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ICE PARTICLES IN CIRCULAR ORBITS AROUND THE SUN

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RESUMEN

Hemos obtenido curvas teóricas de evolución del radio de partículas esféricas compuestas de hielo de agua que describen órbitas circulares o elípticas alrededor del Sol. Estas curvas fueron integradas numéricamente. La eficiencia de la observación se obtuvo usando la teoría exacta de Mie (1908), mientras que la temperatura se calculó usando la ecuación de balance energético. Nuestro cálculo es válido para partículas esféricas de radio entre 0.2 y 100μ a distancias del Sol entre 0.25 y 10 UA. Hemos encontrado un error en un cálculo similar, realizado por Patashnick y Rupprecht (1975); para partículas mayores que 10μ, encontramos temperaturas de equilibrio que son mucho menores, y por lo tanto tiempos de vida mucho más largos que los suyos.

ABSTRACT

We have obtained theoretical curves for the evolution of the radius of spherical particles composed of water ice, following circular or elliptical orbits around the Sun. These curves were integrated numerically. The absorption efficiency was obtained using the exact Mie (1908) theory, while the temperature was calculated using the equation of energy balance. Our computations are valid for spherical particles of radius from 0.2 to 100 μ, and at distances to the Sun between 0.25 and 10 AU. We have found an error in a similar calculation by Patashnick and Rupprecht (1975); for particles larger than 10 μ, we find equilibrium temperatures which are much lower, and thus lifetimes which are much larger than theirs.

Key words: COMETS – SOLAR SYSTEM-GENERAL

I. INTRODUCTION

We shall calculate the history of water ice particles in ircular orbits around the Sun. Water ice was chosen ecause apparently comets are made of a conglomerate f ices, with water in larger proportion (Whipple 1950).)elsemme and Wenger (1970) have proposed that a halo f water ice particles exists around some comets. These articles are carried out to the cometary coma by the ublimating gas from the nucleus; the halo can reach up o several thousand kilometers. Particles absorb and catter solar energy, thus contributing to the total ıminosity of the comet, and affecting it greatly. On the ther hand, the halo partly shields the nucleus from the olar radiation, and thus decreases the nuclear temperaure, which in turn, reduces the number of particles that re fed into the halo. In order to model this complex ehaviour of the halo, it is necessary to know the history f the individual particles as a function of time.

In this work we will consider spherical particles, since he solution of Maxwell's equations for particles of an rbitrary shape is very complicated. Greenberg and Shah 1971) have shown that for spheroidal particles of small ize, the exact shape does not affect the particle emperature greatly.

Patashnick and Rupprecht (1975) have calculated the lifetime of a water ice particle in circular orbit around the Sun. We tried to reproduce their calculations for any value of the parameters involved. We also tried to extend their results to elliptical orbits.

In reproducing their calculations, we found that they had made an invalid approximation, which substantially affects their results.

II. THEORY

The solar energy absorbed by the particle is partly emitted and partly used in sublimation. Both quantities depend on the size of the particle, and since this changes due to evaporation, the problem is a function of time. The absorbed power depends on the distance to the Sun, while the power dissipated by sublimation is a function of vapour pressure. Thus the temperature and the sublimation rate depend on particle size, index of refraction solar distance, and vapour pressure.

a) *Absorbed Power*

For a spherical particle of radius r and complex index of refraction $m(\lambda) = n_1 - i\,n_2$, at a distance R from the Sun, the absorbed power is given by

$$P_{Abs}(R,r,m) = 2.02709 \times 10^6$$

$$\times \frac{A}{R^2} \int_0^\infty \frac{Q_{Abs}(r,m,\lambda)\, d\lambda}{\lambda^5\, [\exp(2.4/\lambda) - 1]} \quad , \qquad (1)$$

where R is in AU, λ in μ and P_{Abs} in erg s^{-1}. A is the surface area of the particle, and the solar emission has been approximated by a black body spectrum at 5850°. $Q_{Abs}(r,m,\lambda)$ was obtained exactly from the equations of Mie (Van de Hulst 1957) when

$$\frac{2\pi r}{\lambda}\, \mathrm{Im}(m) < 100 \quad .$$

Otherwise, we used Van de Hulst approximation (Plass 1966).

The index of refraction was taken from Bertie *et al.* (1969) for wavelengths from 1.25 to 333 μ. For wavelengths between 0.2 and 1.25 μ we used the complex index of refraction of water given by Irvine and Pollack (1968), which served as an approximation.

b) *Emitted Power*

For a temperature T, the emitted power P_{em} is given by

$$P_{em}(r,T) = -A\, 3.7469 \times 10^{11}$$

$$\times \int_0^\infty \frac{Q_{Abs}(r,m,\lambda)\, d\lambda}{\lambda^5\, [\exp(14394/\lambda T) - 1]} \quad , \qquad (2)$$

where P_{em} is given in units of erg s^{-1}, λ is given in μ, and T in °K.

c) *Power Dissipated by Sublimation*

Given the composition of the particle and the temperature, the power dissipated by sublimation is

$$P_s = -A\, H\, \phi(T) \quad , \qquad (3)$$

where H is the latent heat of sublimation and ϕ (T) is the sublimation rate. Since H depends only slightly on temperature, we have taken it as constant. When the temperature of the particle is the same as the temperature of the sublimating vapour, the sublimation rate in gr cm^{-2} s^{-1} is given by

$$\phi(T) = P(T)\, [M / 2\pi R_g T]^{1/2} \quad , \qquad (4)$$

where M is the molecular weight and R_g is the gas constant. The vapour pressure P was calculated using an equation given by Jancso *et al.* (1970). Finally

$$P_s(T) = -6.66774 \times 10^9 \ A\, \frac{P(T)}{T^{1/2}} \quad , \qquad (5)$$

d) *Energetic Balance*

To obtain the equation of conservation of energy, we assume that no energy is stored in the particle. So we require

$$P_{Abs}(R,r) + P_{em}(r,T) + P_s(T) = 0 \quad . \qquad (6)$$

The function T(R,r) which satisfies this equation, represents the equilibrium temperature. We assume that the heating of the particle is isotropic, and that a state of equilibrium is reached rapidly. Once derived the equilibrium temperature, we can find the sublimation rate

$$\phi = 1.494 \times 10^{-11} \ P_s(T) / A \quad . \qquad (7)$$

e) *Decrease of Particle Size as a Function of Time*

Assuming the evaporation takes place uniformly over the whole surface of the particle, we have

$$dm/dt = 4\pi r^2\, \rho\ dr/dt \quad , \qquad (8)$$

and

$$dm/dt = 4\pi r^2\ \phi(T) \quad , \qquad (9)$$

where dm/dt is the mass loss rate, and ρ is the density. We obtain from these equations

$$\rho\, \frac{dr(R,r)}{dt} = \phi(R,r) \quad . \qquad (10)$$

For a circular orbit equation (10) can be integrated to give the life-time of the particle, τ (r)

$$\tau(r) = \int_r^0 \phi(r)\, dr \quad . \qquad (11)$$

For an elliptical orbit, equation (10) can then be written as

$$\phi(R,r) = \rho \frac{dr}{dR} \left[\frac{dR}{dt}\right]$$

$$= \rho \frac{dr}{dR} V_{rad}(R,e,a) \quad , \qquad (12)$$

where $V_{rad}(R,e,a)$ is the radial velocity of the particle, at a distance R from the Sun, in an elliptical orbit of excentricity e, and semimajor axis a. Our results for elliptical orbits will be presented elsewhere.

III. RESULTS

In Figure 1 we present the temperature changes for particles between 0.1 and 50μ, at different distances from the Sun. The temperature obtained depends strongly on the size of the particle and on its solar distance, and decreases rapidly for particles larger than 1μ.

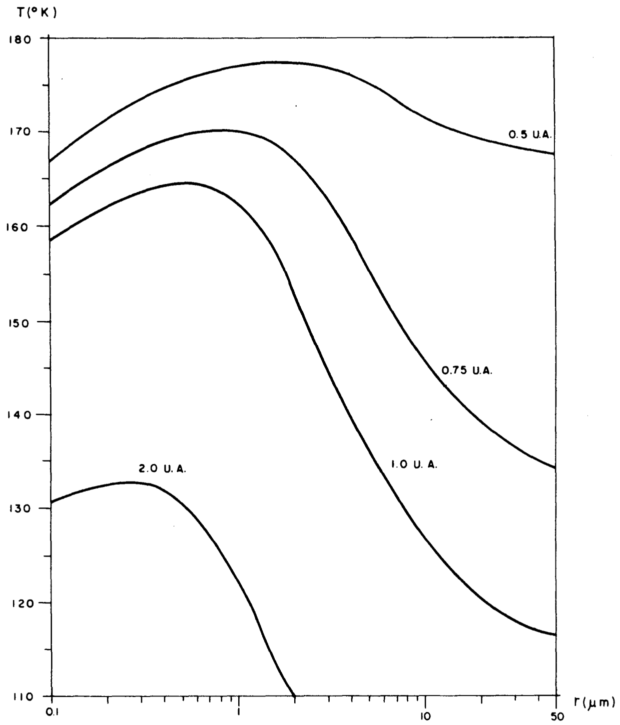

Fig. 1. Temperature as function of particle size for circular orbits at 0.5, 0.75, 1 and 2 AU.

In Figure 2 we show the sublimation rate as a function of particle size. Our computations are shown in solid line, we used Mie calculations. The dashed lines show the results by Patashnick and Rupprecht. There is a strong discrepancy between our results and theirs, for particles larger than 10 μ. They obtain a minimum of the

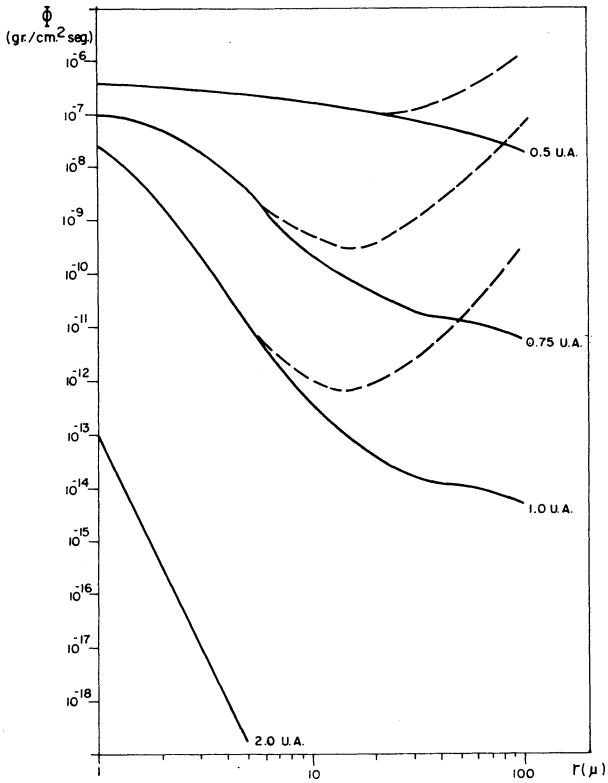

Fig. 2. Sublimation rate as function of particle size for circular orbits at 0.5, 0.75, 1 and 2 AU. Dashed lines are the results by Patashnick and Rupprecht. Discrepancy is discussed in text.

sublimation rate that we cannot reproduce. In trying to understand how their minimum arised, we found that they had used the Rayleigh approximation.

$$Q_{Abs} = -4\,x\,\mathrm{Im}\left[\frac{m^2-1}{m^2+2}\right] \quad , \qquad (13)$$

where $x = 2\pi r/\lambda$. This approximation only holds when

$$x \ll 1 \text{ and } m\,x \ll 1 \quad . \qquad (14)$$

If the Rayleigh approximation is used, we find that our results are not changed substantially, except for $n_2 x < 0.3$, when the minimum they found is reproduced exactly. This is incorrect, and thus their minimum does not have physical significance. The theoretical curves continue to fall as a function of r, beyond 10 μ.

We show in Figure 3 the change in temperature as a function of R, for particles of different sizes. In this figure circles are the results by Lamy (1974). The agreement is excellent.

In Figure 4 the lifetime of the particle is presented as a function of r, for several distances to the Sun. Notice

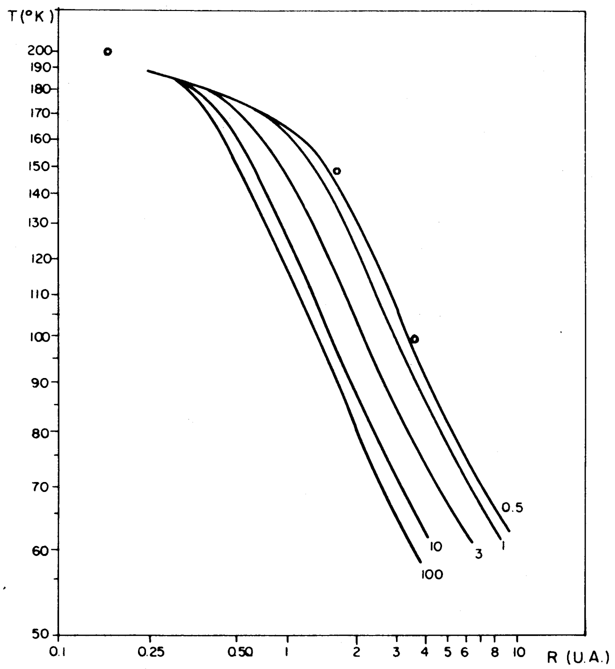

Fig. 3. Particle temperature as function of solar distance for particles of radius 0.5, 1.0, 3, 10 and 100 μ. Circles are the results by Lamy for grains of radius 0.5 μ. Agreement is excellent.

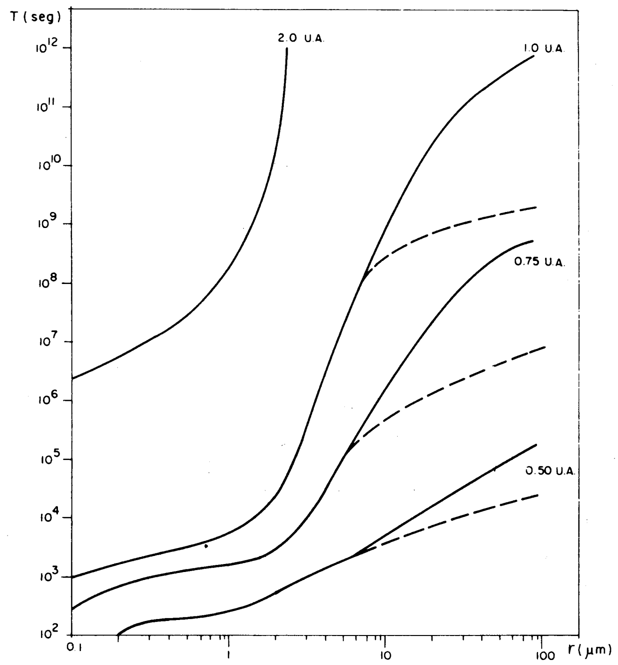

Fig. 4. Lifetime of ice particles in circular orbits at 0.5, 0.75, 1 and 2 AU. Dashed lines are the results by Patashnick and Rupprecht.

the large discrepancy with Patashnick and Rupprecht. For particles larger than 10 μ, we find substantially larger lifetimes.

We plan to incorporate these results into a model of a comet which includes a halo of ice particles. This model will serve to compare with observations and to predict the behaviour of the comet along its orbit. A more detailed treatment of the above problems is given by Parravano (1980).

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DISCUSSION

Sofía: Un posible modelo de un cometa es el de la bola de nieve sucia. ¿Hasta qué punto deben los resultados de hielo puro ser modificados para representar realistamente ese modelo?
Ferrín: Hay que calcular la eficiencia de absorción para la mezcla y dependiendo de la distribución espectral del índice de refracción complejo, la tasa de sublimación será alterada, lo cual también producirá cambios en los resultados.

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