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FRAGMENTATION OF PRESTELLAR CLOUDS BY MOLECULE FORMATION

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RESUMEN

La distribución de la temperatura de los granos en nubes interestelares densas se ha investigado empleando las ecuaciones del balance de energía y de transferencia radiativa, esta última en una aproximación derivada por Giovanelli para medios no-homogéneos y que corresponde a una generalización de la aproximación de Eddington. El objetivo es determinar si inhomogeneidades en densidad pueden producir variaciones espaciales en temperatura las cuales producirían fluctuaciones en la tasa de formación de la molécula de hidrógeno, como lo sugirió Reddish (1975).

El problema del calentamiento radiativo se resolvió analíticamente, en primera aproximación, para una nube modelo semi-infinita con fluctuaciones transversales en la extinción, en función de los siguientes parámetros: el albedo para una dispersión simple $\bar{\omega}$, el parámetro de invernadero η, la amplitud de las fluctuaciones a, y el parámetro $r = 2\pi/\lambda_d\bar{\kappa}$, donde λ_d y $\bar{\kappa}$ son la longitud de onda de las fluctuaciones transversales, y el coeficiente de extinción promedio, respectivamente. Para $r \ll 1$, las fluctuaciones en densidad no están conectadas radiativamente (en el visual) y las soluciones al problema del calentamiento radiativo tienden a las del caso homogéneo. Para $r \gg 1$, las fluctuaciones en temperatura tienden a cero. Hay un fuerte acoplamiento entre las fluctuaciones en temperatura $\Delta T_d/\overline{T}_d$ y la profundidad óptica media $\bar{\tau}$. Fluctuaciones con r tales que $1 \lesssim r \lesssim 2$ (el valor exacto depende del valor de $\bar{\tau}$ al cual se alcanza la temperatura T_{cri}) pueden originar fluctuaciones en temperatura que resulten en una discontinuidad química, en la formación de H_2, y en el inicio del proceso de fragmentación.

ABSTRACT

The distribution of grain temperature through dense interstellar clouds has been investigated using the equations of heat balance and radiative transfer in the Eddington-like approximation derived by Giovanelli for non-homogeneous media. The aim is to determine whether density inhomogeneities can produce spatial temperature variations which would produce fluctuations in the rate formation of hydrogen molecule such as those suggested by Reddish (1975).

The radiative heating problem has been solved analytically, in a first approximation, for a semi-infinite cloud model with transverse fluctuation in extinction, in terms of the following parameters: the albedo for single scattering $\bar{\omega}$, the greenhouse parameter η, the amplitude of the fluctuations a, and the parameter $r = 2\pi/\lambda_d\bar{\kappa}$, where λ_d and $\bar{\kappa}$ are the wavelength of the transverse fluctuations and the mean extinction coefficient, respectively. For $r \ll 1$, the density fluctuations become radiatively disconnected in the visual, and the solutions of the radiative heating problem tend to the homogeneous case. For $r \gg 1$ the temperature fluctuations disappear. There is a strong coupling between temperature fluctuations $\Delta T_d/\overline{T}_d$ and mean optical depth $\bar{\tau}$. Fluctuations with r such that $1 \lesssim r \lesssim 2$ (the exact value depending on the value of $\bar{\tau}$ which T_{cri} is reached) can produce temperature fluctuations which result in a chemical discontinuity, in the formation of H_2, and the onset of the fragmentation process.

Key words: MOLECULAR PROCESSES – INTERSTELLAR-MOLECULES – RADIATIVE TRANSFER

I. INTRODUCTION

It has been suggested by Reddish (1975, 1978) that ie fragmentation process of proto-stellar clouds can be iitiated by the formation of the H_2 molecule. Accordig to Reddish, spatial density fluctuations of the dust omponent lead to spatial fluctuations of the radiation eld which produce spatial fluctuations of the dust emperature T_d; the radiation field being due to star ght. Because the rate of H_2 formation depends critially on T_d (Solomon and Wickramasinghe 1969; Lee 1972, 1975); there are regions where the rate of molecule formation is fast, and there are other regions where this process is slow, or does not occur at all. This phenomenon produces instabilities, Reddish proposed for them a length scale of the order of one unit of optical depth in visual extinction.

A study of Reddish fragmentation process has been done by Ibañez (1979, 1981). In this paper, a model of a non-homogeneous dust cloud will be explored to find the effective scale length at which the dust temperature fluctuates around the critical temperature T_{cri} for H_2 formation on dust grains.

II. FORMALISM

The dust temperature at any point **r** in a cloud is determined by the local energy balance equation.

$$\int_0^\infty n_d(\mathbf{r}) \left[\langle \sigma_d Q_{abs} \rangle_\nu B_\nu(T_d(\mathbf{r})) - \langle \sigma_d Q_{abs} \rangle_\nu J_\nu(\mathbf{r}) \right] d\nu = \frac{1}{4\pi} E(\mathbf{r}), \qquad (1)$$

where $\langle \sigma_d Q_{abs} \rangle_\nu$ is a mean absorption cross section per grain at frequency ν, $B_\nu(T_d(\mathbf{r}))$ the Planck function, $J_\nu(\mathbf{r})$ the local mean intensity of the radiation field at frecuency ν, and $E(\mathbf{r})$ represents the net amount of energy lost by the non-radiative processes: gas-dust collision and H_2 formation. At a gas temperature $T \lesssim 150°$ and a gas density $n < 10^6$ cm^{-3} the transfer of energy by gas-dust collision is negligible in comparison with the radiation heating (Hayashi and Nakano 1965). For $n \lesssim 10^4$ cm^{-3} the energy input by H_2 formation can also be neglected, if the local radiation field can maintain the dust temperature above 9°K (Solomon and Wickramansinghe 1969). Indeed, this limit was obtained by these authors assuming that all the binding energy can be absorbed by the grain. However, according to Hunter and Watson (1978), H_2 probably recombines into high rotational states, and new molecules return to the gas in rotational states $J \gtrsim 7$. Therefore, the above temperature would represent an upper limit and equation (1) becomes

$$\int_0^\infty \langle \sigma_d Q_{abs} \rangle_\nu J_\nu(\mathbf{r})\, d\nu = \int_0^\infty \langle \sigma_d Q_{abs} \rangle_\nu B_\nu(T_d(\mathbf{r}))\, d\nu \ . \qquad (2)$$

The mean radiation field $J(\mathbf{r})$ at a particular point **r** in any cloud is determined by the radiative transport into the cloud of the radiation field reaching the free surface of the cloud. Here the interest is focused on the early stages of evolution of pre-stellar clouds, when the transition of H I to H_2 occurs and therefore no internal sources of radiative energy are expected to be present. According to Zimmerman (1964), and Krishna-Swamy and Wickramasinghe (1968), the diluted mean stellar radiation field (incident field) is well represented by a black-body radiation field corresponding to a temperature of about 10^4 °K but diluted by a factor $w = 10^{-14}$. Strictly speaking, this field is a good representation of the field in the vicinity of the sun. A more sophisticated representation of this field has been discussed by Greenberg (1971).

The radiative transfer equation, along a ray s in th direction Ω, can be written in the form

$$\frac{dI_\nu(s,\Omega)}{ds} = -\kappa_\nu(s)\, I_\nu(s,\Omega) + j_\nu(s,\Omega) \ , \qquad (3$$

where $I_\nu(s,\Omega)$ is the specific radiation intensity a frecuency ν, at point s, in the direction defined by th unit vector Ω, $\kappa_\nu(s)$ is the extinction coefficient, an $j_\nu(s,\Omega)$ is the total emission coefficient.

As it is well known, the radiation field at any poin in a cloud can be represented by the sum of tw components: *a)* The attenuated field, i.e. the inciden field reaching any point s in the cloud without havin suffered any absorption or scattering. *b)* The diffus radiation field, i.e. the field at any point s in the clou originated by scattering and emission processes.

For the diffuse field, Giovanelli (1959, 1963) cor structed a generalization to the Eddington approxima tion for inhomogeneous media. By writing the genera solution of equation (3) in the form

$$I_\nu = \sum_{n=0}^{\infty} \{ I_n P_m(\mu) + \sum_{m=1}^{n} [a_n^m \cos(m\Phi) + b_n^m \sin(m\Phi)]\, P_n^m(\mu) \} \ , \qquad (4$$

where $P_m(\mu)$ and $P_n^m(\mu)$ are the Legendre polynomial and Legendre associated functions; I_n, a_n^m, and b_n^m ar coefficients depending on frequency and position, bu not on direction (μ, Φ). He showed that if only firs order effects in μ and Φ are taken into account, i.e $I_2 = 0$ and

$$(\partial a'_2/\partial x) = (\partial b'_2/\partial y) = 0 \ ,$$

from equations (3) and (4), the following equation i obtained

$$\nabla \cdot \left[\frac{1}{\kappa_\nu(\mathbf{r})} \nabla J_\nu(\mathbf{r}) \right] = 3\,[\alpha_\nu(\mathbf{r})\, J_\nu(\mathbf{r}) - \kappa_\nu(\mathbf{r})\, S_\nu(\mathbf{r})] \qquad (5$$

where $J_\nu(\mathbf{r})$ is the mean intensity, $\alpha_\nu(\mathbf{r})$ is the absorptio coefficient, and $S_\nu(\mathbf{r})$ is a source function without th scattering term corresponding to the diffuse field. Equa tion (5) leads to solutions for 3-D non-uniform media, o the same degree of validity as those obtained from th Eddington approximation in the 1-D case. Unno an Spiegel (1966) derived (5) independently, and the proved formally that their solutions are reasonably accurate over the full range of optical thickness.

The temperature of the grains in dust clouds can be evaluated by considering the cloud to be grey in the visual and infrared regions of the spectrum. Following Wildt (1966), Stibbs (1971) and Morgan (1973), one may define the greenhouse parameter η, by the relation $\eta = \kappa_s/\kappa_p$, where κ_s is a mean extinction coefficient in the visual region, and κ_p is a mean extinction coefficient in the infrared region.

The mean free paths of visual and infrared photons are $\sim 1/\kappa_s$, and $\sim 1/\kappa_p$, respectively. Thus the η parameter is a measure of the relative thickness of the medium, and contains schematically the otherwise very complex frequency dependence, characteristic of dust optics (Andriesse 1977).

It is convenient to define a dimensionless function $\psi(\mathbf{r})$, which contains the spatial dependence of the extinction

$$\kappa_i = \langle Q_{ext}\sigma_d \rangle_i \, \overline{n}_d \, \psi(\mathbf{r}) = \overline{\kappa}_i \, \psi(\mathbf{r}) \quad , \quad i = s,p \quad , \qquad (6)$$

where $\overline{\kappa}_i$ is a mean value of the extinction. Similar relations can be written for the absorption and scattering coefficients α_i and σ_i, respectively.

In the visual region the emission coefficient j_s includes the radiation scattered from the attenuated and diffuse fields, and the thermal emission, assumed to be zero in this region. Therefore equation (5) becomes

$$\nabla \cdot \left[\frac{1}{\psi(\mathbf{r})} \nabla J_s(\mathbf{r}) \right] = 3\overline{\kappa}_s^2 \, \psi(\mathbf{r}) \left[\lambda J_s(\mathbf{r}) - \overline{\omega} J_s^a(\mathbf{r}) \right] \quad , \qquad (7)$$

where $\overline{\omega}$ is the albedo for single scattering σ_s/κ_s, $\lambda = 1 - \overline{\omega}$ the scattering parameter, and J_s^a the mean intensity of the attenuated field.

The emission coefficient in the infrared, j_p, includes four terms: the thermal radiation absorbed and re-emitted, the visual radiation converted into infrared, and the scattered infrared radiation, assumed to be zero. Therefore, equation (5) becomes

$$\nabla \cdot \left[\frac{1}{\psi(\mathbf{r})} \nabla J_p(\mathbf{r}) \right] = -3\lambda\eta^{-1} \, \overline{\kappa}_s^2 \, \psi(\mathbf{r}) \left[J_s(\mathbf{r}) + J_s^a(\mathbf{r}) \right] \quad . \qquad (8)$$

For plane geometry and plane parallel incidence with directions $\cos^{-1}\mu_0$, ϕ_0, and radiance πF in the visual, equations (7) and (8) become, respectively,

$$\nabla \cdot \left[\frac{1}{\psi(\mathbf{r})} \nabla J_s(\mathbf{r}) \right] = 3\overline{\kappa}_s^2 \, \psi(\mathbf{r}) \times \left[\lambda J_s(\mathbf{r}) - \overline{\omega} \frac{F}{4} \exp\left[-\tau_0(s;\mu_0,\Phi_0)\right] \right] \quad , \qquad (7')$$

$$\nabla \cdot \left[\frac{1}{\psi(\mathbf{r})} \nabla J_p(\mathbf{r}) \right] = -3\lambda\eta^{-1} \, \overline{\kappa}_s^2 \, \psi(\mathbf{r}) \times \left[J_s(\mathbf{r}) + \frac{F}{4} \exp - \tau_0(s;\mu_0,\Phi_0) \right] \quad , \qquad (8')$$

where $\tau_0(s;\mu_0,\Phi_0)$ is the optical depth at the point $\mathbf{r}$ along the incident beam.

For the diffuse field, the boundary condition consistent with the approximation (5) is

$$\mathbf{n} \cdot \nabla J_s(\mathbf{r}_0) = -\sqrt{3} \; \overline{\kappa}_s \, \psi(\mathbf{r}_0) \, J_s(\mathbf{r}_0) \quad , \qquad (9)$$

in the visual, and

$$\mathbf{n} \cdot \nabla J_p(\mathbf{r}_0) = -\sqrt{3} \; \eta^{-1} \overline{\kappa}_s \, \psi(\mathbf{r}_0) \, J_p(\mathbf{r}_0) \quad , \qquad (10)$$

in the infrared. Here $\mathbf{r}_0$ represents any point on the boundary surface, and $\mathbf{n}$ is an outward normal unit vector at any point $\mathbf{r}_0$.

If one defines the effective temperature of the incident radiation field by the relation

$$\sigma_0 T_0^{\,4} = \pi F \quad , \qquad (11)$$

the temperature at any point of a particular cloud nomalized to T_0 will be given by

$$T(\mathbf{r}) = \{ J_p(\mathbf{r}) + \lambda\eta \left[J_s(\mathbf{r}) + J_s^a(\mathbf{r}) \right] \}^{1/4} \quad , \qquad (12)$$

where the substitutions $T(\mathbf{r})/T_0 \rightarrow T(\mathbf{r})$, and $J_i/F \rightarrow J_i$ have been done. In the particular case of plane geometry and plane parallel incident radiance πF, (12) becomes

$$T(\mathbf{r}) = \{ J_p(\mathbf{r}) + \lambda\eta \left[J_s(\mathbf{r}) + \frac{1}{4} \exp\left[-\tau_0(s;\mu_0,\Phi_0) \right] \right] \}^{1/4} \quad . \qquad (13)$$

III. ANALYTICAL SOLUTION

A simple model may be proposed to represent schematically a non-uniform cloud and gain some insight into the general behaviour of the radiative field by analytical means, and, in particular, of the dependence of the optical thickness on the inhomogeneities and the temperature fluctuations. In particular, a semi-infinite cloud with a density distribution of the form

$$\psi(x,z) = 1 + a \cos l x \quad , \qquad (14)$$

may be proposed; where $a < 1$, and $l = 2\pi/\lambda_d$ is the wavelength number of the density fluctuation. The free surface is the plane $z = 0$, on which a parallel beam of net flux πF is incident in some specific direction $\cos^{-1}(\mu_0, \Phi_0)$. To represent the nearly isotropic mean incident galactic field, $\mu_0 = 1/\sqrt{3}$ and $\Phi_0 = \pi/4$ will be taken as two cuadrature points.

The dimensionless variables X and Z, and the parameter r are defined as follows

$$X = lx \quad , \quad Z = \overline{\kappa}_s z \quad , \quad r = l/\overline{\kappa}_s = 2\pi/\lambda_{d\,s} \quad , \qquad (15)$$

where, X is measured in units of the wavelength λ_d, Z is the optical depth which corresponds to the optical depth in the homogeneous case ($a = 0$), or the actual optical depth measured at $X = \pi/2$. The parameter r is a measure of the optical thickness of the fluctuation. With the density distribution (14), and in terms of the variables defined above, equation (7′) becomes

$$(1 + a\cos x)\left(r^2 \frac{\partial^2 J_s}{\partial X^2} + \frac{\partial^2 J_s}{\partial Z^2}\right) + r^2 a \sin x \,\frac{\partial J_s}{\partial X} = 3(1 + a\cos x)^3 \times \left[\lambda J_s - \frac{\overline{\omega}}{4}\exp\left[-\tau_0(X,Z;\mu_0,\Phi_0)\right]\right] . \qquad (16)$$

The exponential in (16) can be expanded as a power series, which for $a < 1$ converges very fast, and the error remains small if enough terms are taken in the expansion. In addition, if every power of the functions sin x and cos x is expressed as harmonic sum, it is reasonable to look for solutions of (16) of the form

$$J_s(X,Z) = \sum_{k=0}^{\infty} J_k^s(Z)\cos kx + \sum_{m=1}^{\infty} H_m^s(Z)\sin mx \quad . \qquad (17)$$

Substituting equation (17) into (16) and equating the coefficients for the different harmonics of X, a set of simultaneous differential equations is obtained for the functions $J_k^s(Z)$ and $H_m^s(Z)$. The system of equations, together with the corresponding set obtained from the boundary condition (9), can be solved with the help of an appropriate numerical technique. Analytically, however, only a small number of harmonics can be retained. Fortunately the terms involving harmonics greater than or equal to 2, are small and may be neglected when $a < 1$, in a first approximation, reducing the solution (17) to the form:

$$J_s(Z) = J_0^s(Z) + J_1^s(z)\cos x + H_1^s(Z)\sin x \quad . \qquad (18)$$

The functions J_0^s, J_1^s and H_1^s are solutions of the system of equations

$$J_0^{s\prime\prime}(Z) + \frac{a}{2} J_1^{s\prime\prime}(Z) - 3(1 + \frac{3}{2}a^2)\lambda J_0^s(Z) - [ar^2 + \frac{9}{2}a(1 + \frac{a^2}{4})\lambda]J_1^s(Z) = -\frac{3\overline{\omega}}{4}(1 + \frac{3}{2}a^2)g_0(Z) - \frac{9\overline{\omega}}{8}a(1 + \frac{a^2}{4})g_1(Z) \quad ,$$

$$aJ_0^{s\prime\prime}(Z) + J_1^{s\prime\prime}(Z) - 9a(1 + \frac{a^2}{4})\lambda J_0^s(Z) - [r^2 + 3(1 + \frac{9}{4}a^2)\lambda]J_1^s(Z) = -\frac{9\overline{\omega}}{4}(1 + \frac{a^2}{4})g_0(Z) - \frac{3\overline{\omega}}{4}(1 + \frac{9}{4}a^2)g_1(Z) \quad , \qquad (19)$$

$$H_1^{s\prime\prime}(Z) - [r^2 + 3(1 + \frac{3}{4}a^2)\lambda]H_1^s(Z) = -\frac{3\overline{\omega}}{4}(1 + \frac{3}{4}a^2)h_1(Z) \quad ,$$

where the primes denote derivatives with respect to Z, and the functions $g_0(Z)$, $g_1(Z)$, and $h_1(Z)$ are defined by the relations

$$g_0(Z) = 1 + \frac{a^2}{4}(f_1^2 + f_2^2) + \ldots \quad ,$$

$$g_1(Z) = -af_1\left[1 + \frac{a^2}{8}(f_1^2 + f_2^2) + \ldots\right] , \quad (20)$$

$$h_1(Z) = af_2\left[1 + \frac{a^2}{8}(f_1^2 + f_2^2) + \ldots\right] ;$$

where

$$f_1 = \sin(\delta Z)/\mu_0\delta \ , \quad f_2 = (\cos\delta Z - 1)/\mu_0\delta \ ,$$

$$\delta = r(1/\mu_0^2 - 1)^{1/2} \cos\Phi_0 \ .$$

The boundary condition (9) gives three equations

$$J_0^{s\,\prime}(Z)|_{Z=0} = \sqrt{3}\,[J_0^s(0) + \frac{a}{2}\, J_1^s(0)] \ ,$$

$$J_1^{s\,\prime}(Z)|_{Z=0} = \sqrt{3}\,[J_1^s(0) + a\, J_0^s(0)] \ , \quad (21)$$

$$H_1^{s\,\prime}(Z)|_{Z=0} = \sqrt{3}\, H_1^s(0) \ .$$

The condition $J_s(X,Z)\exp(-Z/\mu_0) \to 0$ when $Z \to \infty$, provides three additional equations. Therefore, the system of equations (19) is completely determined.

The solutions to equations (19) are

$$J_0^s = c_2 \exp(\alpha_2 Z) + c_4 \exp(\alpha_4 Z)$$

$$+ [d_0 + d_1 \cos\delta Z + d_2 \sin\delta Z]\exp(-Z/\mu_0) \ ,$$

$$J_1^s = \beta_2 c_2 \exp(\alpha_2 Z) + \beta_4 c_4 \exp(\alpha_4 Z)$$

$$+ [q_0 + q_1 \cos\delta Z + q_2 \sin\delta Z]\exp(Z/\mu_0) \ , \quad (22)$$

$$H_1^s(Z) = k_2 \exp(-vZ) + [l_0 + l_1 \cos\delta z$$

$$+ l_2 \sin\delta Z]\exp(-Z/\mu_0) \ ,$$

where α_2, α_4, d_0, d_1, d_2, β_2, β_4, q_0, q_1, q_2, v, l_0, l_1, and l_2, are well defined parameters which are functions of a, $\bar{\omega}$, r, μ_0, and Φ_0. The constants c_2, c_4 and k_2 are determined from the boundary conditions.

Once the visual field has been evaluated, the infrared field will be calculated solving equation (8′) with the density distribution given by (14). Proceeding in the same way as described above, equation (8′) can be written as follows

$$(1 + a\cos x)\left(r^2 \frac{\partial^2 J_p}{\partial X^2} + \frac{\partial^2 J_p}{\partial Z^2}\right) + r^2 a \sin x \frac{\partial J_p}{\partial X}$$

$$= -3\eta^{-1}\lambda(1 + a\cos x)^3 \Big[J_s(X,Z) \quad (23)$$

$$+ \frac{1}{4}\exp[-\tau_0(X,Z,\mu_0,\Phi_0)]\Big] \ .$$

The solution to (23) is

$$J_p(X,Z) = J_0^p(Z) + J_1^p(Z)\cos x + H_1^p(Z)\sin x \ , \quad (24)$$

where $J_0^p(Z)$, $J_1^p(Z)$, and $H_1^p(Z)$ are solutions to the following system of differential equations

$$J_0^{p\,\prime\prime} + \frac{a}{2} J_1^{p\,\prime\prime} - ar^2 J_1^p$$

$$= -3\eta^{-1}\lambda\{(1 + \frac{3}{2}a^2)[J_0^s(Z) + \frac{1}{4}g_0(Z)]$$

$$+ \frac{3}{2}a\left(1 + \frac{a^2}{4}\right)[J_1^s(Z) + \frac{1}{4}g_1(Z)]\} \ ,$$

$$aJ_0^{p\,\prime\prime} + J_1^{p\,\prime\prime} - r^2 J_1^p \quad (25)$$

$$= -3\eta^{-1}\lambda\{(1 + \frac{9}{4}a^2)[J_1^s(Z) + \frac{1}{4}g_1(Z)]$$

$$+ 3a\left(1 + \frac{a^2}{4}\right)[J_0^s(Z) + \frac{1}{4}g_0(Z)]\} \ ,$$

$$H_1^{p\,\prime\prime} - r^2 H_1$$

$$= -3\eta^{-1}\lambda(1 + \frac{3}{4}a^2)[H_1^s(Z) + \frac{1}{4}h_1(Z)] \ ,$$

where $g_0(Z)$, $g_1(Z)$, and $h_1(Z)$ are given by (20).

From the boundary condition (10) the following equations are obtained

$$J_0^{p\,\prime}(Z)|_{Z=0} = \frac{\sqrt{3}}{\eta}[J_0^p(0) + \frac{1}{2}aJ_1^p(0)] \ ,$$

$$J_1^{p\,\prime}(Z)|_{Z=0} = \frac{\sqrt{3}}{\eta}[J_1^p(0) + aJ_0^p(0)] \ , \quad (26)$$

$$H_1^{p\,\prime}(Z)|_{Z=0} = \frac{\sqrt{3}}{\eta} H_1^p(0) \ ,$$

which together with the condition

$$J_p\exp(-Z/\mu_0) \to 0 \text{ when } Z \to \infty \ ,$$

provide the necessary conditions to determine the solutions of equations (25), which are

$$J_0^p(Z) = \frac{P_2}{\alpha_2^2}\exp(\alpha_2 Z) + \frac{P_4}{\alpha_4^2}\exp(\alpha_4 Z) + \frac{a}{2(1-a^2)} C_2 \exp(-RZ) + [V_0 + V_1 \cos\delta Z + V_2 \sin\delta Z]\exp\left(-\frac{Z}{\mu_0}\right) + C_4 \ ,$$

$$J_1^p(Z) = C_2\exp(-RZ) + S_2\exp(\alpha_2 Z) + S_4\exp(\alpha_4 Z) + [Q_0 + Q_1\cos\delta Z + Q_2 \sin\delta Z]\exp\left(-\frac{Z}{\mu_0}\right) \ , \quad (27)$$

$$H_1^p(Z) = K_2\exp(-rZ) + L_3\exp(-vZ) + [L_0 + L_1\cos\delta Z + L_2\sin\delta Z] \ \exp\left(-\frac{Z}{\mu_0}\right) \ ,$$

where the parameters P_2, P_4, V_0, V_1, V_2, S_2, S_4, Q_0, Q_1, Q_2, L_0, L_1, L_2 and L_3 are well defined functions of a. $\bar{\omega}$, r, η; and μ_0, Φ_0. C_2, C_4, and K_2 are constants of integration evaluated from the boundary conditions.

Equations (18) and (24) are the solutions for the diffuse field in a semi-infinite cloud with sinusoidal transverse density fluctuations. The temperature distribution can be obtained straightforwardly by substituting these equations into (13).

IV. RESULTS

The basic parameters defining the semi-infinite cloud model are: the amplitude of the density fluctuation a, the albedo for single scattering $\bar{\omega}$, the greenhouse parameter η, and the parameter r which measures the optical thickness of the density fluctuations. Each set of values (a, $\bar{\omega}$, η, r) determines a particular cloud model. Let us fix $a = 0.1$ (this value ensures the rapid convergence of the power series of exponentials introducing only small errors), $\bar{\omega} = 0.5$ (a typical value for interstellar dust clouds of interest in this study), and $50 \leqslant \eta \leqslant 10^4$.

From equation (13) it follows that the grain temperature is practically determined by the attenuated field close to the boundary, i.e. $0 \leqslant Z \lesssim 1.1$. At greater depths, $1.1 < Z \lesssim Z_D$, where $3.0 \lesssim Z_D \lesssim 5$, the diffuse field dominates the infrared. Deep into the cloud, $Z > Z_D$, the temperature is determined by the infrared field as is expected from simple physical considerations.

The temperature T(X, Z) at $X = \pi/2$, which is a measure of the mean temperature, decreases with depth up to values of Z at which the temperature become: completely controlled by the infrared field. For th rather extreme values of $\eta = 10^4$, this happens at depth: of the order of 10. For values of $\eta \simeq 10^2$ the value o $T(\pi/2,Z)$ changes by a factor of 2.9 between the surface $Z = 0$, and depths $Z \gtrsim 10$. This value compares well wit that obtained from formal homogeneous models.

Figures 1 to 4 show the relative temperature fluctua tion as a function of r at mean optical depths Z = 1.0 2.0, 3.0 and 5.0, where the diffuse field is dominan over, or comparable to, the attenuated or infrared fields In these figures the horizontal line marked $\Delta\gamma/\gamma = 0.9$ indicates the temperature fluctuation required to sepa rate the region where the rate of H_2 formation is 99% from that where the rate is 1%.

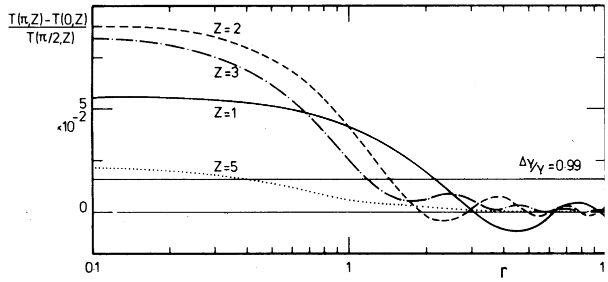

Fig. 1. The temperature fluctuation $[T(\pi,Z) - T(0,Z)]/T(\pi/2,Z$ as a function of r for several values of Z. The values $\eta = 50.0$ $\bar{\omega} = 0.5$, $a = 0.1$, $\mu_0 = 1/\sqrt{3}$ and, $\Phi_0 = \pi/4$ have been assumed The horizontal line marked $\Delta\gamma/\gamma = 0.99$ corresponds to th temperature fluctuation required to produce a change in the rat of H_2 formation of 99% if $T(\pi/2,Z) = T_{cri}$.

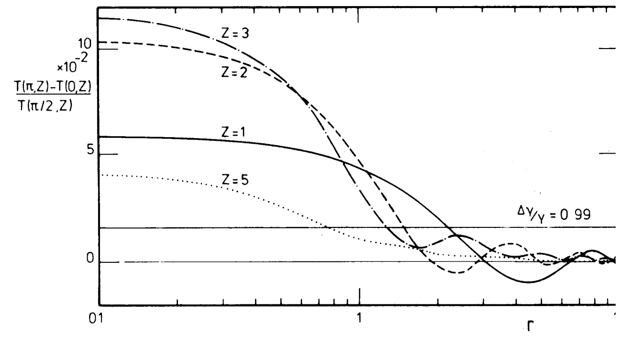

Fig. 2. As Figure 1 for $\eta = 10^2$.

There is a strong coupling between the temperatur fluctuation and the mean optical depth Z. However fluctuations with $r > 0$ (the exact value depends on η are unable to produce appreciable temperature fluctua tions at any depth > 1.0. In particular if similarly t

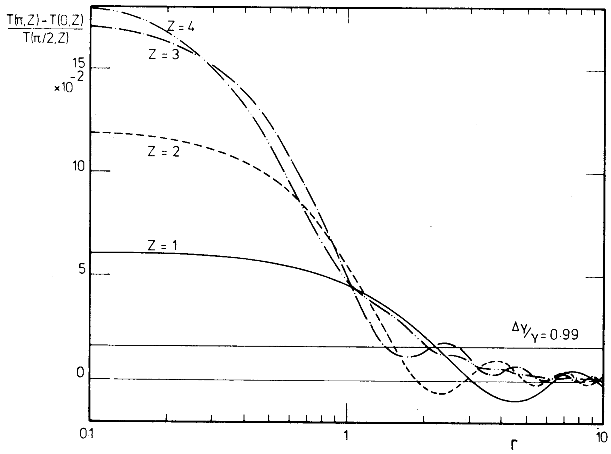

Fig. 3. As Figure 1 for $\eta = 10^3$.

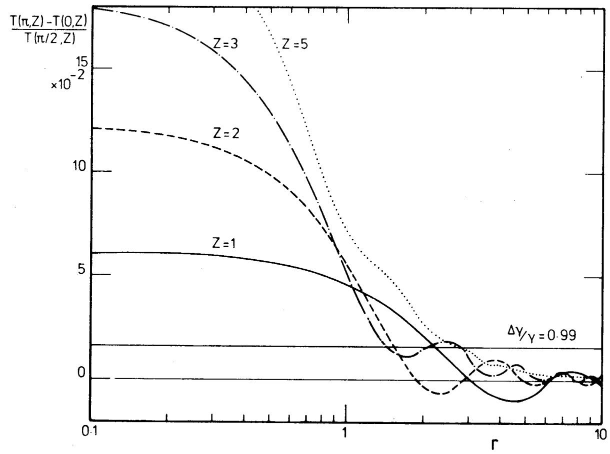

Fig. 4. As Figure 1 for $\eta = 10^4$.

formal homogeneous models by Leung (1975), it is assumed that at mean optical depth of 3 the critical temperature for H_2 formation on graphite grains is reached, fluctuations with $r < r_c$ would be able to produce a change in the rate of H_2 formation of 99%. The value of r_c is very insensitive to the particular value of η for the range of interest, i.e. $\eta \gtrsim 50$, as can be seen from Table 1 extracted from Figures 1 to 4.

TABLE 1

CRITICAL RADIUS AS A FUNCTION OF THE GREENHOUSE PARAMETER

r_c	η
1.1	50
1.3	10^2
1.4	10^3
1.4	10^4

A reasonable value to adopt would be $r_c \cong 1.3$, which would give a thickness for the "radius" of the cell, where H_2 formation proceeds without difficulty, of $\bar{r} = \lambda_d \bar{\kappa}_s/4 = \pi/2.6$.

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DISCUSSION

Calvet: ¿Qué intervalo de espesores ópticos encuentras? ¿Qué errores introduce el uso de la aproximación de Eddington?

Ibañez: Consideramos nubes con espesores ópticos mayores que una unidad óptica de extinción visual. En la aproximación de Eddington los peores errores ocurren hacia la frontera $\tau \lesssim 0.5$. En la generalización empleada es de esperarse lo mismo. Sin embargo, para $\tau > 1$ los errores son del orden de 10%, los cuales no afectan los resultados. En este modelo simple solo intentamos encontrar una escala aproximada para la variación de la temperatura del polvo en la nube interestelar modelo.

Torres-Peimbert: ¿Incluyes el tiempo característico de la formación de H_2 en tus cálculos? y ¿cuál es?

Ibañez: Aquí sólo hemos tratado un aspecto del problema, i.e., el del calentamiento radiativo. Los tiempos característicos de los procesos termodinámicos y termoquímicos son varios órdenes de magnitud menores que la escala de tiempo para reestablecer el equilibrio radiativo. Por lo tanto, la distribución de temperatura de los granos en la nube se deriva de las ecuaciones de transferencia y calentamiento radiativos independientes del tiempo.

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