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A SIMPLE METHOD OF PHOTOGRAPHIC PHOTOMETRY DATA REDUCTION

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RESUMEN

En este artículo se presenta una variante de los métodos estándar de reducción de medidas fotométricas fotográficas. Se combinan el ajuste mínimo cuadrático, rápido y compacto, mediante polinomios ortogonales y el análisis de varianza, para obtener curvas de calibración libres de efectos de magnitud y color. Se incluye, asímismo, un ejemplo de aplicación a cuatro placas *R* (Sistema RGU de Becker) del cúmulo abierto IC 2581.

ABSTRACT

In this paper we present a variant of the standard methods in photographic photometry data reduction. The compact and fast least squares fitting with orthogonal polynomials has been combined with the analysis of variance, in order to obtain calibration curves corrected for magnitude and color effects. An example of application to four *R* plates (Becker's RGU system) is also included.

Key words: DATA REDUCTION – LEAST SQUARES – PHOTOGRAPHIC PHOTOMETRY

I. INTRODUCTION

The problem of data reduction in photographic photometry is old, and various techniques have been proposed in the past. The procedure we describe here, which has already been tested, shows some advantages which can improve the quality of the results, concerning specially the statistical treatment of data.

The first point to consider in reducing photographic measurements when a photoelectric sequence is available in the field under study, as in our case, is the derivation of calibration curves. These curves relate photometer readings (iris readings in our case) and magnitudes of the stars of the photoelectric sequence. The next step consists in analyzing the residuals of the previous calibration to test magnitude and color effects, in order to correct them if they are significant. In what follows we will discuss both topics.

II. DESCRIPTION OF THE METHOD

a) *Calibration Curves*

The relation between iris readings and magnitudes is obtained either graphically or by least squares numerical methods. The latter procedure is used most frequently given its greater accuracy and the easy access to fast and precise computation tools (Burkhead and Seeds 1971; Butler 1972). The available standard sequences show frequently the following deficiencies:

a) The number of standard stars in the sequences is relatively small.

b) The limiting magnitude of the standard sequences is lower than the limiting magnitude accessible with photographic emulsions, i.e., it does not reach a sufficiently faint magnitude as would be desirable in stellar statistics studies based on three color photometry.

c) Generally the stars of the sequences are not equally distributed within each magnitude interval.

The solution of these problems is purely observational, so we are forced to a certain extent to use methods of analysis capable of extracting more meaningful information from the available data. Taking all these into account we have resolved to use orthogonal polynomials for fitting calibration curves and apply an analysis of variance to choose the best adjustment and test magnitude and color effects.

The polynomial equation is

$$y = \sum_{j=0}^{m} A_j \cdot P_j(x) \qquad (1)$$

where $P_j(x)$ are polynomials of degree j satisfying

$$\sum_{i=1}^{n} p_j(x_i) \cdot P_k(x_i) = \begin{cases} 0, \text{if } j \neq k \\ 1, \text{if } j = k \end{cases}$$

and n is the number of points to be fitted. The coefficients A_j represent the average values –average slope, average curvature, etc.– of the data (they are referred to the "center of gravity" of the data-bulk). These coefficients are independent of: a) the choice of the coordinate system, b) the degree of fitted polynomials, and represent physical characteristics of the data. This is not the case in normal polynomial adjustment where all information is referred to one point which may not be representative of the data (Bevington 1969).

On the other hand, as orthogonal polynomials are independent of one another and, at the same time, the sum of the squared residuals is very easy to compute using

$$\sum_{i=1}^{n} [y_i - \sum_{j=0}^{m} A_j \cdot P_j(x_i)]^2 = \sum_{i=1}^{n} y_i^2 - \sum_{j=0}^{m} A_j^2 , \quad (2)$$

an analysis of variance can be carried out immediately among the orthogonal terms to check by means of an F-test (3) if the contribution of each one is significant or not. In the last case

$$F^m_{calc.} = \frac{A_m^2 \cdot (n - m - 1)}{\sum_{i=1}^{n} y_i^2 - \sum_{j=0}^{m} A_j^2} \quad (3)$$

The non-significant terms can be discarded without recalculating all previously obtained coefficients. These F-tests are isolated as in easier cases of analysis of variance (Crow *et al.* 1960).

Two different algorithms have been tried to calculate the polynomials. One of them based on Crout's matrix factorization (Acton 1966) and the other as described by Peterson (1979), which is specially suited for small computers and requiring single precision arithmetic only. The same results (polynomial coefficients and their errors) were reached with both methods; however the latter is of easier application entailing a considerable economy of time and memory. Therefore, we decided to adopt it.

In stellar statistical procedures (namely three color photometry of open clusters and galactic star fields) four or more plates of each color are needed to control the effects of systematic errors and to maintain them between reasonable limits (Becker 1972). In addition to that, the calculation of a calibration curve for each plate allows a better control of individual errors (Tammann 1963).

b) *Analysis of calibration residuals*

From a physical point of view, a plot of iris readings versus photoelectric magnitudes gives only an approximation of the calibration curve. A detailed analysis of residuals may show a significant dependence on magnitude and color. Once magnitude and color equations, if any, are determined, the photoelectric magnitudes can be transferred to the photographic system and a new calibration curve may be drawn. If a further analysis of residuals reveals no more magnitude or color dependent terms this curve is taken as the final relation between iris readings and magnitudes; otherwise we proceed iteratively (Stock and Williams 1969).

Normally it is not found necessary to allow for quadratic color dependent terms, therefore we have adopted a linear model for these corrections. For each given color there are p X q residuals $(m_{pg} - m_{pe})$, where p is the number of stars in the standard sequence and q is the number of plates measured in the chosen color. We begin then the analysis of variance with the following model (Acton 1966):

$$r_{ij} = (m_{pg} - m_{pe})_{ij} = \mu + \zeta_i + \eta_j + \epsilon_{ij} \quad (4)$$

where we are assuming that each residual is made up of at least four parts:

μ = a mean value common to all the residuals.
ζ_i = a value common to every residual (of the same star) in all the plates.
η_j = a value common to all the residuals in the same plate.
ϵ_{ij} = a random perturbation applied to every residual with zero mean and unknown variance σ^2.

Later on we introduce explicitely a straight line (or lines) in the model to account for magnitude and color effects. The resulting table of variances informs us about the degree of significance of these plural effects allowing us, at the same time, to consider qualitatively and quantitatively similarities and discrepancies among the plates of the same color. This consideration is rather important for the later analysis of field errors.

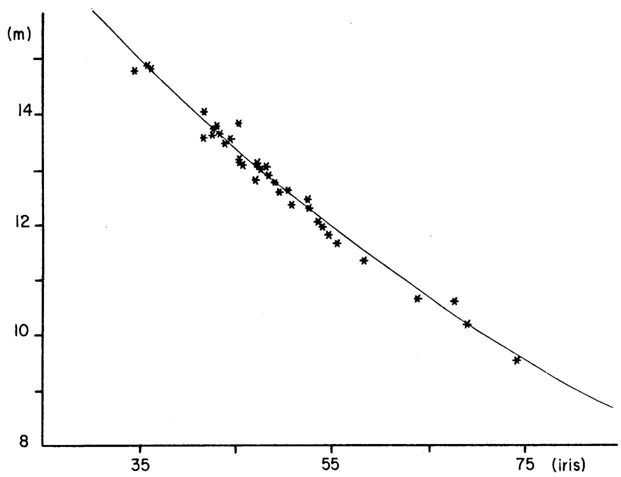

Fig. 1. Calibration curve for the first plate.

III. EXAMPLE OF APPLICATION

The method outlined previously has been programed in BASIC language for a 9845 A Hewlett-Packard mini-computer in our Institute. Input values (iris readings, photoelectric magnitudes and color indices) are stored on magnetic tape. The interactive working mode of the computer allows us to choose the most convenient procedures to follow during execution. The programs are available on request.

As a practical application we give in what follows the analysis of four R plates (Becker's RGU system). The photoelectric sequence (open cluster IC 2581, Evans 1969) shows the difficulties mentioned above. (Figure 1). The characteristics of the photographic material are shown in Table 1, while the photoelectric magnitudes, color indices and iris readings are presented in Tables 2 and 3.

The limiting degree of the polynomials is ten. The

TABLE 1

PHOTOGRAPHIC MATERIAL USED IN THIS DISCUSSION

Plate No.	Emulsion	Filter	Seeing	t_{exp}	Quality	Remarks
(R1) RCS 11	103a – E	RG 1	good	15^m	good	elong. image
(R2) RCS 10	"	"	"	"	"	" "
(R3) RCS 07	"	"	medium	"	medium	very elong. image
(R4) RCS 08	"	"	"	"	"	elong. image

TABLE 2

STANDARD PHOTOELECTRIC SEQUENCE

No.	No. (Evans)	V	$B-V$	$U-B$	$E(B-V)$	G	$G-R$	$U-G$
1	4	9.60	0.20	−0.62	0.43	9.79	0.69	0.63
2	5	9.77	−0.03	−0.23	0.13	9.75	0.33	1.00
3	6	11.06	0.19	−0.58	0.43	11.24	0.68	0.67
4	11	11.95	0.06	−0.44	0.13	12.03	0.45	0.78
5	13	12.29	0.17	0.10	0.13	12.43	0.55	1.37
6	16	10.89	1.15	0.84	0.13	11.66	1.58	2.46
7	20	11.78	0.24	−0.38	0.43	11.99	0.73	0.89
8	21	12.97	0.27	0.17	0.13	13.19	0.66	1.46
9	25	12.26	0.23	−0.41	0.43	12.46	0.72	0.86
10	26	13.28	0.23	−0.18	0.43	13.47	0.72	1.10
11	33	13.55	0.21	0.03	0.13	13.73	0.60	1.30
12	36	12.82	0.24	−0.31	0.43	13.02	0.72	0.97
13	37	13.57	0.29	−0.14	0.43	13.41	0.78	1.15
14	42	13.22	0.23	−0.34	0.43	13.42	0.72	0.93
15	59	13.56	0.37	0.14	0.43	13.85	0.86	1.46
16	68	12.96	0.86	0.43	0.13	13.71	1.33	1.78
17	74	12.35	0.06	−0.05	0.13	12.39	0.42	1.21
18	100	13.39	0.81	0.27	0.13	14.11	1.28	1.60
19	101	13.09	0.72	0.20	0.13	13.73	1.18	1.52
20	103	14.18	0.89	0.45	0.13	14.96	1.36	1.80
21	105	13.67	0.97	0.80	0.13	14.49	1.44	2.19
22	107	13.63	0.32	−0.18	0.43	13.90	0.82	1.11
23	108	14.09	0.34	0.00	0.43	14.36	0.83	1.31
24	109	14.05	0.41	0.23	0.13	14.40	0.82	1.53
25	111	14.45	1.62	1.23	0.13	15.62	2.10	2.92
26	114	14.91	0.69	0.13	0.13	15.53	1.15	1.44
27	115	14.22	0.68	0.09	0.13	14.83	1.13	1.40
28	116	15.25	0.52	0.19	0.13	15.70	0.94	1.50
29	117	14.33	0.45	0.06	0.43	14.70	0.95	1.38
30	118	15.35	0.77	0.34	0.13	16.02	1.22	1.68
31	119	13.44	0.53	0.02	0.13	13.92	0.97	1.31
32	120	14.94	1.60	1.71	0.13	16.06	2.06	3.43
33	121	13.98	0.35	0.07	0.43	14.26	0.84	1.38
34	123	15.48	0.59	0.46	0.43	15.95	1.10	1.82
35	128	14.85	2.21	1.62	0.13	16.54	2.76	3.38
36	129	13.06	1.18	0.93	0.13	13.85	1.61	2.83
37	130	13.93	1.43	1.16	0.13	14.93	1.88	2.83
38	131	11.41	1.49	1.49	0.13	12.44	1.94	3.19

TABLE 3

IRIS READINGS OF STANDARD STARS IN *R* FILTER[a]

Star No.	*R1*	*R2*	*R3*	*R4*
1	81.7	81.8	83.0	79.2
2	74.6	75.6	77.2	73.7
3	64.3	65.2	68.1	63.8
4	56.0	58.3	60.3	57.4
5	54.5	55.6	57.2	54.9
6	69.4	70.4	73.2	68.3
7	58.8	60.1	62.8	58.9
8	50.2	51.5	52.9	50.7
9	55.2	55.6	58.3	54.6
10	47.7	48.9	49.9	48.1
11	46.0	47.1	47.6	46.4
12	51.4	51.8	53.6	51.0
13	46.4	46.9	48.0	46.0
14	49.6	50.7	51.6	49.4
15	48.7	49.5	50.4	49.1
16	53.1	54.2	56.1	53.6
17	54.1	55.0	56.3	53.7
18	49.0	49.9	50.6	49.2
19	51.0	51.9	52.9	50.3
20	44.0	44.1	45.8	44.6
21	47.8	48.3	50.0	48.3
22	46.0	46.2	47.7	46.0
23	42.3	43.1	43.5	42.9
24	43.2	43.8	44.6	43.6
25	45.2	46.1	47.2	45.8
26	38.7	39.7	--	39.0
27	43.3	44.1	44.8	43.1
28	35.1	35.0	34.3	35.6
29	43.6	44.0	44.4	42.9
30	36.8	37.0	36.2	36.9
31	48.0	49.0	50.1	47.9
32	42.4	42.6	42.1	42.2
33	44.5	45.3	44.9	43.8
34	36.5	36.5	34.9	35.9
35	46.0	46.1	48.0	45.6
36	53.2	54.1	56.2	53.4
37	48.0	48.9	50.0	48.1
38	68.1	68.6	71.0	66.3

a. Number identification as in Table 2.

computer program finds the best polynomial fitting through an F-test (95% confidence limit) up to the maximum degree to be considered (Table 4 and Figure 1). Photographic magnitudes of standard stars calculated with this polynomial are stored and the residuals (m_{pg} – m_{pe}) are obtained for each plate. With these a first evaluation of data variability is carried out displaying a bifactorial ANOVA (ANalysis Of Variance) table (rows = stars, columns = plates) where SS = sum of the corresponding squared residuals, DF = degrees of freedom, and MS = variances or mean squared residuals (Table 5). It can be seen immediately that the most important contribution to the total SS comes from the rows. To extract the information contained in them we introduce the dependence of the residuals upon color and magnitude, adopting, as we said above, a linear model. The results of the magnitude equation can be seen in Table 6 and the corresponding ANOVA in Table 7. It is noted that the magnitude equation does not significantly contribute to the total SS, being "Other row effects" the most relevant term. However, when analyzing the color equation (Tables 8 and 9, Figure 2) we see that the average regression accounts for almost 2/3 of the row-variability; an F-test applied to these values is quite significant. In the same ANOVA table the term "Dif. between slopes" does not pass an F-test, indicating a slight additional contribution to the total variability.

TABLE 4

BEST FITTED POLYNOMIAL FOR *R1* PLATE

Polynomial degree: 2

P(0) =	2.19952 E + 01	(± 7.8 E – 01)
P(1) =	–2.26493 E – 01	(± 2.9 E – 02)
P(2) =	7.85339 E – 04	(± 2.7 E – 04)

Variance of fitting = 2.95 E – 02
F (calculated) = 8.573
Degrees of freedom = 34

TABLE 5

VARIABILITY OF THE RESIDUALS[a]
ANOVA – 1 (Data variability) (0)

	SS	DF	MS
Files (stars)	3.3722	35	0.0963
Columns (Pl.)	0.0000	3	0.0000
Interaction	0.2162	105	0.0021
Total	3.5884	143	

a. Linear model not yet introduced.

According to the results obtained above the photoelectric magnitudes of the standard stars were corrected for color with an equation of the form

$$m^*_{pg} = m_{pe} + a\,C_{pe} + b. \qquad (5)$$

With the magnitude obtained in this way new calibration curves were computed (Figure 3) and the whole process

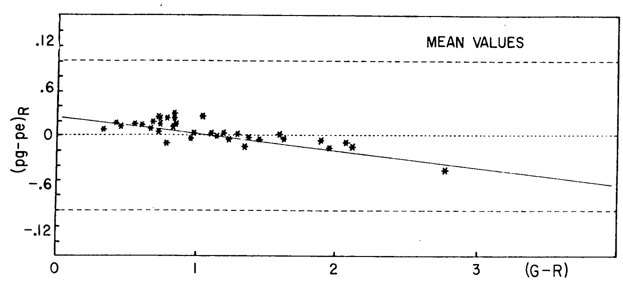

Fig. 2. Color equation for the average of the four *R* plates.

TABLE 6

LINEAR MAGNITUDE EQUATIONS FOR THE FOUR *R* PLATES AND THE AVERAGE VALUES
Average + individual regressions (magnitude equation) – (0)

	Y	b	X	SSReg[a]	SSR[b]
Mean values	0.000	–0.016	12.718	0.0145	0.8286
Plate nr. 1	–0.001	–0.018	”	0.0178	0.9613
Plate nr. 2	0.001	–0.015	”	0.0127	0.8397
Plate nr. 3	0.000	–0.017	”	0.0159	0.9184
Plate nr. 4	0.000	–0.015	”	0.0118	0.8108

a. Sum of squared residuals accounted for the regression.
b. Sum of squared residuals after regression.

TABLE 7

VARIABILITY OF THE RESIDUALS[a]
ANOVA – 2 (magnitude equation) (0)

	SS	DF	MS
Dif. between means (columns)	0.0000	3	0.0000
Average regression	0.0579	1	0.0579
Dif. between slopes	0.0004	3	0.0001
Other row effects	3.3143	34	0.0975
Residual variability	0.2158	102	0.0021
Total	3.5884	143	

a. Introducing magnitude equations.

repeated. The results are shown in Tables 10 and 11, where it can be seen that a new linear correction is not necessary.

The term "Other row effects" is what Acton (1966) designates as the "common snakiness", which measures how much the average residuals oscillate around their common (average) fitted line; "Residual variability" measures the degree of dispersion of individual residuals around their respective regression lines.

The program enables us to analyze also the effects of several pooled plates in the residual variability. In this way we have at our disposal an operative index about differences and similitudes among plates. This is also important for the subsequent analysis of the possible field errors, which would be the next step in usual photographic photometry data reduction (Table 12).

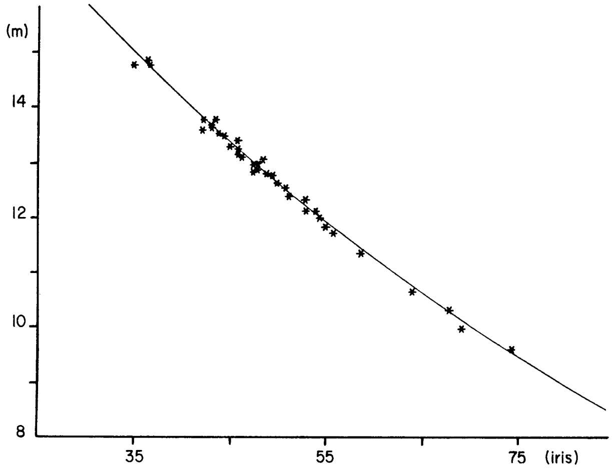

Fig. 3. Calibration curve for the first plate corrected of color equations.

TABLE 9

VARIABILITY OF THE RESIDUALS[a]
ANOVA –2 (color equation) (0)

	SS	DF	MS
Dif. between means (columns)	0.0000	3	0.0000
Average regression	2.1388	1	2.1388
Dif. between slopes	0.0133	3	0.0044
Other row effects	1.2334	34	0.0363
Residual variability	0.2029	102	0.0020
Total	3.5884	143	

a. Introducing color equations.

TABLE 8

LINEAR COLOR EQUATIONS FOR THE FOUR *R* PLATES AND THE AVERAGE VALUES
Average + individual regressions (color equation) – (0)

	Y	b	X	SSReg	SSR
Mean values	0.000	–0.226	1.086	0.5347	0.3083
Plate nr. 1	–0.001	–0.244	”	0.6184	0.3606
Plate nr. 2	0.001	–0.207	”	0.4472	0.4052
Plate nr. 3	0.000	–0.245	”	0.6259	0.3084
Plate nr. 4	0.000	–0.210	”	0.4605	0.3621

TABLE 10

VARIABILITY OF THE RESIDUALS OBTAINED FROM THE NEW CALIBRATION CURVES[a]
ANOVA – 2 (magnitude equation) (1)

	SS	DF	MS
Dif. between means (columns)	0.0000	3	0.0000
Average regression	0.0087	1	0.0087
Dif. between slopes	0.0001	3	0.0000
Other row effects	1.1831	34	0.0348
Residual variability	0.2104	102	0.0021
Total	1.4023	143	

a. Introducing magnitude equations.

TABLE 11

VARIABILITY OF THE RESIDUALS OBTAINED FROM THE NEW CALIBRATION CURVES[a]
ANOVA – 2 (color equation) (1)

	SS	DF	MS
Dif. between means (columns)	0.0000	3	0.0000
Average regression	0.0019	1	0.0019
Dif. between slopes	0.0140	3	0.0047
Other row effects	1.1899	34	0.0350
Residual variability	0.1965	102	0.0019
Total	1.4023	143	

a. Introducing color equations.

TABLE 12

VARIABILITY OF THE RESIDUALS FOR TWO GROUPS OF PLATES: (*R1, R3*) AND (*R2, R4*)
ANOVA – 3 (color equation) (1)

Variability ascribable to	SS	DF	MS
Means:			
Dif. between (1,3) & (2.4)	0.0000	1	0.0000
Dif. (+) between (1) & (3)	0.0000	1	0.0000
Dif. (+) between (2) & (4)	0.0000	1	0.0000
Slopes:			
Common for all plates	0.0019	1	0.0019
Sep. for (1,3) & (2,4)	0.0138	1	0.0138
Sep. for (1) & (3)	0.0000	1	0.0000
Sep. for (2) & (4)	0.0001	1	0.0001
Residuals:			
Common snakiness about Y	1.1899	34	0.0350
Sep. lin. (1,3) & (2,4)	0.0607	34	0.0018
Sep. lin. (1) & (3)	0.0681	34	0.0020
Sep. lin. (2) & (4)	0.0677	34	0.0020
Total	1.4023	143	

IV. CONCLUSIONS

The method above described allows: 1) To derive in a systematic way calibration curves free of magnitude and color effects; 2) the control of partial results at each stage testing them statistically.

The point of view kept in mind along the whole reduction procedure is that any successful analysis of data demands a careful choice of the underlying mathematical model, a point often forgotten in other classical reduction methods of photographic photometry. Our proposed mathematical model is shown to work well, and it is quite adequate for photographic photometry data reduction as the authors have widely confirmed in two other papers (in preparation) on: *RGU* photometry of a star field in Carina and *RGU* photometry of the open cluster NGC 2141.

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