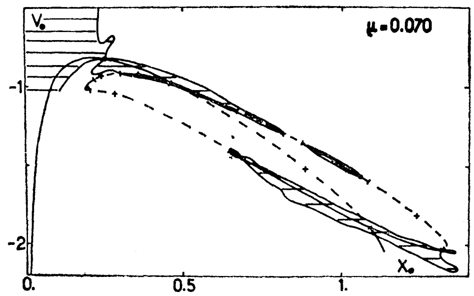

FIG. 6-f

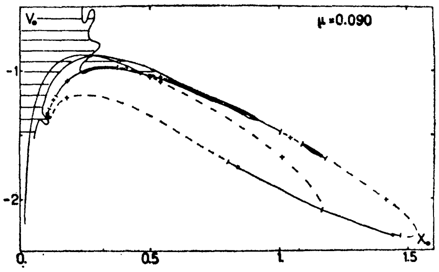

FIG. 6-g

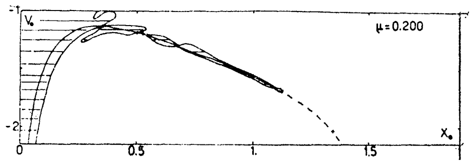

FIG. 6-k

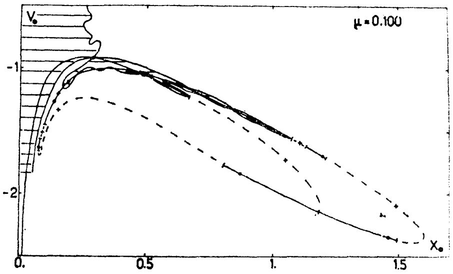

FIG. 6-h

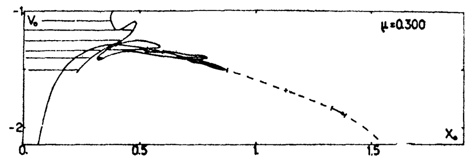

FIG. 6-l

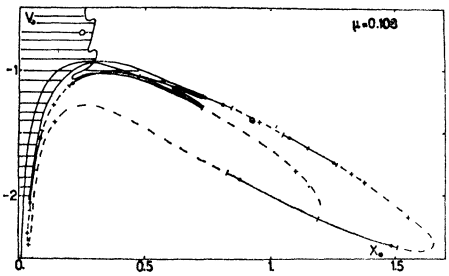

FIG. 6-i

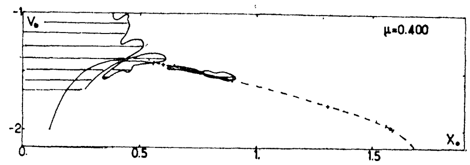

FIG. 6-m

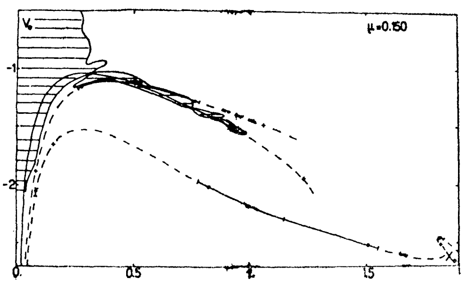

FIG. 6-j

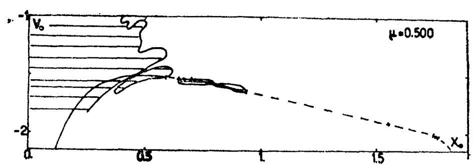

FIG. 6-n

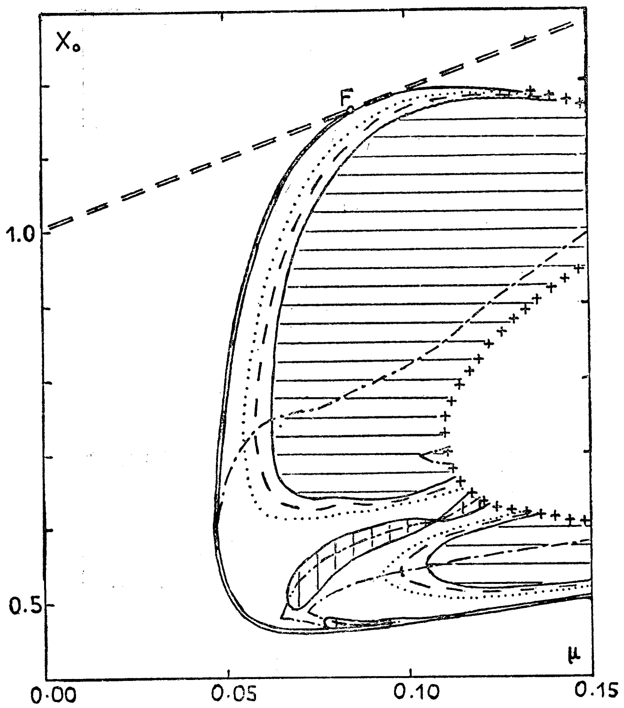

FIG. 5. Stability of family ϕ: full lines: critical orbits ($|a| = 1$); dashed lines: $a = -0.5$; dotted lines: $a = 0$; horizontal hatchings: $a < -1$; vertical hatchings: $a > +1$; — . — . — : a is minimum; — . . — . . — : a is maximum; crosses: limits of good orbits; double line: critical orbits of family f; double dashed line: ejection orbits for family f.

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