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A STELLAR WIND MODEL FOR BIPOLAR NEBULAE

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RESUMEN

Se analiza la configuración de estado estacionario producida por una estrella con viento estelar y sumergida en una nube interestelar con forma de disco. El viento se supone isotrópico. En particular, el modelo es aplicado a un disco autogravitante, infinito e isotérmico. También se presenta la velocidad radial del material emisor como función de su distancia a la estrella. Se encuentra que el modelo concuerda satisfactoriamente con las formas observadas ópticamente en las nebulosas bipolares; especialmente se explica de forma natural los extremos puntiagudos que se observan en varias de ellas. Más aún, se concluye que estos extremos puntiagudos representan la marca distintiva de las nebulosas producidas por vientos estelares.

ABSTRACT

The steady flow pattern around a wind-producing star immersed in an interstellar disk-shaped cloud is analyzed. The wind is taken as isotropic. In particular, we apply the model to an infinite, isothermal and self-gravitating disk. The radial velocity of the emitting material is also presented. The model is found to account satisfactorily for the observed shapes of optical Bipolar Nebulae; especially for the acute tips which can be seen in several of them. Further, it is concluded that such acute tips represent the hallmark of wind-produced nebulae.

Key words: HYDRODYNAMICS – SHOCK WAVES – STELLAR WINDS – BIPOLAR NEBULAE.

I. INTRODUCTION

Group IV of Cometary Nebulae (Dibai 1970; Cohen 1974), more commonly referred to as Bipolar Nebulae (BN), consist of two, frequently symmetrical, bright lobes with a star or star-like object between them. They show, in general, well defined structures, with dimensions rarely exceeding two minutes of arc.

Historically, a bright nebula has been classified as bipolar just by its photographic or visual appearance. It is likely, therefore, that they do not form an homogeneous group (Fix and Mutel 1977). Some BN can be regarded as pure reflection nebulae, while others are likey to represent the early stages of a planetary nebula or to be evolved planetary nebulae (Calvet and Cohen 1978). Several authors have discussed a possible evolutionary link between bipolar reflection nebulae and planetary nebulae (see Schmidt, Angel and Beaver 1978).

BN have been the subject of extensive photographic (Cohen *et al.* 1978; Elliott and Meaburn 1977; Meaburn and Walsh 1980 *a, b),* spectroscopic and IR (Cohen *et al.* 1975; Cohen and Kuhi 1977; Cohen 1977; Cohen *et al.* 1978; Calvet and Cohen 1978; Eiroa *et al.* 1979), line profile and radial velocity (Maucherat 1975; Elliott and Meaburn 1977; Meaburn and Walsh, 1980 *a,b*; Pişmiş and Hasse 1980; Hippelein and Münch 1980) and molecular line observations (Little *et al.* 1979; Arny and Bechis 1980; Cantó *et al.* 1981). These observations reveal two important features. First, the presence of either molecular or partially ionized interstellar disks (with sizes comparable with that of the nebula itself) around the waist of the nebulae. Second, neutral and ionized material moving at supersonic velocities into two antiparallel streams, each stream being related to a lobe. We have to mention here that additional objects showing a bipolar-type of flow motions have recently been reported (Snell *et al.* 1980 for L1551; Rodríguez *et al.* 1980*a* for Cep A; Lada and Harvey 1980 for GL490;

and Rodríguez *et al.* 1980*b* for GGD12-15 and GGD27-28). In those cases, broad-wing CO emission of the cloud material surrounding the objects is found, the blue-shifted and red-shifted wings of the spectra peaking at different positions, clearly suggesting a bipolar outflow.

Elliott and Meaburn (1977) and Calvet and Cohen (1978) have suggested that an energetic stellar wind, from a star embedded in a dense disk, could produce many of the observed features in BN. This suggestion is strengthened by the remarkable similarities between the radial velocity field observed in NGC 6302 (Elliott and Meaburn 1977; Meaburn and Walsh 1980 *a,b*) and that expected when a wind producing star is surrounded by a medium with a pressure gradient (Cantó 1980).

If a star, losing mass isotropically, were immersed in an homogeneous medium, the static configuration would be a spherical cavity filled by unshocked stellar wind and bounded by a shock in the wind (Cantó 1978). However, if the pressure distribution around the star is disk-like, the wind will be able to sweep up disk material farther in directions perpendicular to the disk rather than parallel to it. The static configuration will therefore consist of two ovoid cavities, also bounded by a shock in the wind, symmetric with respect to the disk plane. Since the shock is now oblique, the wind is refracted across it, retaining a large fraction of its initial momentum and kinetic energy, and sliding along the walls of the ovoid cavities toward the less dense outer regions (Cantó 1980; Cantó and Rodríguez 1980).

In the present paper we shall consider a star which blows off material in the form of an isotropic stellar wind, and is immersed in a dense interstellar disk of gas and dust. For simplicity, the disk is assumed infinite, isothermal and in hydrostatic equilibrium due to self-gravity. The resulting flow pattern and the radial velocity of the emitting material in the stationary configuration are presented (§II). In §III the results are discussed, while in §IV we summarize our results and give our conclusions.

Application of this model to a particular nebula will be the subject of a future paper.

II. THE MODEL

Let us consider a gaseous flat disk, in hydrostatic equilibrium due to self-gravity. The disk is assumed to

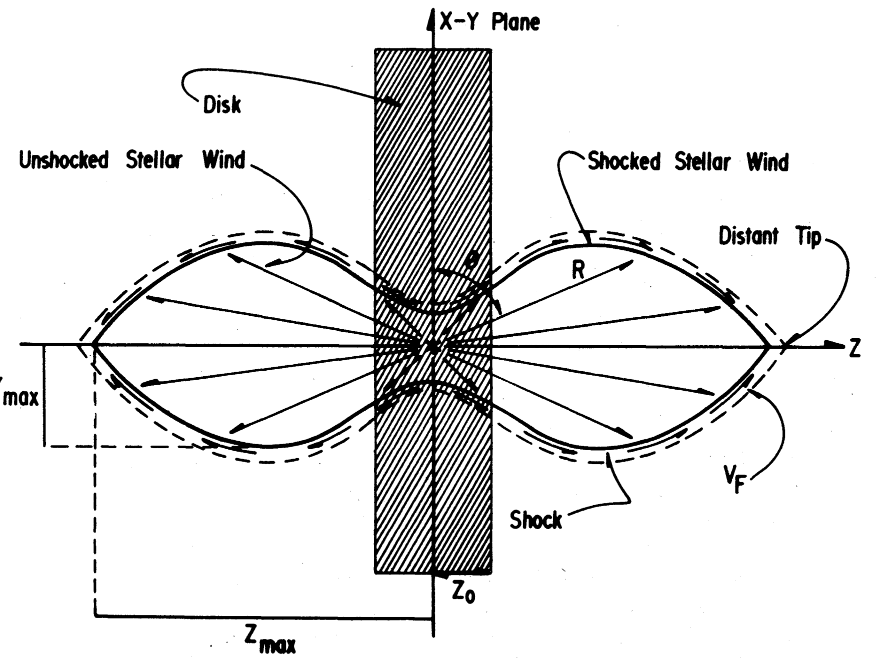

Fig. 1. A scheme showing the idealized flow structure. The dashed zone represents a dense infinite disk. The flow pattern consists of two ovoid cavities filled by unshocked stellar wind and bounded by a shock in the wind. The shocked stellar wind slides along the walls of the cavities toward the *acute* distant tips of the configuration.

be infinite, with uniform temperature T (sound speed c) and surface mass density σ_o and surrounded by a tenuous medium of uniform pressure P_o. A wind-producing star is located in the middle plane of the disk and an orthogonal frame, with its origin at the star, has its Z-axis perpendicular to the disk (Figure 1). Let θ be the angle between the radius vector (with modulus R) and the X-Y plane. Then, the pressure distribution around the star can be represented by

$$P = \begin{cases} P_c f(R \sin\theta) & \text{if } R\sin\theta < Z_o \\ P_o & \text{if } R\sin\theta \geqslant Z_o, \end{cases} \tag{1}$$

where P_c is the pressure in the X-Y plane, and 2 Z_o is the thickness of the disk. Also,

$$\begin{aligned} & f(Z) \equiv \mathrm{sech}^2 (Z/\Lambda), \\ & \Lambda \equiv j\sigma_o c^2/2(1+j)^{1/2} P_o, \\ & P_c = (1+j)P_o/j, \\ & j \equiv 2P_o/\pi G\sigma_o{}^2, \\ & Z_o \equiv \Lambda \text{ arc tanh } (1/(1+j)^{1/2}). \end{aligned} \tag{2}$$

With this pressure distribution around the star, the flow configuration produced by an isotropic wind will not be spherical. Instead, the static configuration achieved by the flow pattern will consist of two ovoid cavities (symmetric with respect to the disk plane) filled by unshocked stellar wind and bounded by a shock in the wind (see Cantó 1980, for details). This flow pattern is schematically represented in Figure 1.

Since the shock will be in general oblique, the shocked stellar wind material will be refracted across the shock instead of being practically stopped by it. The shocked wind then slides along the walls of the cavities toward the distant tips of the configuration (see Figure 1).

The locus of the shock is determined by pressure balance, a condition given by

$$\rho_\omega V_{*n}^2 + P_{CE} = P, \tag{3}$$

where ρ_ω is the mass density of the wind at the shock, V_{*n} is the normal component of the wind velocity with respect to the shock surface and P_{CE} is the centrifugal correction (Cantó 1980).

According to Cantó (1980), eq. (3) can be written as,

$$P/P_c = \frac{1}{(r^2 + r'^2)} \left\{ 1 + \frac{r^2 + 2r' - rr''}{(r^2 + r'^2)^{1/2}} \; \frac{1}{(r\cos\theta)} \int_0^\theta \frac{r'}{(r^2 + r'^2)^{1/2}} \cos\theta \, d\theta \right\} \tag{4}$$

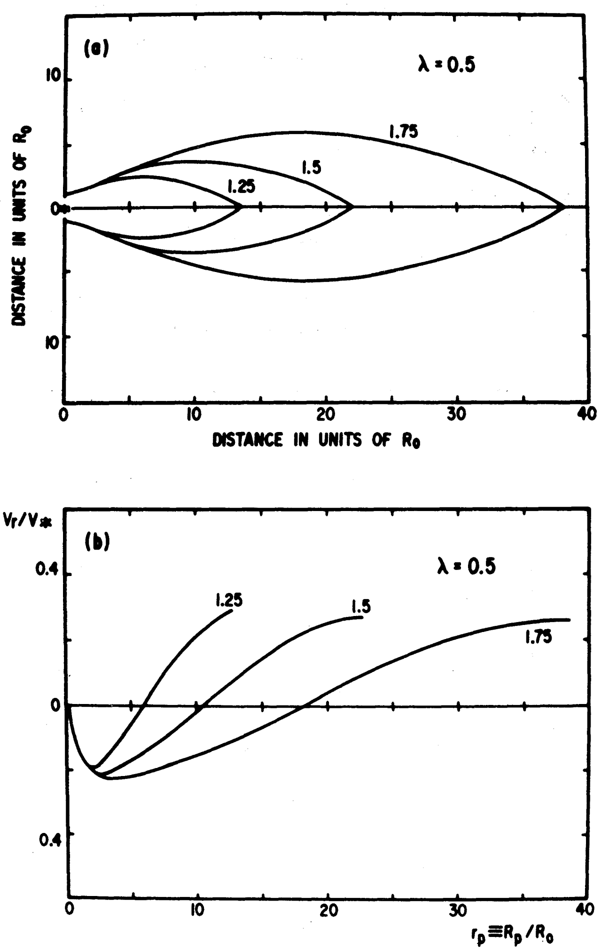

Fig. 2. (*a*) The locus of the shock surface is units of R_o (the shock distance parallel to the disk). (*b*) The radial velocity of the emitting material (in units of V_*), for the nearest part to the observer of the shock surface, as a function of its projected distance from the star. Labels indicate the values of z_o (the disk half-thickness in terms of R_o).

where, $r \equiv R/R_o$ and R_o is the shock radius for $\theta = 0$. R_o is given by the condition

$$\dot{M}_* V_* / 4\pi R_o^2 = P_c. \tag{5}$$

Here, $\dot{M}_*$ and V_* represent the mass loss rate in the wind and its terminal velocity, respectively. Thus, the pressure distribution function (eq. (1)) is replaced by

$$P/P_c = \begin{cases} \mathrm{sech}^2 (r\sin\theta/\lambda) & \text{if } r\sin\theta < z_o \\ j/(1+j) & \text{if } r\sin\theta \geqslant z_o, \end{cases} \tag{6}$$

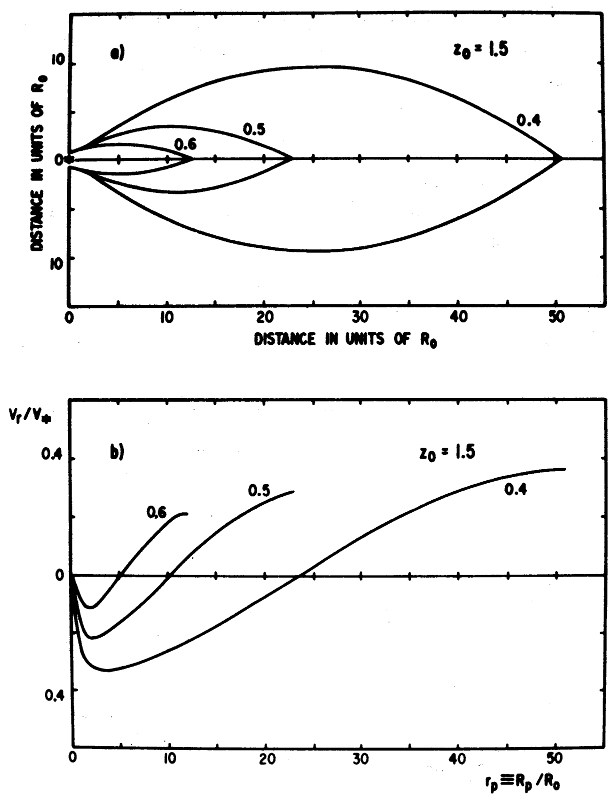

Fig. 3. Same as in Figure 2. Labels indicate the values of λ.

where

$$\lambda \equiv \Lambda/R_o = c^2\ (2/G\dot{M}_*V_*)^{1/2}$$

and

$$z_o \equiv Z_o/R_o = \lambda \text{ arc tanh } (1/(1+j)^{1/2}) \tag{7}$$

Equation (4) is subject to the following boundary conditions at $\theta = 0$:

$$\begin{aligned} r &= r_o = 1 \\ r' &= r_o' = 0 \\ r'' &= r_o'' = [(1 - 12\, d_o'')^{1/2} - 1]/6, \end{aligned} \tag{8}$$

where $d_o'' = d^2(P/P_c)/d(r \sin\theta)^2|_{\theta=0}$. According to eq. (6), $d_o'' = -2/\lambda^2$, and therefore

$$r_o'' = [(1 + 24/\lambda^2)^{1/2} - 1]/6, \tag{9}$$

Equation (4) was integrated numerically, with (P/P_c) given by eq. (6) and initial conditions given by eqs. (8) and (9), for several values of λ and z_o. Results for the shock locus, for selected values of λ and z_o, are given graphically in Figures 2*a* and 3*a*. They are discussed in the following section.

The shocked stellar wind velocity, V_f, (Figure 1) will be a function of its distance to the star. Since transfer of mass (and momentum) from the shocked wind stream to the surroundings is expected to be negligible (Kahn 1980; Cantó 1980), V_f can be evaluated directly from conservation of momentum parallel to the shock. From Cantó's (1980) results, this is given by

$$V_f/V_* = \frac{1}{\sin\theta}\int_0^{\theta} \cos\theta\ \frac{r'}{(r^2+r'^2)^{1/2}}\ d\theta. \tag{10a}$$

Therefore, V_f is an increasing function of the distance to the star.

A line of sight crossing one of the lobes will intersect the annular flow of emitting material (the shocked wind) in two points; one at each side of the lobe. Since at each of these points both V_f and its angle with respect to the line of sight are in general different, double-peaked line profiles are expected. The velocity difference between the peaks is a function of the distance to the star. Let θ_1 be the angle between the observer's line of sight and the X-Y plane. Then the predicted radial velocity for a point (at the near side of the cavity), with projected distance R_p to the star, will be given by the parametric equations

$$\frac{V_r}{V_*} = \frac{V_f}{V_*}\,\frac{1}{(r^2+r'^2)^{1/2}}\Big[r'\cos(\theta-\theta_1) - r\sin(\theta-\theta_1)\Big]$$

$$\text{and } r_p \equiv \frac{R_p}{R_o} = r\sin(\theta-\theta_1). \tag{10b}$$

Results for each model are also shown in Figures 2*b* and 3*b* for $\theta_1 = 0$, and are discussed below.

III. RESULTS AND DISCUSSION

According to eq. (6) the assumed pressure distribution function within the disk ($r \sin\theta < z_o$) is independent of j. Therefore, the flow configuration within the disk is identical for models with the same value of λ, whichever the value of j. The properties of the flow within the disk and their dependence on λ can then be investigated for a particular value of j without any loss of generality. Let us consider $j = 0$ (i.e.,$P_o = 0$ or $z_o = \infty$).

First, consider the curvature of the shock locus. This is given by

$$k(\theta) = \frac{r^2 + 2r'^2 - rr''}{(r^2 + r'^2)^{3/2}} \qquad (11)$$

Substitution of eqs. (8) and (9) into (11) gives, for $\theta = 0$,

$$k(0) = 1 - r_0'' = [7 - (1 + 24/\lambda^2)^{1/2}]/6. \qquad (12)$$

From the diagrams shown in Figure 1 of Calvet and Cohen (1977), and the photographs by Meaburn and Walsh (1980*a,b*), it is apparent that for most BN $k(0) < 0$. This restricts the range of possible values of λ to be $< 1/\sqrt{2}$.

Substituting $c^2 \equiv kT/\bar{m}$, where $\bar{m}$ is the average mass per particle ($\cong 2 \times 10^{-24}$ gm) in eq. (7), we have

$$\lambda \simeq 4.74 \times 10^{-2} \times \frac{T}{[(\dot{M}_*/10^{-7} M_\odot\, yr^{-1})\,(V_*/100\ km\ s^{-1})]^{1/2}} \qquad (13)$$

According to the restriction ($\lambda < 1/\sqrt{2}$), the exciting stars in BN have to blow-off winds with "strength":

$$(\dot{M}_*/10^{-7} M_\odot\, yr^{-1})\,(V_*/100\ km\ s^{-1}) > 4.5 \times 10^{-3}\, T^2, \qquad (14)$$

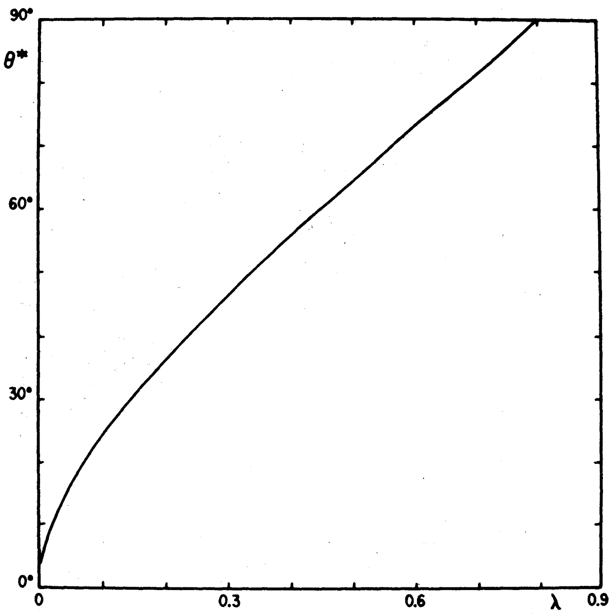

Fig. 4. Dependence on λ of the asymptotic angle θ^* (see text).

in order to produce the observed shapes.

Note that the above result is independent of either the disk density or its thickness, but depends (strongly) on the assumed disk temperature. This follows from the assumption that the disk is in hydrostatic equilibrium due to self-gravity.

Another important result follows from the model. Numerical integration of eq. (4) for several values of λ (and $j = 0$) shows that for $\lambda < 0.8$, the shock radius (and its first and second derivatives) increases without limit as θ approaches a particular value $\theta^* = \theta^*(\lambda)$. It appears then, that the solution of eq. (4) has an asymptotic behavior provided $\lambda \lesssim 0.8$. Figure 4 shows θ^* as a function of λ. This result has important observational implications.

Assume a BN with its axis perpendicular to the line of sight to the observer. If the angle θ^* can be estimated from its shape, use of Figure 4 provides λ directly. As above, if λ is known, an estimate of the wind strength can be obtained, provided a disk temperature T is assumed.

Furthermore, combining eqs. (5) and (7), we find that

$$\lambda = c/(2\pi G \rho_c)^{1/2}\, R_o, \qquad (15)$$

where $\rho_c \equiv P_c/c^2$ is the mass density in the middle plane of the disk. Since both c and ρ_c are expected to follow from direct observations of the disk, eq. (15) can provide an estimate of the nebular distance if λ and R_o (in angular units) are known, This is an important and useful result, since distances to BN are, in general, poorly known. Notice, however, that eq. (15) was derived under the assumptions that, *i)* the disk is infinite, isothermal and in hydrostatic equilibrium due to self-gravity, and *ii)* the flow configuration has achieved its steady state. If one of these assumptions does not apply for a particular BN, then the distance to the nebula derived from eq. (15), will be in error.

Finally, let us briefly consider models with $j \neq 0$. In contrast with those with $j = 0$, these do not show the asymptotic behavior described above. Instead the annular flow of shocked wind is always forced to turn toward the nebular axis. This is a consequence of the increasing diference in pressure between the inner and the outer sides of this flow. The latter is constant once the flow runs into the uniform medium which surrounds the disk, while the former decreases as $\sim 1/R^2$.

This focusing effect makes the annular stream of shocked wind narrow down and shock again but now against itself, at the distant tip of the cavity. Since the flow carries a large fraction of the momentum and kinetic energy of the stellar wind, a "bright spot" is then expected at this point. This mechanism has been

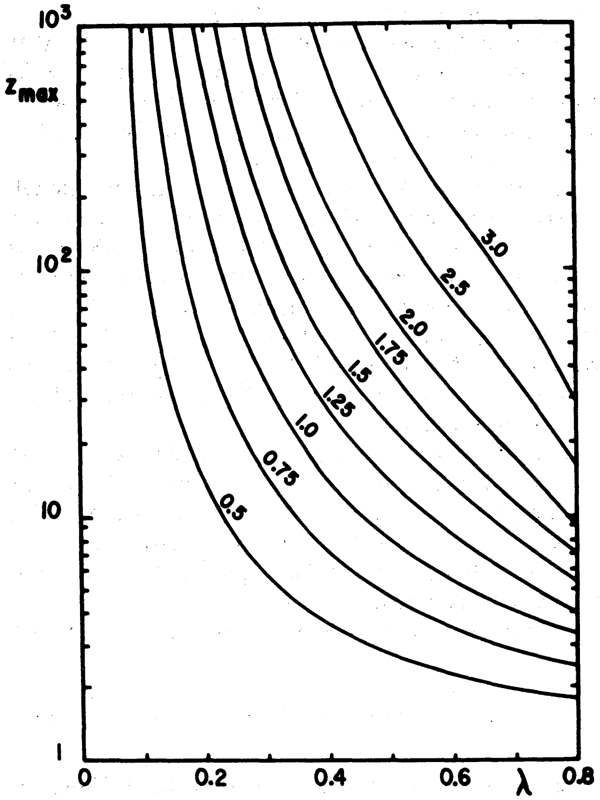

Fig. 5. Dependence on λ of the distance between the star and the distant tip of the cavity (see Figure 1), Z_{max}, in terms of R_o ($z_{max} \equiv Z_{max}/R_o$). Labels indicate the values of z_o (the disk half-thickness in units of R_o).

recently proposed as an explanation for Herbig-Haro objects by Cantó and Rodríguez (1980).

The above mentioned "bright spot" can clearly be seen in the photograph of NGC 6302 shown by Meaburn and Walsh (1980*b*) (knot 1 in that paper). Also, the unexpected high electron density in this knot, as reported by Meaburn and Walsh (1980*b*), can be explained in a natural way under the present model (Cantó and Rodríguez 1980). Furthermore, the line-splitting accross the nebula as predicted by the model (Figures 2 and 3), has also been observed by Meaburn and co-workers in NGC 6302 (Elliott and Meaburn 1977; Meaburn and Walsh 1980 *a, b*), and by Hippelein and Münch (1980) in S106.

Perhaps, the most remarkable result of the present study lies in the model's prediction of the acute ending tips that can be observed in several BN (see for instance the diagrams of BN in Calvet and Cohen 1978, the photograph of NGC 6302 in Meaburn and Walsh 1980*b*, and the photograph of Mz3 in Cohen *et al.* 1978). This peculiar shape of ending tip is essentially due to the inclusion of the obliquity of the shock into the pressure balance equation. Since pure reflection or photoionization models (Kandel and Sibille 1978) can hardly account for such tips (unless a very peculiar density distribution is supposed to exist), they can be regarded as the hallmark of wind-produced nebulae.

Several useful parameters can be derived from numerical solutions of eq. (4). Namely, *i)* the distance between the star and the distant tip of the cavity, Z_{max}, in terms of R_o (that is, $z_{max} \equiv Z_{max}/R_o$) as a function of λ (Figure 5), *ii)* z_{max}/z_o ($\equiv Z_{max}/Z_o$), where Z_o is the half-thickness of the disk, as a function of λ (Figure 6), and *iii)* the maximum distance from the nebula edge to the axis, Y_{max}, in terms of R_o, that is, $y_{max} \equiv Y_{max}/R_o$ also as a function of λ (Figure 7).

In Figures 5, 6 and 7 we present different curves for selected values of z_o.

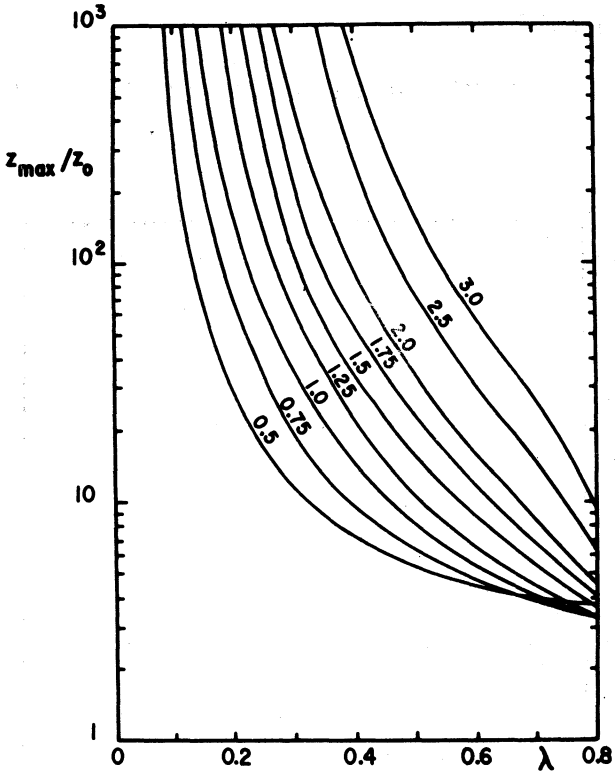

Fig. 6. Dependence on λ of the ratio z_{max}/z_o ($\equiv Z_{max}/Z_o$). See text and Figure 1.

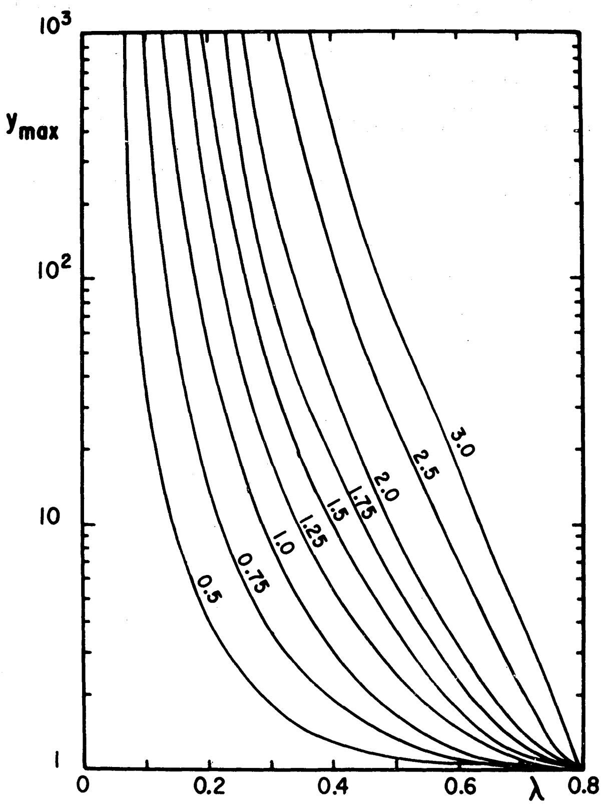

Fig. 7. Dependence on λ of the maximum distance from the nebula edge to the nebula axis, y_{max}, in terms of R_o ($y_{max} \equiv Y_{max}/R_o$). See text and Figure 1.

IV. CONCLUSIONS

Although Bipolar Nebulae have either been regarded as pure reflection nebulae or been related to the planetary nebula stage, there has recently been increasing observational evidence for the existence of interstellar gas disks around them, and of ionized and neutral material moving at high velocities. This latter piece of evidence has led several authors to propose that the exciting stars in BN are strong producers of stellar winds.

In this paper we present the results of a steady-state model in which a wind-producing star is immersed in an interstellar disk. The wind is taken as isotropic and the disk is assumed infinite, isothermal and self-gravitating.

The main results of this model can be summarized as follows:

1) The static configuration achieved by the flow pattern consists of two ovoid cavities, symmetric with respect to the disk plane, filled by unshocked stellar wind and bounded by a shock in the wind.

2) The shocked stellar wind material slides along the walls of the cavities, forming an annular stream toward the distant tips of the configuration. The velocity of this stream is comparable with the wind velocity, even in the static configuration.

3) Double-peaked line profiles in the nebula are expected, with the velocity difference between the peaks being a function of their distance to the star.

4) In contrast with other models (pure reflection or photoionization), the present model is able to account, naturally, for the acute ending tips observed in several bipolars. Further, it is suggested that such acute tips represent the hallmark of wind-produced nebulae, since pure reflection or photoionization models can hardly account for them.

5) Finally, a minimum value for the wind "strength" in BN,

$$(\dot{M}_*/10^{-7}\ M_\odot\ yr^{-1})\ (V_*/100\ km\ s^{-1}) > 4.5 \times 10^{-3}\ T^2,$$

(where T is the disk temperature) is suggested.

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