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TRANSITION REGION MODELS FOR Be STARS

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RESUMEN

Utilizando hipótesis simplificativas y considerando la pérdida de energía por radiación, la aceleración radiativa en forma aproximada, y la rotación, se ha tratado de elaborar un modelo preliminar que reproduzca las características generales de los anchos equivalentes y los perfiles de líneas correspondientes a iones tales como C IV y Si IV que se observan en el espectro ultravioleta de muchas estrellas, en particular de estrellas tempranas que experimentan pérdida de masa. Para lograr un acuerdo en cuanto al orden de magnitud con las observaciones, resultó imprescindible postular, en las ecuaciones de conservación de impulso radial y angular, la existencia de algún mecanismo de frenamiento.

ABSTRACT

A preliminary model that reproduces the general characteristics of equivalent widths and profiles of ultraviolet spectral lines corresponding to ions such as C IV and Si IV, observed in a number of stars, particularly in early type objects undergoing mass loss, has been worked out. We make simplifying assumptions and consider radiative energy loss, radiative acceleration in an approximate form, and rotation.

Agreement, at least as to the order of magnitude, between the model and the observations is found if we postulate the existence of some kind of braking mechanism.

Key words: STARS-Be – STARS-CORONAE

I. INTRODUCTION

Ultraviolet stellar spectra often show the presence of absorption lines that correspond to ions such as C IV, Si IV and N V, frequently displaying asymmetric profiles. These may be formed in an expanding chromosphere-corona-like region.

In order to describe the temperature and density structure of such a region, two approaches have been used so far: *a)* to fit the observed profiles by means of radiative transfer calculations, where the relevant coefficients (opacity and emissivity) are decoupled from the temperature, for instance, by assuming that scattering is the only mechanism that contributes to the source function (Hamann 1979); and *b)* to build a theoretical model on the basis of a certain hypothesis on the energy balance, and by using a simplified treatment of the radiation transport such as the Sobolev approximation.

The first approach does not yield a complete physical description of the region but only a relation between some parameters such as the distance to the star's center and the velocity. Moreover, the solution is strongly conditioned by the assumptions that are adopted, as for instance the monotonic increase of velocity with distance.

The second approach has been followed by Castor, Abbot and Klein (1975), who assumed radiative equilibrium. This fails to explain the ionization of the material unless one postulates a sophisticated mechanism such as Auger ionization (Cassinelli *et al.* 1978*a, b*).

In the present paper we propose to follow a sort of modified second approach which considers terms due to the kinetical and potential energies, in addition to radiative losses to solve for the energy balance equation. In a first approximation we will show how the kinetic energy can account for the heating of the material up to temperatures compatible with the formation of ions like C IV, and Si IV by collisional processes.

II. FORMALISM

We assume that the physical parameters depend only on one coordinate which we call R and we adopt the classical equations of conservation of mass, momentum and energy, in a stationary or quasi-stationary state, and as a consequence, we neglect the time derivatives.

If we consider a flux tube of cross section S, of density ρ, moving with velocity v in the direction of R, mass conservation implies that

$$\rho \, S \, v = A \quad , \tag{1}$$

where A is constant, and S is a function of R.

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Solving for S as a function of R requires finding the solution for the flux in three dimensions. However, we will not consider such a solution since we have adopted a one dimensional dependence of the physical parameters.

If one assumes that streamlines have a particular symmetry that may be spherical ($S \propto R^2$), axial ($S \propto R$) or plane ($S = \text{constant}$), then we can write that $S \propto R^{\beta}$.

The equation of conservation of energy is written by considering a kinetic energy per unit mass.

$$v \frac{d}{dR} \left(\frac{5}{2} \mathcal{R} T + \frac{v^2}{2} + \frac{w^2}{2} \right) = -Q \qquad (2)$$

where $\mathcal{R}$ is the gas constant, T the temperature, Q the kinematic energy loss per unit mass per unit time, and w the rotational velocity. The first term in the parenthesis gives the enthalpy per unit mass for an ideal gas, and Q is accounted for by sources of energy other than the kinetic energy of particles. Here we have assumed that these sources of energy are proportional to the mass, which is reasonable if one considers radiation losses, potential energy or dissipation of waves.

The equation of conservation of momentum per unit mass can be written as

$$v \frac{dv}{dR} = -\frac{1}{\rho} \frac{dp}{dR} + f_R \ , \qquad (3)$$

and the component of the velocity w (perpendicular to v)

$$v \frac{dw}{dR} = f_w \ , \qquad (4)$$

where p is the pressure, f_R the force along R, and f_w the force in the direction of w, both forces are expressed per unit mass. In equations (3) and (4) f includes the corresponding terms due to the system of coordinates chosen (for instance, the centrifugal and Coriolis forces).

The other equation that we take into account is the equation of state for an ideal gas, namely

$$p = \mathcal{R} \rho T \ , \qquad (5)$$

which is assumed to be valid for a gas composed mainly of protons and electrons such as the one we are dealing with. In order to integrate equations (1) to (5) we adopt the logarithmic form; then

$$d(\ln p) - d(\ln \rho) - d(\ln T) = 0 \ , \qquad (6)$$

$$d(\ln \rho) + d(\ln v) = -\beta \, d(\ln R) \ , \qquad (7)$$

$$d(\ln T) + \frac{2 v^2}{5 \mathcal{R} T} d(\ln v) + \frac{2 w^2}{5 \mathcal{R} T} d(\ln w) = -\frac{2 Q R}{5 \mathcal{R} T v} d(\ln R) \ , \qquad (8)$$

$$d(\ln v) + \frac{p}{\rho v_2} d(\ln p) = \frac{R f_R}{v^2} d(\ln R) \ , \qquad (9)$$

and

$$d(\ln w) = \frac{R f_w}{v w} d(\ln R) \ . \qquad (10)$$

Equation (7) follows from mass conservation for the cases where β is constant. If β were not constant it could be redefined as

$$\beta = \frac{R}{S} \frac{dS}{dR} \ .$$

Equation (10) has two components that correspond to the two components of w.

The form of equations 6-10 does not depend on the choice of the coordinate system; however, the right-hand side terms do have a specific expression for each system.

These equations are a first order differential system whose solution is determined by either initial values or boundary conditions. We have integrated them by specifying initial values for all the variables and a fixed step $\Delta \ln R$. The coefficients were evaluated for each step at the geometrical mean point by the Newton-Raphson method, which in this case converges quite fast. The resulting model is very stable, except for some singular (critical) points.

In considering the region where the lines of C IV and Si IV are formed in the Be stars, no simple symmetry can be assumed, because of the high rotational velocities of these objects. So if one choses spherical coordinates all physical variables will depend on at least the coordinates ρ and θ.

In order to get a first order approximation to the structure of the region we solve equations (6) to (10) for the equatorial plane, assuming that the physical parameters are independent on the ϕ and θ coordinates.

In such a case, the energy loss is given by

$$Q = v\frac{C^*}{R^2} - v\frac{4\pi}{c}\int \chi_\nu H_\nu \, d\nu + \int 4\pi \; (\eta_\nu - \chi_\nu J_\nu)\, d\nu \quad , \qquad (11)$$

where C* is the mass of the star multiplied by the constant of gravitation, c is the velocity of light, χ_ν and n_ν are the absorption and emission coefficients per unit mass at the frequency ν and H_ν and J_ν are, respectively, the radiation flux and mean intensity.

If account is taken of the centrifugal force, the radial force is given by the expression

$$f_R = -\frac{C^*}{R^2} + \frac{w^2}{R} + \frac{4\pi}{c}\; \chi_\nu \, H_\nu \, d\nu + f_a \quad , \qquad (12)$$

and the angular force, taking consideration of the Coriolis term is

$$f_w = -\frac{v\,w}{R} + f_b \quad . \qquad (13)$$

In (12) and (13), f_a and f_b are additional terms to which we will refer later.

To obtain a first estimation of the radiative terms, we assume that the radiation field is the one emitted by the stellar photosphere and diluted by the geometrical distance. This implies that we do not consider in this approximation the effect on the radiation field of the absorption or emission of the material, and this is not a bad approximation for the cases where the residual intensities of the lines are not smaller than, say, 50%.

According to this approximation if R* is the photospheric radius,

$$H_\nu = \frac{I_\nu}{4}\left(\frac{R^*}{R}\right)^2 \quad ,$$

$$J_\nu = \frac{I_\nu}{2}\left[1 - \sqrt{1 - \left(\frac{R^*}{R}\right)^2}\,\right] \quad ,$$

where I_ν is the emergent intensity from the stellar photosphere.

To evaluate the absorption and emission coefficients we consider only the resonance lines of many ions such as C IV, Si IV, N V, etc., at frequencies near $\nu_0 = 3 \times 10^{15}$ Hz. As a first approximation, we write

$$4\pi \int (\eta_\nu \, \chi_\nu \, J_\nu)\; d\nu = \rho \, q\, (1 - J_{\nu_0}/B_{\nu_0}) \quad , \qquad (14)$$

$$4\pi \int \chi_\nu \, H_\nu \, d\nu = G\, q\, H_{\nu_0} \quad . \qquad (15)$$

This form is suggested by the solution of the statistical equilibrium equation, when we consider a two level atom with a transition at frequency ν_0. In this case, q and G are coefficients which depend on the atomic parameters, temperature and density. In particular, G depends on the ratio of the collisional to radiative transition rates.

The adopted values of q were taken from Cox and Tucker (1969) because their values reflect the radiative energy losses for an optically thin plasma with no incident radiation ($J_{\nu_0} = 0$); the function G was taken from Mihalas (1978).

These formulae provide a first approximation to the radiative energy loss and the radiative momentum loss. Formula (14) gives the correction for the Cox and Tucker calculations, in the case where there exists a radiation field, while formula (15) is an expression compatible with the first one.

To do the calculation in a more accurate way one has to solve the statistical equilibrium equations for all the levels of all the ions present, which requires knowledge of all the atomic parameters, and the radiative transfer equations for all the frequencies involved.

As a first step, we have computed models with $f_a = f_b = 0$. They yield a steep decrease in temperature and a steep increase in velocity. The computed resonance lines of C IV and Si IV in absorption yield equivalent widths that are too small because of the very small optical thickness of the region at which the temperature permits the formation of these ions. We introduced the forces f_a and f_b in order to increase the equivalent widths of the lines to values comparable to the observed ones, and to produce a run of the velocities and temperatures compatible with the observations.

For these forces we adopt a form suggested by the hydrodynamical "drag force" (cf. Landau and Lifshitz 1958), namely

$$f_a = -\, v^2 \; (1/L_a)$$

$$f_b = -\, v\, w\, (1/L_b) \quad ,$$

where L_a and L_b are coefficients with dimensions of length.

We have calculated models by assuming a constant value for $L_a = L_b = L$, and also by assuming

$$\frac{1}{L} = \frac{\rho\, v}{D\, T^{5/2}} \quad ,$$

where D is a constant.

This last expression was introduced because it depends strongly on T, in the same way as the viscosity and thermal conduction coefficients.

The models show a remarkable difference depending on whether or not we consider a dependence of (1/L) on temperature and velocity.

If we analize equations 6-10 we can see that there are points at which they are not independent, this occurs when

$$v^2 = \frac{5}{3}\ \mathcal{R}\ T = v_c^2$$

i.e., when the velocity reaches the adiabatic speed of sound.

This is the critical point and if the temperature and velocity are required to be continuous at this point, it is necessary that

$$\frac{5}{2}\ \frac{R\,f_R}{v_c^{\,2}} + \beta + \frac{RQ}{v_c^{\,3}} - \frac{w^2}{v_c^{\,2}} + \frac{R\,w\,f_b}{v_c^{\,3}} = 0 \quad .$$

If this were not the case, a shock is produced at the critical point, and equations (6) will not describe the situation any longer.

III. RESULTS

In our calculations we have always assumed that β = constant, and we have checked that the value of β doe not affect significantly the results in the region we ar interested in.

In Figures 1 and 2 we have plotted the velocity and the temperature as functions of height from the startin point, for a set of models defined by the different value of L and D indicated in Table 1, for a star with radiatio temperature T_r = 18 000 °K, radius equal to 5 $R_\odot$, an mass equal to 10 $M_\odot$. For the starting point we hav adopted: $\rho = 4 \times 10^{-13}$ g cm^{-3}, $T = 8 \times 10^4$ °K $v = 100$ km s^{-1}, and $w = 200$ km s^{-1}.

In Figure 3 we show predicted profiles of the C IV λ1548 line, if one assumes a model to be valid for al angles ϕ and θ. We can see the variations that the profil undergoes when we modify the value of L. In all case the velocity and the temperature behavior depen strongly on 1/L. In some cases we obtain a temperatur maximum (models 103, 107, 201, 203, 205 and 207) and in other cases we obtain a maximum in velocity (models 106, 202 and 204). Models 205 and 206 ar approaching a critical point with increasing height Model 104 corresponds to 1/L = 0 (i.e., $f_a = f_b = 0$).

In Figures 4 and 5 we show models for a star o T_r = 30 000°K, a radius of 6.7 $R_\odot$, and a mass of 22.

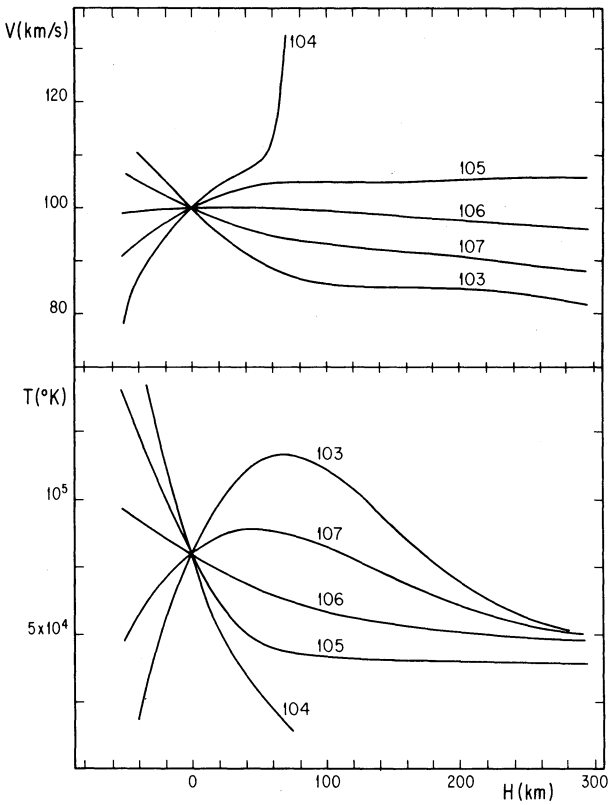

Fig. 1. Models with (1/L) constant.

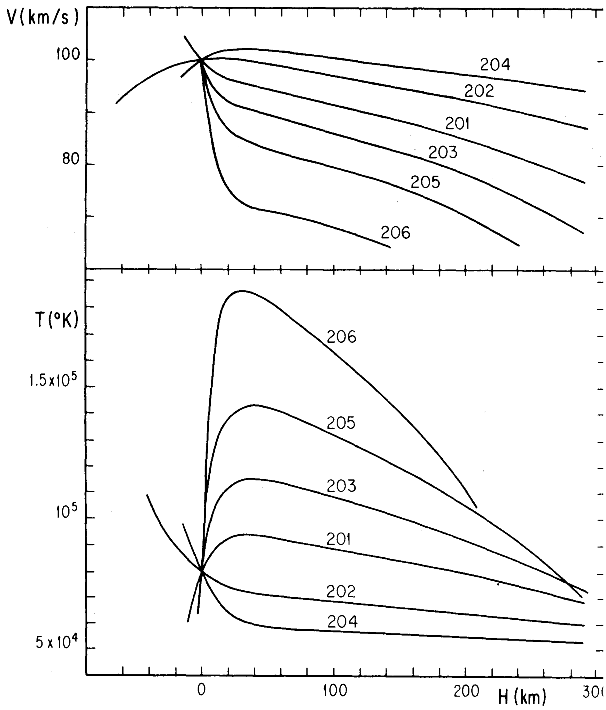

Fig. 2. Models with D constant.

TABLE 1

BRAKING COEFFICIENTS AND D FACTORS

Model	L (km)	D ($\times 10^{10}$)
For $T_I = 18000°K$, $R = 5 R_\odot$ and $M = 10 M_\odot$		
103	1000	28.73*
107	1333	38.35*
106	2000	57.46*
105	4000	114.92*
104	∞	∞ *
206	90.5*	2.60
205	226 *	6.50
203	445 *	13.00
201	905 *	26.00
202	2263 *	65.00
204	4525 *	130.00
For $T_I = 30000°K$, $R = 6.7 R_\odot$ and $M = 22.5 M_\odot$		
304	160 *	6.5
301	320 *	13.0
302	640 *	26.0
303	1600 *	65.0
305	3200 *	130.0
401	59 *	1.0
402	147 *	2.5
403	294 *	5.0

* Value at the starting point.

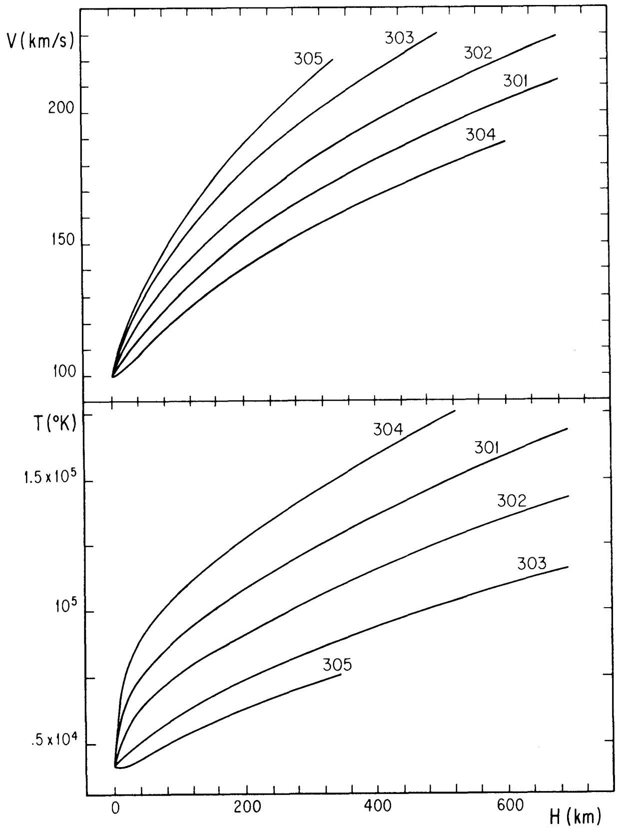

Fig. 4. Models with D constant.

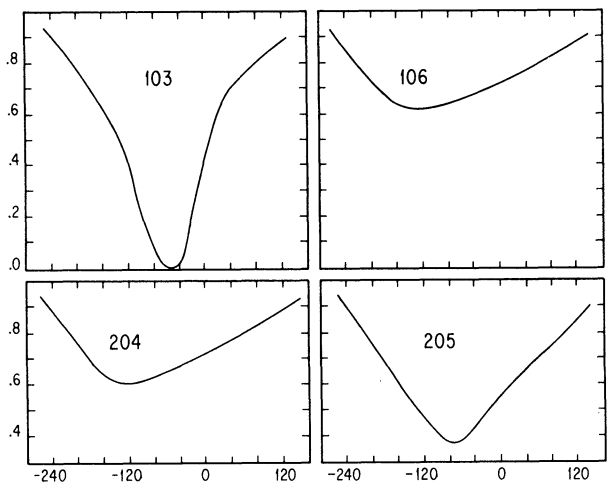

Fig. 3. C IV λ1548, line profiles.

$M_\odot$, with a starting point defined by: $v = 95$ km s^{-1}, and $w = 100$ km s^{-1}. For the models in Figure 4, $\rho = 10^{-13}$ g cm^{-3}, and $T = 4 \times 10^4$ °K, and for the models in Figure 5, $\rho = 10^{-12}$ g cm^{-3}, and $T = 5 \times 10^4$ °K. In Table 1 we list the adopted values of L and D. We see that the velocity and the temperature increase with height for models 301, 302, 303, 304 and also for model 305, except at the very beginning. On the other hand, model 403 has a maximum temperature and a maximum velocity, and model 401 has a maximum in temperature and a decreasing velocity. In Figure 6 we show predicted profiles for two of the models in Figures 4 and 5. The starting point was assumed to have a temperature adequate to bring the resultant models to the region of temperatures in which the C IV ion can be formed (Jordan 1969). On the other hand, we assume that this point is close to the star's surface (Ringuelet *et al.* 1981), and that R and w are close to the photospheric equatorial values that correspond to main sequence objects. The velocity of the starting point was assumed to be supersonic in all cases.

IV. CONCLUSIONS

In an attempt to explain the heating of the material surrounding the star up to a temperature compatible with the formation of ions like C IV, Si IV, N V etc., by collisional processes, we suggest that the dissipation of energy due to expansion and/or rotation velocities is enough to balance the radiation losses. It is not

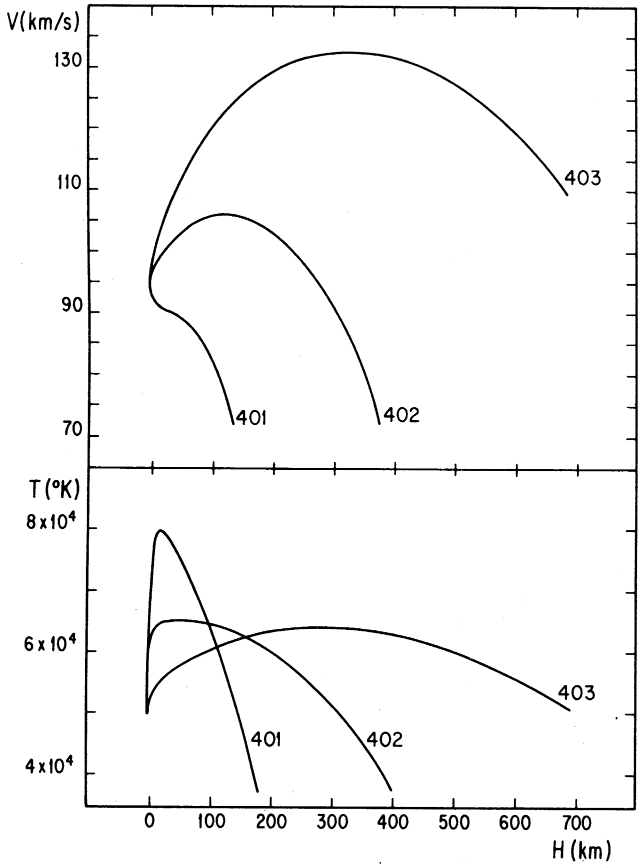

Fig. 5. Models with constant D.

necessary to postulate the existence of other sources of energy, but only to find a mechanism to dissipate the energy contained in the expansion and/or rotation. This can be accomplish in case *a)* by a deceleration of the velocities, or in case *b)* only by a decrease in the acceleration of the expansion velocity. In case *a)* the energy source (expansion and rotational velocities) vanishes with increasing distance, so that we can expect a limited region of high temperature, close to the star, and a cold envelope farther out. On the other hand, in case *b)* one can expect that the temperature stays high as long as the acceleration mechanism (which can be radiation forces) is at work.

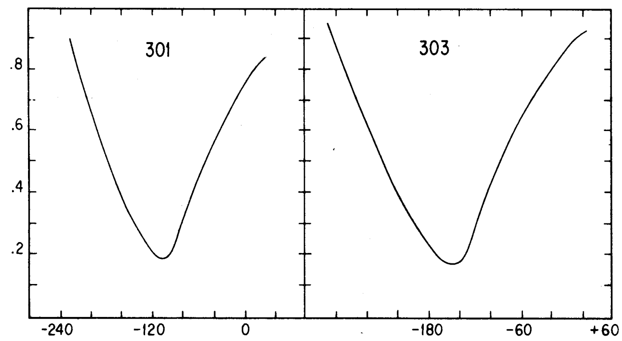

Fig. 6. C IV λ1548, line profiles.

One must keep in mind that since we are dealing with a highly ionized plasma, and since in our case the dominant force is the radiation pressure, which acts selectively on different particles (ions, electrons), there could exist magnetic fields, inhomogeneities, etc., not included in our equations. These processes could conceivably generate additional terms, not only in the energy equation, but also in the force equations, and thus give rise to the postulated braking forces f_a and f_b. In particular, indication of chaotic motions is found by Lamers and Rogerson (1978) in the case of τ Scorpii.

The results of our models show that the behavior of velocity and temperature depend not only on the stellar mass and luminosity but also on the braking coefficient (1/L) and on the initial values, which are not necessarily the same for stars having the same mass and luminosity.

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