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ON THE EVOLUTION OF Hα/N II IN SUPERNOVA REMNANTS

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RESUMEN

Se ha observado que en remanentes de supernova el cociente de líneas Hα/N II es función de la velocidad de expansión. Este efecto es explicado suponiendo que la abundancia de nitrógeno con respecto a hidrógeno en la masa eyectada por la supernova, o por la actividad de la estrella precursora, es mayor que la abundancia que caracteriza al medio circundante. De este modo el cociente de líneas irá aumentando a medida que la abundancia relativa de nitrógeno evoluciona hacia su valor cósmico como resultado de la acreción de material interestelar que va realizando la onda de choque. El comportamiento de Hα/N II como función del radio del remanente y como función del cociente de líneas λλ6717/6731 de [S II] puede también ser explicado mediante este modelo.

ABSTRACT

The line ratio Hα/N II in supernova remnants has been observed to evolve as a function of the expansion velocity. This is explained by assuming that the abundance of nitrogen with respect to hydrogen in the mass ejected by the supernova explosion or, prior to it, by the presupernova star, is higher than the average ambient value. The line ratio will increase as the relative abundance of nitrogen evolves towards the normal cosmic value as a result of accretion of interstellar material by the shock wave. The behavior of Hα/N II as a function of the radius of the supernova remnant and as a function of the [S II] line ratio λλ6717/6731 can also be explained with this model.

Key words: **NEBULAE-SUPERNOVA REMNANTS**

I. INTRODUCTION

Daltabuit, D'Odorico, and Sabbadin (1976) discovered empirically that in most supernova remnants (SNRs) the line ratio I(Hα)/I([N II] = 6548 + 6584 A), hereafter Hα/N II, seems to be correlated with the expansion velocity (as determined from optical observations), the radius and the S^+ line ratio λλ6717/6731. The only optical galactic remnants in which these correlations are not found, are Cas A (Chevalier and Kirshner 1978) and MSH 11-54 (Goss *et al.* 1979), where both N and H are depleted, and Tycho (Chevalier and Raymond 1978) and SN1006 (Lasker 1981), where the optical emission is produced by the interaction of a shock wave with a partially neutral medium. But in all the other SNRs Hα/N II is contained within the interval [0.1, 2.5] (Sabbadin and D'Odorico 1976) and it is in these "normal" objects where the correlations mentioned above are found, so that we will concentrate our attention on them.

The correlations indicate that a large value of Hα/N II occurs in those objects where the radius is large, the expansion velocity small and the S^+ line ratio λλ6717/6731 corresponds to low electron densities. However, some doubts have been cast on the reality of these correlations specifically the one involving the radius, due to the existence of a nitrogen abundance gradient in several spiral galaxies, among them our own (Hawley 1978; Peimbert 1979), M 31 (Blair, Kirshner, and Chevalier 1982) and M 33 (Dopita, D'Odorico, and Benvenuti 1980). The existence of this gradient (with N/H decreasing as the galactocentric distance increases) has been verified through the observation and modelling of H II regions and large SNRs, where any contribution to the chemical composition by the ejected material is expected to be negligible. In the case of M 31 both approaches lead to very similar results (Blair *et al.* 1982), whereas no correlation is found between the remnant's diameter and Hα/N II, which is not surprising since the observed objects are sufficiently large so as to be dominated (at least from the chemical point of view) by the surrounding medium. Yet, it is possible to argue that these correlations are the result of an observational selection effect combined with this undisputed gradient. For instance, such an effect might be that most large low surface brightness SNRs are observed in the anticenter direction, leading in this way to a predominance of large values of Hα/N II in large objects. Though this effect would account for the scarcity of large remnants with small values of Hα/N II, it would still have to explain why no small objects show large values of Hα/N II, which is the main qualitative aspect of these correlations. In any case, the diameter of most SNRs, specially those from our own galaxy, is a poorly known quantity, so that a relative large dispersion can be expected at least from this circumstance. Theoretically, the radius of a remnant depends strongly and simultaneously on such parameters as the interstellar density, the energy and the ejected mass, leading in such a way to an increased dispersion. It is then no wonder that the most disputed correlation is the one involving the radius, though almost as many problems can be found in relation to the expansion velocity, which has been estimated in only a few objects. This is not so with the S^+ line ratio $\lambda\lambda 6717/6731$, which can be determined more easily and with less observational uncertainties. This is reflected in what seems to be a better defined relation (see Figure 4), so that this quantity should provide a better criterium to test the reality of the correlations. In any case, in order to discover if there is an element of truth in them, it is necessary to make abundance gradient corrections for those galaxies where the existence of this gradient is certain. This has been done in this paper and no substantial change has been found to occur in the proposed correlations.

In order to explain the observed behaviour of Hα/N II Dopita (1977*b*) assumed that nitrogen is being enhanced through the destruction of grains by the shock wave. Faster shock waves should be more effective in stripping nitrogen atoms from the surface of grains, increasing the abundance of this element in the gaseous phase and thereby decreasing the value of the line ratio. Several sublimation models were considered, but the results were not particularly encouraging. Since nitrogen is one of the less depleted elements (Spitzer and Jenkins 1975), this is hardly surpising.

In this paper, the theoretical basis for the observed relationships is examined further. It is found that the observations can be fitted only by involving a higher than normal abundance of nitrogen in the SNRs. The source of nitrogen overabundance is assumed to be the supernova (SN) explosion itself. As the remnant ploughs into the interstellar medium, the enriched ejected mass is diluted into it and the relative abundance of nitrogen with respect to hydrogen (N/H) evolves from its value in the supernova ejecta towards the normal cosmic value.

An overabundance of nitrogen in the mass ejected by supernovae has been expected for some time (Shklovsky 1968), and detailed theories of late stages of stellar nucleosynthesis and SN explosions show that nitrogen can be enhanced through the CNO cycle (Lamb, Iben, and Howard 1976; Lamb 1978). Furthermore, spectroscopic observations of Type I SN indicate that N/H is above the cosmic value in these objects (Mustel 1974; Bychkova and Bychkov 1977). Similarly, N/H is at least 10 times larger in Puppis A than in the Sun, and it is thought that the origin of this overabundance is a nitrogen rich ejected mass (Dopita, Mathewson, and Ford 1977). On the other hand, N/H is relatively normal in all the other remnants where it has been possible to determine its value. This is not surprising in old objects, where the ejected mass is very diluted. But in the Crab, the only young SNR where the relative abundance of these two elements has been estimated, N/H seems to be very close to the solar value (Miller 1978; Davidson 1979). Thus, there is evidence indicating that nitrogen is overabundant with respect to hydrogen in at least one type of supernovae and in some but not all, SNRs.

To calculate the line ratio we will assume that the only energy source is collisional excitation at the shock front. The line ratio is calculated by including emission from the cooling region behind the shock front and from the medium surrounding the remnant, which from now on will be known as the pre-ionized region (see Figure 1). We will also assume that hydrogen is fully ionized in this region. Shull and McKee (1979) have shown that a shock wave expanding with a velocity larger than 100 km s^{-1} is capable of keeping hydrogen (in the surrounding medium) fully ionized. Since the recombination timescale of hydrogen is $10^5/N_e$ yr, it follows that a remnant expanding with a smaller shock velocity may still be encountering a fully ionized interstellar medium. At large velocities there may be some problems to ionize the surrounding medium, since the cooling function can be considerably smaller. It can be shown that the shock front will not over-run the ionization front as long as the shock velocity is approximately smaller than 900 km s^{-1}. In younger remnants the medium may already be ionized through the activity of the pre-supernova star (an O7 star will ionize a sphere of 35 pc in 2×10^6 yr; Osterbrock 1974) or by the ionizing radiation produced at the time of the SN explosion. Yet, there seems to be at least two young remnants, Tycho and SN1006, where

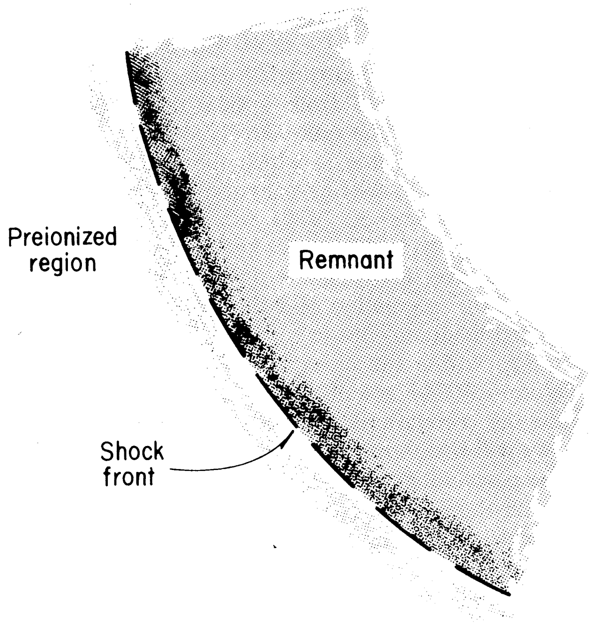

Fig. 1. A scheme showing the geometrical distribution of the two emitting regions (the preionized region and the remnant) which are separated by the shock front.

this has not happened (Chevalier and Raymond 1978). Thus, it seems reasonable to assume that hydrogen is fully ionized in the medium surrounding those remnants where the shock velocity is smaller than 900 km s^{-1} and larger than 70-80 km s^{-1}. In faster remnants the medium will be ionized only if the pre-supernova star was of a convenient type or if the explosion produced sufficient ionizing radiation.

This simple emission model is obviously not intended as an alternative to the substantially more sophisticated works of Dopita (1977*a*), Raymond (1979) or Shull and McKee (1979). These models refer to a stationary situation, whereas our purpose is to describe in a simple manner an evolutionary one, namely, the evolution of Hα/N II. Such a description is beyond the possibilities of more detailed formulations. Yet, it is worth mentioning that the intensity of Hα obtained in this work is, within a factor of 2, very similar to the theoretical results of the above mentioned shock wave emission models and to observations of Cas A and IC443.

II. THE MODEL

a) *Hα Emission*

In the pre-ionized region the temperature ($\sim 10^4$ K) is sufficiently low for collisional excitation to be neglected, so that the Hα line is predominantly formed by recombination. Hence,

$$I(H\alpha)_a = \int_{V_a} N(H^+)\, N_e\, \alpha_{32}\, h\nu_\alpha\, dV \quad \text{erg s}^{-1}, \qquad (1)$$

where $N(H^+)$ and N_e are the proton and electron number densities in the pre-ionized region, α_{32} is the effective recombination coefficient of Hα and V_a is the volume of the pre-ionized region. If the hydrogen number density and temperature are uniform,

$$I(H\alpha)_a = N(H^+)\, N_e\, \alpha_{32}\, h\nu_\alpha\, V_a \quad . \qquad (2)$$

The ionizing radiation is generated by those atoms excited by the shock wave as they recede towards the cold recombined region, and these particles are produced at a rate given by

$$4\pi R_0^2\, N_0\, V_s \quad , \qquad (3)$$

where N_0 is the atom number density in the pre-ionized region, R_0 is the radius of the remnant and V_s is the shock wave velocity.

Let each excited atom produce S_{UV} ionizing photons as it moves from the shock front to the cold recombined region. Some of these photons, P_{UV}, will be sent ahead of the shock front to ionize the surrounding medium. The total number of photons for this region is then equal to

$$\phi = 4\pi R_0^2\, N_0\, V_s\, P_{UV} \;\; s^{-1} \quad . \qquad (4)$$

This is the production rate of ionizing photons for a shell that was absorbed by the shock front when it had a radius R_0, a velocity V_s, and a cooling function leading to a particular value of P_{UV}. These parameters do not remain constant, and it is necessary to add the contribution of a sequence of shells with different initial conditions to obtain the total emission of ionizing photons. If the cooling timescale for any shell is shorter than the evolutionary timescale for the shock wave, i.e., if each shell cools down to the recombination temperature before the shock wave preceding it changes its velocity and radius appreciably, then the initial conditions in the shells that make up the cooling region of the SNR are uniform and parameters such as ϕ, P_{uv} and others that will be defined below, will be the same throughout the cooling region. We assume that this is the case, and we will show later on that this assumption has no major consequences on the line ratio.

The other source of ionization in the pre-ionized region will be the internally produced Lyman continuum photons. We assume that these photons are absorbed within the region. Since the volume of the pre-ionized region is such that the total number of ionizing photons emitted by the source is equal to the total number of recombinations to the excited states, we have that

$$\phi = N(H^+)\, N_e\, \alpha_B\, V_a \quad , \qquad (5)$$

where α_B is the recombination coefficient to all levels of hydrogen except the ground level. Substituting equations (4) and (5) into equation (2) we have that the emission of Hα in the pre-ionized region is

$$I(H\alpha)_a = 4\pi R_0^2\, N_0\, V_s\, h\nu_\alpha\, \alpha_{32}/\alpha_B \quad . \qquad (6)$$

According to Mallik (1974) $\alpha_{32} = 1.17 \times 10^{-13}\, T_4^{-0.95}$ cm^3 s^{-1}, where T_4 is the temperature in units of 10^4 K. From Osterbrock (1974) $\alpha_B = 2.6 \times 10^{-13}$ cm^3 s^{-1} for $T_4 = 1$, which will be the temperature that we will assume for the pre-ionized region. The value of P_{UV} will be given below.

Let us now calculate the emission of Hα produced by the remnant. If, as we assume, the main bulk of hydrogen ionization occurs in the pre-ionized region, the intensity of the Hα line is proportional to the absorption rate of ionized hydrogen by the shock wave, given by

$$4\pi R_0^2 N(H^+) V_s \quad , \tag{7}$$

where $N(H^+)$ is the number density of the absorbed protons. If it takes a time δt for hydrogen to move from the post-shock zone to the region where it is recombined, and since the H lines in the remnant are essentially formed by recombination, we have that the intensity of the line will be given by

$$I(H\alpha)_b = 4\pi R_0^2 N(H^+) V_s h\nu_\alpha N_e \alpha_{32} \delta t \quad , \tag{8}$$

with α_{32} already defined above. Let H_{rec} be the average number of recombinations each H^+ atom undergoes during the time δt. If so, it is given by

$$H_{rec} = N_e \alpha_A \delta t \quad , \tag{9}$$

where α_A is the total recombination coefficient of hydrogen. On the other hand we have that for the material being absorbed by the shock wave $N_0 \simeq N(H^+)$, so that

$$I(H\alpha)_b = 4\pi R_0^2 N_0 V_s h\nu_\alpha \quad \alpha_{32}/\alpha_A H_{rec} \quad . \tag{10}$$

Since the shell does not evolve at constant temperature, but it cools down, we have to take the mean value of α_{32} and α_A. Most of the emission is produced at temperatures smaller than 2×10^4 K and since the two recombination coefficients appear as a ratio, the mean value is hardly sensitive to the temperature. We will take this temperature to be $T_4 = 1$. At this temperature $\alpha_A = 4.18 \times 10^{-13}$ $cm^3 s^{-1}$ (Osterbrock 1974). The value of H_{rec} will be given below.

b) *The Emission of N II*

These two nitrogen lines, 6548 and 6584 A, are formed by the transitions $^1D_2 \rightarrow {}^3P_1, {}^3P_2$ of singly ionized nitrogen. As long as the density is not too large the ^1D level is populated basically through electron collisions with level ^3P, so that the intensity of the sum of the two lines in the pre-ionized region is,

$$I(N\,II)_a = q_{14} (P_{43} h\nu_{43} + P_{42} h\nu_{42}) N(N^+) N_e V_a \quad . \tag{11}$$

Where q_{14} is the collisional excitation rate for the transition $^3P \rightarrow {}^1D$, P_{43} and P_{42} are the branching ratios for the radiative decay into the lines at 6584 and 6548 A, $h\nu_{43}$ and $h\nu_{42}$ the energy of these lines, and $N(N^+)$ the number density of singly ionized nitrogen in the emitting region. We are assuming that all of N^+ is in the ^3P state. Let $Q = P_{43} h\nu_{43} + P_{42} h\nu_{42}$ and rewrite equation (11) in the following form,

$$I(N\,II)_a = Q\, q_{14} \frac{N(N^+)}{N(N)} \frac{N(N)}{N(H^+)} N(N^+) N_e V_a \quad . \tag{12}$$

Since most of hydrogen in the pre-ionized region is ionized, we have that $N(N)/N(H^+)$ is practically equal to the interstellar abundance of nitrogen relative to hydrogen. Let A_{IS} be this number, and let f_a be the fraction of singly ionized nitrogen with respect to the total number of nitrogen atoms in the pre-ionized region. The product $N(H^+) N_e V_a$ is already given in equation (5), so that

$$I(N\,II)_a = 4\pi R_0^2 N_0 V_s Q \frac{q_{14}}{\alpha_B} P_{uv} f_a A_{IS} \quad . \tag{13}$$

From Seaton (1975), with $\gamma = 2.7$, we have that

$$q_{14} = 2.59 \times 10^{-8} T_4^{-0.5} \exp\text{-}2.2/T_4 \ cm^3 s^{-1} .$$

On the other hand $P_{43} = 0.75$ and $P_{42} = 0.25$ (Allen 1973).

To calculate the N II emission from the remnant we will consider separately the two sources of excitation: collisions at the shock front and photoionization followed by recombination. The collisional contribution is given by

$$\delta I(N\,II)_b = q_{14}\, Q \frac{N(N^+)}{N(N)} \frac{N(N)}{N(H^+)} N(H^+) N_e 4\pi R_0^2 \delta X \quad , \tag{14}$$

where δX is the region where singly ionized nitrogen is present, so that if we assume that in this region there is no other nitrogen ion we have that $N(N^+)/N(N) = 1$.

Since the ionization potential of hydrogen is smaller than that of nitrogen, we expect the former to be fully ionized in this region. Consequently, we have that $N(N)/N(H^+) \simeq N(N)/N(H)$, which is the relative abundance of nitrogen with respect to hydrogen in the optical region of the SNR, and will be written as A_{SNR}. Assume that $N(H^+)N_e \simeq N^2$, where N is the particle density in this region. Since $N_0/N << 1$, it follows from momentum conservation that

$$N \simeq \frac{16}{3} \frac{N_0 T_s}{T} \quad , \tag{15}$$

where T_s is the post-shock temperature and T is the one in the N^+ region, which is bounded by the temperature at which nitrogen is doubly (T_1) and singly ionized (T_0). If within this region there is a relation $X = G(T)$ between the distance, X, behind the shock front and the temperature at that point, the total emission is then given by

$$I(\mathrm{N\,II})_b = Q \left(\frac{16\, N_0\, T_s}{3}\right)^2 4\pi R_0^2 A_{SNR} \times \int_{T_1}^{T_0} \frac{q_{14}}{T^2} \frac{dG(T)}{dT}\, dT \quad . \qquad (16)$$

The form of this function will be given below.

To calculate the emission of the nitrogen lines formed by photoionization we must follow the same procedure as with nitrogen emission in the pre-ionized region. Three differences have to be considered: the fraction of singly ionized nitrogen can be different, and will be given in this case by f_b, the number of ionizing photons is $S_{uv} - P_{uv}$, and the relative abundance of nitrogen is that of the remnant, A_{SNR}. Taking these into account we have that

$$I(\mathrm{N\,II})_c = 4\pi R^2_0\, N_0\, V_s\, Q \frac{q_{14}}{\alpha_B} (S_{uv} - P_{uv}) \times f_b A_{SNR} \quad . \qquad (17)$$

The ratio q_{14}/α_B will be calculated for a temperature of 10^4 K.

c) *The Line Ratio*

From equations (6), (10), (13), (16) and (17), we find that the ratio is given by

$$\frac{\mathrm{H}\alpha}{\mathrm{N\,II}} = \left\{ h\nu_\alpha \left(\frac{\alpha_{32}}{\alpha_B} + \frac{\alpha_{32}}{\alpha_A} \frac{H_{rec}}{P_{uv}} \right) \right\} Q\, f_a\, A_{IS} \left[\frac{q_{14}}{\alpha_B} + \frac{A_{SNR}}{f_a A_{IS}} \frac{N_0}{V_s} \left(\frac{16\, T_S}{3}\right)^2 \frac{1}{P_{UV}} \int_{T_1}^{T_0} \frac{q_{14}}{T^2} \frac{dG(T)}{dT}\, dT + \frac{q_{14}}{\alpha_B} \frac{f_b A_{SNR}}{f_a A_{IS}} \left[(S_{uv} - P_{uv})/P_{uv} \right] \right] \quad . \qquad (18)$$

Hence, when the line ratio is considered, the set of functions describing some of the aspects of the physical behavior of the cooling region are only included dividing each other. Regardless of the physical state of each shell in the remnant we expect that the ratio H_{rec}/P_{uv} remains roughly constant throughout it, since H_{rec} represents all hydrogen recombinations and P_{uv} includes Lyman continuum photons as an obviously important part. Furthermore, for a remnant where hydrogen is predominant, we expect this ratio to be close to unity. The ratio of the inward to the outward going ionizing photons, $(S_{uv} - P_{uv})/P_{uv}$, will also be a quantity close to unity in any shell. Finally, it is immediately apparent that the integral term is proportional to the number of photons produced in the region of the remnant where nitrogen is fully ionized. Again, regardless of the physical structure of the cooling region, we expect that the ratio of the number of these photons to P_{uv} will remain essentially constant. All these implies that even if the intensity of each line is a function of the model used to describe the cooling region behind the shock front, the Hα/N II line ratio will be relatively independent of it and it shows that the assumption made above on the uniformity of the initial conditions in the shells that make up the cooling region, has a minor effect on the line ratio. It is also clear that, as long as there is enough energy to ionize nitrogen and hydrogen, the evolution of the line ratio is mainly determined by the evolution of the relative abundance of nitrogen to hydrogen in the SNR. This has already been pointed out by Dopita (1977*a*), who showed that, over a wide velocity range, Hα/N II has a very slight velocity dependence (though he underestimates the ratio at low velocities; Shull and McKee 1979) and that the strength of the [N II] lines is directly dependent on the abundance of nitrogen.

Since the line ratio is relatively independent of the model used for the cooling region, we will use the results of Cox (1972) for the set of functions defined in the previous paragraphs. He obtained the following:

$$P_{uv} = 1.7\,(V_7{}^2 - 0.41)$$
$$S_{uv} = 4.0\,(V_7{}^2 - 0.35)$$
$$H_{rec} = 2.7\,(V_7{}^2 + 0.1)$$
$$G(T) = X_c[\,1 - (T/T_S)^2\,]$$
$$X_c = 4 \times 10^{16}\, V_7/N_0 \quad , \qquad (19)$$

where V_7 is the shock wave velocity in units of 100 km s^{-1}. These coefficients are valid as long as $V_7 > 0.65$. For smaller velocities there is not enough energy to ionize nitrogen or the surrounding medium. Let us now proceed to calculate the integral term in equation (18) by substituting the expression for G(T) given in equation (19). We find,

$$\frac{N_0}{V_s}\left(\frac{16\, T_s}{3}\right)^2 \int_{T_1}^{T_0} \frac{q_{14}}{T^2} \frac{dG(T)}{dT}\, dT = 2.3 \times 10^{11} \int_{T_0}^{T_1} \frac{q_{14}}{T}\, dT \quad . \qquad (20)$$

In equilibrium, nitrogen is ionized at $T_0 \simeq 1.5 \times 10^4$ K and doubly ionized at $T_1 \simeq 5 \times 10^4$ K (Cox and Tucker 1969; Shapiro and Moore 1976). But non-equilibrium shock wave calculations indicate that elements tend to be over-ionized in the proximity of the cold recombined region. This is also apparent from the observations of IC443 (Fesen and Kirshner 1980), which indicate that the mean temperature of the N^+ region is approximately equal to 10^4 K. Since the same effect will occur with N^{+2}, and since the largest fraction of the emission will arise from the densest region, we will assume that $T_0 = 10^4$ K and $T_1 = 3.5 \times 10^4$ K. In any case a realistic variation of the limiting temperatures indicates that this term will not change by a factor larger than 1.5 due to this assumption. With the value of q_{14} given in Section II*b* we find that the integral is given in terms of two incomplete gamma functions. With the expressions for P_{uv} and H_{rec}, the values of the integral, and of the atomic coefficients (at $T = 10^4$ K), the following equation is found for the line ratio:

$$\frac{H\alpha}{N\,II} = \frac{0.81 \times 10^{-4}}{f\,A_{IS}} \left(V_7^2 - 0.15\right) \Big/ \left[V_7^2 - 0.41 + 1.35\,\frac{A_{SNR}}{A_{IS}}\left(V_7^2 - 0.31 + 0.07/f\right)\right] . \quad (21)$$

The fraction of singly ionized nitrogen in the pre-ionized region is a function of the shock wave velocity, as has been shown by Shull and McKee (1979), who found that $f_a \gtrsim 0.5$ as long as $V_7 \in [0.85, 1.05]$ and zero elsewhere (the interval where $f_a \simeq 1$ is broadened by taking into account the recombination timescale of N^+ and N^{+2}). Similar calculations are not available for the fraction of singly ionized nitrogen in the remnant (f_b). Opting for the simplest possible analytical expression, and since in any case we would have to deal with P_b as a free parameter, we decided to assume that $f_a = f_b = f$ and to take this parameter as a constant throughout the evolution of the remnant. The possible values of f will be adjusted by comparing our model with the results obtained by the more sophisticated models and with the observations.

We must now provide a relationship between A_{SNR}, the abundance of nitrogen in the optical region, and any one of the variables characterizing the remnant. This is a difficult problem, since our knowledge on the interaction between the ejected mass and the surrounding medium is still very limited. The interaction is idealized either through the introduction of a surface of discontinuity separating both components or through the elimination of any distinction between them. The initial step towards the solution of this question cannot be other but the theoretical discovery of realistic transport coefficients in a plasma, since strong shock waves show how inadequate are the classical transport coefficients that are so frequently used in astrophysics. But the innumerable difficulties faced by collisionless theories of strong shocks seem to indicate that the solution of this problem is still not in sight. Therefore we are forced to use an idealized hypothetical scenario in order to describe the behavior of A_{SNR}.

Let A_{SN} be the relative abundance of nitrogen to hydrogen in the mass ejected (M_e) by the SN explosion. If the ejected mass is not strongly decelerated during the initial stages of the process, so that it is permanently trailing behind the shock front, then the mean value of the abundance of nitrogen in the remnant is roughly equal to

$$\langle A_{SNR} \rangle \simeq \frac{A_{SN}\, M_e + A_{IS}\, \langle \rho \rangle\, V_0}{M_e + \langle \rho \rangle\, V_0} , \quad (22)$$

where $\langle \rho \rangle$ is the mean mass density of the interstellar material swept up by the remnant and V_0 is the volume defined by the shock wave. This equation will not hold if the models elaborated by Gull (1973*a*, 1973*b*, 1975) and Chevalier (1975) are valid, since in these cases a substantial fraction of the ejected mass is left behind, in the vicinity of the SN explosion. This led Chevalier to predict that there is a diffuse X-ray emission characterized by heavy element overabundance in the central region of old remnants. Soft X-ray observations of Cygnus gave no support to this idea (Rappaport *et al.* 1979). Furthermore, there are at least two remnants, Puppis A and W50, where it seems that the ejected mass has not been left behind (Dopita *et al.* 1977; Murdin and Clark 1980). Finally, the evolutionary trend of Hα/N II also suggests that the presence of the ejected mass can be detected beyond the expectations of the theoretical works of Gull and Chevalier. Thus, there are some observational arguments supporting the assumption behind equation (22) which seems to be a plausible approximation in a considerable number of cases.

The relative abundance of nitrogen to hydrogen in the optical region (A_{SNR}) will be somewhat different to the mean value given in equation (22). Nevertheless, we will assume that the ejected mass and the swept up gas are uniformly mixed, so that $A_{SNR} = \langle A_{SNR} \rangle$. In young SNRs the optical emission is localized in regions where a sizable proportion of the ejected mass is concentrated with a high density. Thus, the optical regions of young remnants are characterized by a high concentration of ejected gas, so that $A_{SNR} \sim A_{SN}$. But the radii of these objects hardly ever exceed 3 parsec, so that the swept up mass is never larger than 3 $M_\odot$ (for $\langle \rho \rangle \simeq 2 \times 10^{-24}$ gr cm^{-3}) which is roughly equal to the mass ejected by a Type II SN and maybe 5 times as large as the mass ejected by a Type I SN (Chevalier 1977). Thus, as long as $A_{SN} \lesssim 20\, A_{IS}$, we find that $\langle A_{SNR} \rangle \simeq 1/4\, A_{SN}$ at the very least. Consequently, in young remnants we may underestimate A_{SNR} by a factor not larger than 4, but usually by less than a factor of 2. Conversely, in older objects the brightest optical regions (and the first to be formed) are those where the shock has encountered a diffuse cloud in the interstellar medium, so that the optical region is characterized by an interstellar density that is larger than the mean value, implying that in this case $A_{SNR} < \langle A_{SNR} \rangle$. If there are 8×10^4 diffuse clouds per cubic kiloparsec (Spitzer 1978), and if these objects have the same spatial distribution as SNRs, then it follows that, on the average, a remnant will encounter a diffuse cloud when its radius is at least 15 pc. At this point the remnant has swept up at least 350 $M_\odot$ of interstellar gas (if $\langle \rho \rangle = 2 \times 10^{-24}$ gr cm^{-3}), so that $\langle A_{SNR} \rangle / A_{IS} \simeq 4/3$ at the very most. Since the minimum value of A_{SNR} is A_{IS}, it follows that in older objects we may overestimate A_{SNR} by a factor of 4/3 if we assume uniform mixing. Consequently, it seems likely that within a factor of 2, and with the exception of a few objects $A_{SNR} \sim \langle A_{SNR} \rangle$.

Thus, defining $Z = \langle \rho \rangle V_0 / M_e$, we find that the line ratio is given by

$$\frac{H\alpha}{N\,II} = \frac{0.81 \times 10^{-4}}{f\,A_{IS}} \left[V_7^2 - 0.15 \right] \Big/ \left\{ V_7^2 - 0.41 + 1.35 \left[\frac{Z + A_{SN}/A_{IS}}{Z+1} \right] \times \left(V_7^2 - 0.31 + 0.07/f \right) \right\} . \tag{23}$$

The variable Z stands for the cube of the radius of the shock wave, so that we now need a relation between the radius and the velocity of a SNR. The evolution of a remnant has been described qualitatively by Woltjer (1972). As long as $M_e >> \langle \rho \rangle V_0 (Z << 1)$ the SNR expands with constant velocity and most of its energy is kinetic. When $\langle \rho \rangle V_0 >> M_e (Z >> 1)$ and radiative losses are not important, its kinetic energy is constant and equal to 28% of the total energy (Sedov 1959). Radiative losses become important when the shock velocity is smaller than 150–200 km s^{-1}, and in this case the evolution of the remnant is described better by the equation of momentum conservation. Finally, when the expansion velocity equals the sound speed, the SNR disappears as such and is dissolved into the interstellar medium. We will assume that the evolution of a remnant with no radiative losses can be described by

$$1/2\,(M_e + \langle \rho \rangle V_0/\alpha)\,U^2 = E \ , \tag{24}$$

where U is the material expansion velocity, E is the energy of the remnant and α is a constant that should be equal to 0.28 if equation (24) is to fit Sedov's phase. We will assume that equation (24) also describes those remnants where radiative losses are important (V < 150 km s^{-1}), bearing in mind that in this case the expansion velocity is overestimated. Immediately after the gas crosses the shock front its expansion velocity is equal to $(\gamma - 1)\,V/(\gamma + 1)$. As it moves away from the shock front its expansion velocity approaches V, which is the isothermal limit. Since the optical expansion velocity is close to this limit, we will assume that U = V, so that

$$V_{07}^2 = 280 \ \frac{E_{50}}{M_e/M_0} \left(\frac{1}{Z + 0.28} \right) \ , \tag{25}$$

where E_{50} is the energy of the SNR in units of 10^{50} erg.

Notice that, in general, this shock velocity is larger than the one appearing in equation (23). The latter refers to the shock preceding the optical regions of SNRs and it is rarely larger than 500 km s^{-1}. On the other hand, equation (25) defines the mean shock velocity of the remnant, which can be as large as 10000 km s^{-1}. But we will assume that as long as the mean shock velocity is large so is the velocity of the shock preceding the optical region. That is, we will assume that as long as $V_{07} >> 1$ so is V_7, which means that, in the context of equation (23), we can take $V_{07} \sim V_7$. This has no major effects on the line ratio and it can be done simply because quantities such as H_{rec}/P_{uv} are relatively model independent as long as $V_7 > 1$.

The energy and the ejected mass can be as low as 5×10^{49} erg and 0.3 $M_\odot$ (Utrobin 1978) or as large as 10^{51} erg and 5 $M_\odot$ (Chevalier 1977) so that, in both cases, we approximately have

$$V_7^2 = 500/(Z + 0.28) \ . \tag{26}$$

III. DISCUSSION

Equations (23) and (26) determine the evolution of Hα/N II as a function of the expansion velocity. Alternatively, once the mean interstellar density and the ejected mass are known, Hα/N II can be calculated as a function of the radius. Finally, since the sulphur line ratio 6717/6731 is a known function of the expansion velocity (Daltabuit *et al.* 1976; Cantó 1977), Hα/N II can also be computed as a function of this quantity. From equation (23) we find that Hα/N II has the following qualitative behavior:

i) Hα/N II $\simeq$ constant as long as $Z << 1$. The level of the curve is determined by the values of f and A_{SN}/A_{IS}.

ii) When $Z \simeq 1$ the line ratio will increase only if $A_{SN}/A_{IS} > 1$.

iii) When $Z >> 1$ and $V_7 > 1$ the line ratio is constant again, but this time at a larger this time being determined by f.

iv) There will not be enough energy to ionize nitrogen when $V_7 \lesssim 1$, so that Hα/N II will now increase very rapidly.

Hence, the scale height of the curve is set both by f and A_{SN}/A_{IS}. But its peculiar shape is basically determined by the second quantity, and in order to explain the behavior of Hα/N II it is necessary to assume that $A_{SN}/A_{IS} > 1$, as will be evident from the Figures 2, 3 and 4.

According to Mustel (1974) the relative abundance of nitrogen to hydrogen in Type I SN is at least 10 times larger than the cosmic value. This is consistent with the models of Lamb (1978). On the other hand, N/H is approximately 10 times larger in Puppis a (a relatively evolved object) than in the interstellar medium (Dopita *et al.* 1977). Consequently we will assume that A_{SN}/A_{IS} ranges from 5 to 20.

The interstellar abundance of nitrogen with respect to hydrogen (A_{IS}) is assumed to be 10^{-4}.

The data plotted in Figures 2, 3 and 4 are given in Tables 1, 2 and 3. We decided to separate IC443 into a north and south component since these are expanding at different velocities (Lozinskaya 1979). The radii of the two components were calculated considering that, according to the model elaborated by the author, R(south) = 1.55. As for N135, Mathewson and Clarke (1973) provide some good evidence indicating that this object is composed of two SNRs. Dopita *et al.* (1977) do not point out which of the two was observed by them,

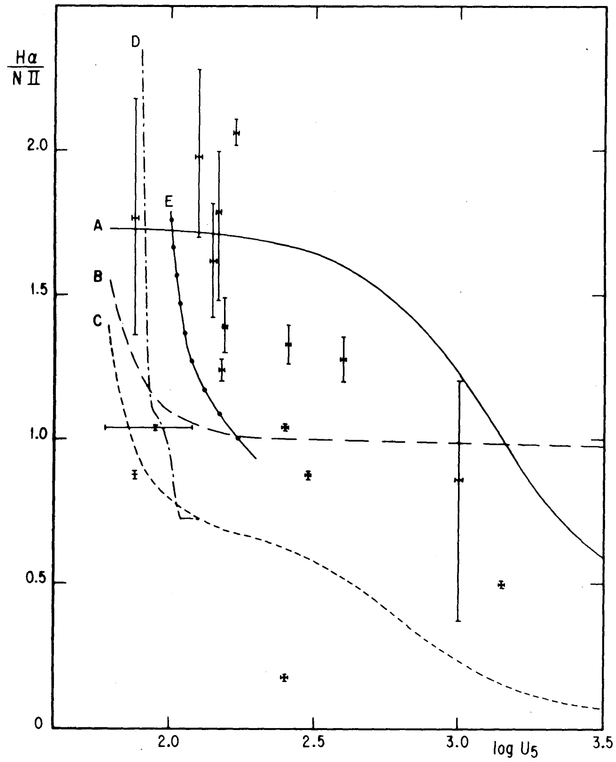

Fig. 2. The evolution of Hα/N II as a function of the optical expansion velocity, U_s, for three sets of parameters for (A_{SN}/A_{IS}, f): (5, 0.2) in curve A, (1, 0.35) in curve B and (20, 0.5) in curve C. Curve D corresponds to models C, D, E, F and G of Shull and McKee (1979) and curve E to models E, F, G, H and I of Raymond (1975).

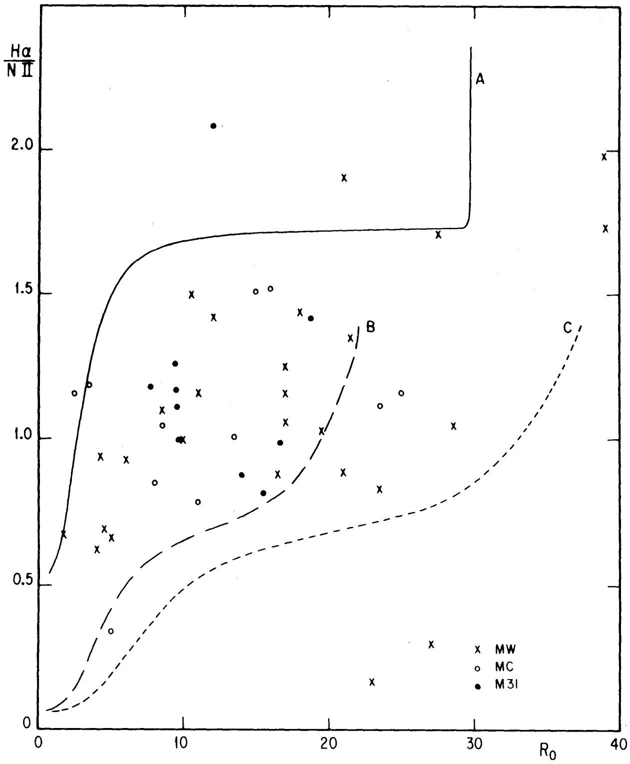

Fig. 3. The evolution of Hα/N II as a function of the radius for three sets of values for (A_{SN}/A_{IS}, f, $M_e/M_\odot$); curve A is for (5, 0.2, 1), curve B for (20, 0.5, 1) and curve C for (20, 0.5, 5). Crosses correspond to remnants from our galaxy, circles to those from the Magellanic Clouds, and dots to those of M 31.

but we decided to ascribe the northernmost position to N135A. The data that has been plotted in these figures has been corrected by taking into account the observed abundance gradient and the different mean nitrogen abundance occurring in different galaxies. The correction formula due to the abundance gradient is

$$\left(\frac{N}{H}\right)_D = -a(D - D_0) + 1 \quad , \qquad (27)$$

where D is the galactocentric distance at which the remnant is given in kpc, D_0 is the distance at which the abundance is normalized and *a* is the observed gradient. This formula is not the customary way in which abundance gradients are presented (log (N/H) vs. D/D_0). Yet, it is a better representation of the results of Blair *et al.* (1982) in M 31, who have measured this gradient over a fairly extended galactocentric distance range. Based on their work we took the following values for (*a*, D_0); (0.06,10) in our galaxy, (0.05,10) in M 31 and (0.03,2.5) in M 33. The corrected line ratio is then equal to (Hα/N II)/$(N/H)_D$. Another correction was applied by taking into account that nitrogen is 3.4 and 15.3 times less

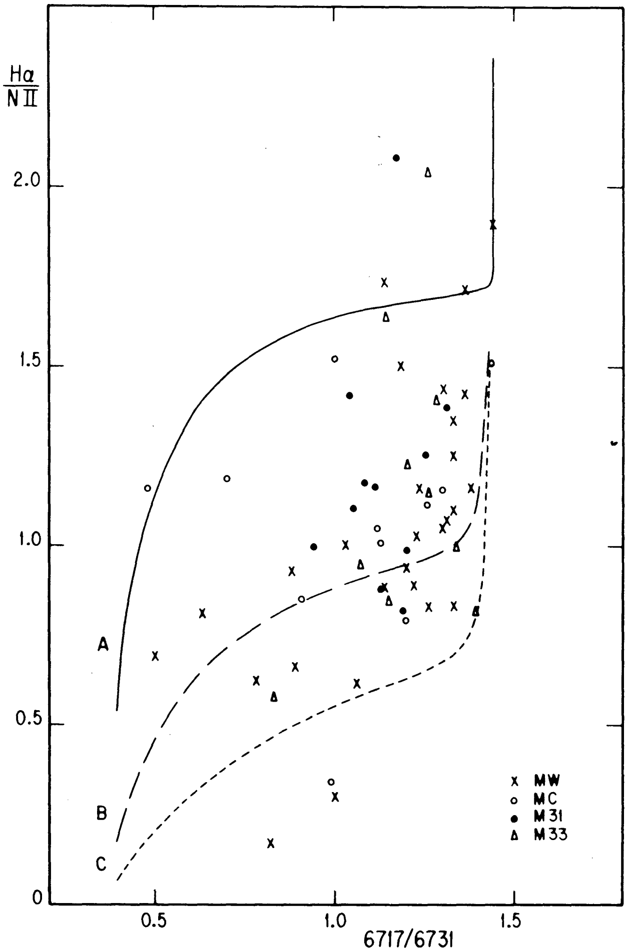

Fig. 4. The evolution of Hα/N II as a function of the S^+ line ratio λλ6717/6731 for three sets of values for (A_{SN}/A_{IS}, f): curve A is for (5, 0.2), curve B for (10, 0.35) and curve C for (20, 0.5). Crosses correspond to SNRs from our galaxy, circles to those from the Magellanic Clouds, dots to those of M 31 and triangles to those from M 33.

abundant in the Large and Small Magellanic Clouds (Peimbert and Torres-Peimbert 1974; 1976) twice less abundant in M 33 (Dopita *et al.* 1970) and 1.5 times more abundant in M 31 (Blair *et al.* 1982). The corrected line ratios are also shown in Tables 1, 2 and 3. As stated previously, it is evident from Figures 2, 3 and 4 that the correlations originally proposed by Daltabuit *et al.* (1976) are not greatly affected by the existence of a nitrogen abundance gradient. At this point it is worthwhile mentioning that these differences in pre-shock abundances will have no major effect in the structure of the cooling region in the SNR, for nitrogen is not one of the major cooling agents.

The intensity of Hα in the remnant, given by equation (10) and with H_{rec} (as given in equations 19), is probably the best suited to compare this model with the intensity predicted by other models and the observations. With the expansion velocity within the interval (60,200) km s^{-1} we found that this model leads to I(Hα) not smaller than 0.75 and not larger than 2.1 times the values found by Raymond (1979), and roughly twice as strong as those predicted by Shull and McKee (1979). On the other hand, using the cloud collision model (McKee and Cowie 1975), we find that in the quasi-stationary flocculi of Cas A the cloud density is roughly 650 cm^{-3} and $V_7 \simeq 2$. Assuming that the size of the emitting region is roughly equal to the size of the cooling region ($R_0 = V t_{cool}$; hence $R_0 \simeq 3 \times 10^{16}$ cm) we obtain I(Hα) $\simeq 2 \times 10^{33}$ erg s^{-1}; whereas McKee and Cowie report that in one of the brightest condensations I(Hα) $\simeq 1.2 \times 10^{33}$ erg s^{-1}. Finally, our theoretical line intensities are within 0.3 and 1.5 times the values observed by Fesen and Kirshner (1980) in several filaments of IC443. It is then evident that this approach leads to an Hα intensity which is not a factor of 2 larger than other theoretical predictions and well within the observational data.

In Figure 2, Hα/N II is plotted as a function of log U_5, where U_5 is the optical expansion velocity in km s^{-1}. In this figure we also plot the evolution of Hα/N II as predicted by Raymond (1979) and Shull and McKee The difference between these two models is partly due to different pre-shock conditions, especially on the abundance, and partly on the theoretical treatment of the problem. These models are used to estimate a reasonable set of values for f, since this parameter alone determines the scale height of the evolutionary curve predicted by our model once the effect of the ejected mass is minimal. We find that f has to be equal to 0.5 if our model is to be close to the results of Shull and McKee and equal to 0.35 if we are to compare it favorably with Raymond's predictions. Notice that, in the velocity range where any comparison is possible, all evolutionary curves have a very similar shape, the main difference being that in the more complex models the line ratio "bends" upwards at larger velocities. But Figure 2 also shows that both sophisticated approaches are unable to account for the rather large values of Hα/N II found in most low velocity remnants. If we are to include these objects within our framework we have to take f = 0.2. Thus, comparison with the observations and other theoretical models leads us to three values for the parameter f: 0.2, 0.35 and 0.5. Three sets of values have been used for A_{SN}/A_{IS} and f: (5, 0.2), (20, 0.5) and (1,0.35). The first two sets of values represent the upper and lower limits of the evolutionary curves of Hα/N II, and it can be seen that only Puppis A, RCW 103 and HB 3 lie definitely outside the region bounded by them, though OA184 might also be marginally out. In HB 3 and OA184 both Hα/N II and Hα/S II, where S II = I([S II] = 6717+6731 A), are exceptionally large, suggesting that there may be a contribution to the optical emission from a nearby H II region (as in DR 4, Baars, Dickel, and Wendker 1978).

TABLE 1

DATA FOR GALACTIC REMNANTS

	Velocity (km s^{-1})	Radius (pc)	D_{gal} (kpc)	Hα/N II Observed	Hα/N II Gradient Corrected	6717/6731	References
Kepler	1400	4.5*	3.6	0.5	0.69	0.5	van den Bergh *et al.* 1973
W28	75	10.0*	7.7	0.86-0.89	0.98-1.01	0.96-1.10	Lozinskaya 1980; Dopita *et al.* 1977
W50	>150	27.0	7.9+	0.17-0.36	0.21-0.40	1.0	Murdin and Clark 1980
3C400.2	...	4.3*	8.1	0.80-0.88	0.90-0.98	1.15-1.25	Sabbadin and D'Odorico 1976
G65.3+5.7	60-120	28.5*	9.6+	1.03-1.05	1.04-1.06	1.25-1.36	Rosado 1981; Sabbadin and D'Odorico 1976
CTB80	...	16.5	9.4+	0.85	0.88	1.14	Angerhofer *et al.* 1980
Cygnus	300	21.0*	9.8	0.86-0.89	0.87-0.90	1.14-1.31	Kirshner and Taylor 1976; Miller 1973
W63	...	18.0	9.9	1.29-1.57	1.30-1.58	1.25-1.35	Sabbadin 1976
CTB1	150	21.5*	12.8	1.42-1.82 <1.62>	1.15-1.55 <1.35>	1.30-1.44 <1.33>	Lozinskaya 1980; D'Odorico and Sabbadin 1977
CTA1	...	23.5	10.7	0.87	0.83	1.27	Fesen *et al.* 1981
G126.2+1.2	...	45.0	13.2+	1.03	0.83	1.33	Blair *et al.* 1980
3C58	<100	>10.0*	19.2	0.47-1.42 <1.07>	0.01-0.96 <0.61>	0.89-1.39 <1.06>	Kirshner and Fesen 1978
HB3	170	21.0*	11.4	2.02-2.11	1.85-1.93	1.43-1.44	Lozinskaya 1980; D'Odorico and Sabbadin 1977
HB9	260	17.0*	11.0	1.26-1.40	1.18-1.32	1.26-1.40	Lozinskaya 1980; D'Odorico and Sabbadin 1977
VRO4205.01	150	12.0*	13.5	1.48-2.00 <1.79>	1.11-1.63 <1.42>	1.30-1.45 <1.36>	Lozinskaya 1980; D'Odorico and Sabbadin 1977
OA184	125	39.0*	12.1	1.79-2.28 <1.98>	1.54-2.03 <1.73>	1.10-1.21 <1.14>	Lozinskaya 1980; D'Odorico and Sabbadin 1977
S147	150	17.0*	10.9	1.20-1.28	1.12-1.20	1.36-1.40	Lozinskaya 1980; D'Odorico and Sabbadin 1977
Crab	1000	1.7	13.4	0.37-1.20 <0.86>	0.18-1.01 <0.67>	0.63-0.89 <0.81>	Lozinskaya 1980; Parker 1967
IC443 (North)	150	11.0	12.8	1.30-1.49 <1.39>	1.06-1.25 <1.15>	1.10-1.38 <1.23>	Lozinskaya 1979; Fesen and Kirshner 1980
IC443 (South)	400	17.0	12.8	1.20-1.36	0.96-1.12	1.24-1.38	Lozinskaya 1979; Fesen and Kirshner 1980
Monoceros	75	27.5*	10.6	1.36-2.18	1.30-2.12	1.33-1.40	Lozinskaya 1979; D'Odorico and Sabbadin 1977
Puppis A	250	23.0*	10.7	0.18	0.17	0.82	Elliott 1978; Osterbrock and Costero 1973
Vela	250	19.5*	10.1	1.04	1.03	1.23	Lozinskaya 1975; Dopita *et al.* 1977
MSH 61-A	...	10.5	9.7	1.47	1.50	1.18	Elliott and Malin 1979
G296.1-0.7	...	8.5	9.8	1.09	1.10	1.33	Longmore *et al.* 1977
RCW86	...	6.0*	8.1	0.62-1.14 <0.84>	0.71-1.23 <0.93>	0.78-0.97 <0.88>	Ruíz 1981
RCW89	...	5.0*	7.2	0.52-0.66 <0.57>	0.61-0.75 <0.66>	0.85-0.96 <0.89>	Dopita *et al.* 1977
RCW103	70	4.0*	4.6	0.45	0.62	0.78	Westerlund and Mathewson 1966; Dopita *et al.* 1977

* The result of taking the mean angular diameter given by van den Bergh *et al.* (1973) - with the exception of Kepler, for which we used the radio diameter and distance to the object reported by Clark and Caswell (1976). The result of the Σ-d relation was only used whenever the object was not used as a distance calibrator. The distance to 3C400.2 was taken from Ilovaisky and Lequeux (1972). The reported radius of CTB80 corresponds to the radio object since the optical region seems excessively small. All the galactocentric distances, with the exception of those marked with a +, are from Clark and Caswell (1976).

TABLE 2

DATA FOR SNRs FROM THE MAGELLANIC CLOUDS

	Radius (pc)	Hα/N II Observed	Hα/N II Corrected*	λλ6717/6731	Reference
			LMC		
N11L	8.5	3.57	1.05	1.12	Dopita *et al.* 1977
N86	15.0	5.14	1.51	1.43	Dopita *et al.* 1980
N120	13.5	3.45	1.01	1.13	Dopita *et al.* 1977
(N49)	5.0	1.15	0.34	0.99	Same as above
N49	8.0	2.88	0.85	0.91	Osterbrock and Dufour 1973
N206	24.5	3.95	1.16	1.30	Dopita *et al.* 1980
N63A	3.5	4.03	1.19	0.70	Dopita *et al.* 1977
N135A	16.0	5.18	1.52	1.00	Same as above
N135B	23.5	3.82	1.12	1.26	Same as above
N103B	2.5	3.95	1.16	0.48	Same as above
			SMC		
N19	11.0	12.08	0.79	1.20	Dopita *et al.* 1977

* The correction factors applied to Hα/N II are 3.4 in the Large Magellanic Cloud and 15.3 in the Small Magellanic Cloud. The radii for LMC are from Mathewson and Clarke 1973; for SMC from Mathewson and Clarke 1972.

TABLE 3

DATA FOR SNRs FROM M 31 AND M 33

M31	Radius (pc)	D_{gal} (kpc)	Hα/N II Observed	Gradient Correction	Abundance Correction	6717/6731
BA 22	16.6	11.8	0.84	0.76	0.99	1.20
BA 23	14.0	5.0	0.54	0.68	0.88	1.13
BA 55	9.7	3.6	0.58	0.77	1.00	0.94
BA160	8.7	11.2	0.97	0.91	1.18	1.08
BA337	23.5	11.7	2.31	2.11	2.74	1.00
BA370	12.0	13.9	2.01	1.60	2.08	1.17
BA416	15.5	5.4	0.51	0.63	0.82	1.19
BA449	9.5	7.3	0.79	0.90	1.17	1.11
BA474	18.7	13.9	1.37	1.09	1.42	1.04
BA490	48.5	19.3	2.06	1.07	1.39	1.31
BA521	9.5	4.8	0.67	0.85	1.11	1.05
BA581	9.4	13.6	1.20	0.97	1.26	1.25
M33						
-2	3.5	4.2	4.30	4.08	2.04	1.26
-5	8.5	3.1	2.50	2.46	1.23	1.20
-6	4.5	2.0	1.87	1.90	0.95	1.07
-7	3.0	1.6	1.12	1.15	0.58	0.83
-8	5.5	0.8	1.62	1.70	0.85	1.15
-9	3.5	1.6	1.59	1.63	0.82	1.39
-11	3.5	1.3	1.92	1.99	1.00	1.34
-14	23.0	2.9	2.85	2.82	1.41	1.28
-16	3.0	3.0	3.32	3.27	1.64	1.14
-18	18.5	3.4	2.36	2.30	1.15	1.26

D gal stands for galactocentric distance. The gradient correction was done using equation (27), with the constants as given in the text. The abundance correction was realized to normalize all data to our own galaxy. The radii and galactocentric distances of the M 31 objects are from Blair *et al.* (1981) and the spectroscopic data from Blair *et al.* (1982). All the data of the M 33 SNRs is from Dopita *et al.* (1980).

With this model we find that the abundance of nitrogen in Puppis A is enhanced 5-10 times over the cosmic value, which is very similar to the result obtained by Dopita *et al.* (1977). If this overabundance of nitrogen was produced by the supernova, then A_{SN} must have been considerably larger than 20. Curve C is for $A_{SN}/A_{IS} = 1$ and it shows how difficult it is to reproduce the observed behavior of Hα/N II in this case. By contrast, the other two curves have a shape that is somewhat similar to the path described by joining the observed mean values.

The behavior of Hα/N II as a function of the SNR radius also depends on the ejected mass and the mean interstellar density. Consequently, the observed points are expected to be more scattered in this case than in the former one. Figure 3 shows that this is indeed the case. The radius of the remnant is given by

$$R_0 = 2.01 \left(\frac{Z\, M_e/M_o}{N_o} \right)^{1/3} \quad \text{pc} \qquad (28)$$

and we have assumed that the mass per particle in the interstellar medium is 2×10^{-24} g. Hα/N II has been plotted as a function of the radius in Figure 3. In this figure we also plot the corrected data from our galaxy, M 31 and the two Magellanic Clouds. We decided to exclude M 33 because we suspect that the size of the remnants from this galaxy has been heavily underestimated. In most M 33 SNRs the reported size is less than 6 pc (Dopita *et al.* 1980). On the other hand these objects have been identified using the two line ratio criteria proposed by Sabbadin and D'Odorico (1976). These criteria are unreliable for young and consequently small remannts, as the observations of Cas A, Tycho and SN1006 show. Thus, it follows that the size of these objects is likely to be considerably larger than the reported value, unless the mean interstellar density in M 33 is much higher than in our own galaxy. The extremely large dispersion shown by the data in this figure explains why there are so many doubts on an evolutionary explanation to the behavior of Hα/N II, and shows how inappropriate it is to use this ratio as a criterium to estimate the diameter of a SNR. Yet, it should be noticed that small remnants ($R \lesssim 10$ pc) are likely to have smaller values of Hα/N II, and a mean value of 0.9 is obtained for these objects as compared with 1.2 for the larger remnants. Assuming that $N_0 = 1\ cm^{-3}$ we have used three sets of values for (A_{SN}/A_{IS}, f, $M_e/M_\odot$), namely (5, 0.2, 1), (20, 0.5,1) and (20, 0.5, 5). Only five remnants lie well outside the region defined by the two extreme curves; BA 337, BA 370, HB 3, W50 and Puppis A. In the first three objects we suspect that there is some contamination from a nearby H II region, a suspicion shared by Blair *et al.* (1982) in the case of BA 337 and BA 370. W50 has been associated with the peculiar X-ray source SS 433 and the small value of Hα/N II is probably due to a high abundance of nitrogen (Murdin and Clark 1980), as is the case in Puppis A, for which we find again an overabundance of nitrogen 5-10 times over the cosmic value. Notice as well that more than 80% of the objects in our sample are in the rather restricted region contained by the two curves for which $M_e = 1\ M_{\odot}$, suggesting that, on the average, the mass ejected by SNRs is not too large.

From the equations of mass and momentum conservation, and in the absence of a transverse magnetic field, one can show that (Cantó 1977)

$$N_0\ U_5^2 = 155\ N_e\ T_4 \quad ; \qquad (29)$$

where N_e and T_4 are the density and temperature (in units of 10^4 K) of a region behind the shock wave where the temperature is approximately equal to the temperature in the pre-ionized region. This condition is satisfied by the region where sulphur is singly ionized. Thus, the electron density in this region, and consequently the S^+ line ratio 6717/6731, is correlated with the energy density in the SNR ($N_0\ U_5^2$) and, through this quantity, with Hα/N II. In Figure 4, we have plotted Hα/N II as a function of 6717/6731.

This line ratio has been calculated as a function of the electron density using the latest collision strengths (Pradhan 1978) with a 5 level atom model and for an electron temperature of 10^4 K.

Notice that in this case there is a better defined correlation with the distribution of observed points being somewhat less scattered. This is, partially at least, a consequence of better observational conditions, but it may also indicate that Hα/N II is essentially, a function of the energy density and not of the expansion velocity. If so, its evolution is mainly due to the accretion of mass by the shock wave (as in this model) and not to effects relating to the strength of the shock, such as sublimation from the surface of grains. Notice as well that only 15% of those objects for which 6717/6731 < 1 are such that Hα/N II > 1 (these objects are N63A and N103B), compared to 72 % in those for which 6717/6731 > 1, indicating that, at least concerning these two quantities, there is an evolutionary effect. The model curves for the evolution of Hα/N II as a function of 6717/6731 are for $N_0 = 1\ cm^{-3}$ and three sets of values for (A_{SN}/A_{IS}, f); (5, 0.20), (10, 0.35) and (20, 0.5). With the exception of Puppis A, (N49), W50 and OA184, BA 337, BA 370 and M 33-2, all the other 53 SNRs are within the region defined by these curves. In the anomalous objects we must either consider a very large nitrogen abundance (as in Puppis A, W50 and N49) or argue that there is some contamination from a nearby H II region (as in OA184, BA 337, BA 370 and M 33-a). But more important than this is the circumstance that the theoretical curves, and especially the mean theoretical curve, are strikingly similar to the path drawn by joining the observed mean values, more so that in the case where Hα/N II was plotted as a function of the logarithm of the expansion velocity, where the correlation is not as good.

IV. CONCLUSIONS

Figures 2, 3 and 4 show that this model is consistent with the observations. It is especially encouraging to find that the best fit occurs in the plot of Hα/N II as a function of 6717/6731 and, at the same time, that the best correlation occurs between these two quantities, a fact suggesting that the evolution of Hα/N II is mainly due to the accretion of interstellar material by the shock wave, as is assumed in this paper. Another encouraging element is that the set of values of (A_{SN}/A_{IS}, f) needed to account for the observations is identical in all cases, namely, in relation to the radius, the expansion velocity and 6717/6731.

There should be some doubts as to the applicability of the model to young remnants, such as the Crab, Tycho and SN1006, where the excitation mechanism is of a different nature. It is, nevertheless, interesting that the line ratio in the Crab has an "adequate" value, suggesting that Hα/N II is practically independent of the model used for the optical region and that, as long as there is sufficient energy to ionize nitrogen, its evolution will depend almost exclusively on the relative abundance of these two elements.

The mixing model that we have used probably is the simplest possible one. We believe that the results are sufficiently encouraging so as to invite a more sophisticated approach, especially if it can lead to the determination of the parameter f_b, so that A_{SN}/A_{IS} can be directly estimated. But this mixing scenario is not the only possible alternative to Dopita's (1977*b*) grain destruction model. An evolutionary line ratio can also occur when there is a composition gradient in the medium surrounding the SN. Such a medium can be produced by the activity of the pre-supernova star, either by a permanent stellar wind or through the violent ejection of material, as with the quasi-stationary flocculi of Cas A (Chevalier and Kirshner 1978). In this case the evolution of Hα/N II depends on the function describing the composition gradient, which will in turn depends on the activity of the pre-supernova star. But in terms of the evolution of Hα/N II this scenario is not essentially different to the one described above as long as the gradient is steep enough and the abundance anomaly is confined within a sphere that is smaller or comparable to the size of the youngest observable SNR (1 or 2 parsec).

Equation (23) can be used to determine A_{SN}/A_{IS} as a function of f provided that the values of Hα/N II, the expansion velocity and Z are known. To determine Z, it is necessary to know the interstellar density around the object, which can be found from the S^+ line ratio 6717/6731 and equation (29), the radius and the ejected mass. With the exception of the ejected mass all the other parameters can be determined directly through observation. By assuming that Kepler and the Crab are products of supernova with slow energy pumping. Utrobin (1978) estimated from the observed light curves that the ejected mass in these two SNRs is 0.23 and 0.73 $M_\odot$ respectively. Assuming that this model can be applied in these objects we find that in Kepler $A_{SN}/A_{IS} \cong$ 2-4 as f decreases from 0.5 to 0.2 (with Hα/N II $\cong$ 0.65, $N_0 = 0.45$ cm^{-3} U_5 = 1450 and $R_0 = 4.5$ parsec), whereas in the Crab $A_{SN}/A_{IS} \cong$ 1-2.5 as f decreases from 0.5 to 0.2 (with Hα/N II $\cong$ 0.8, $N_0 = 0.25$ cm^{-3}, U_5 = 1000 and R_0 = 1.7 pc). This indicates that nitrogen is not particularly overabundant in these two objects, a result that was already known in the case of the Crab, for which a more suitable model leads to no nitrogen enhancement (Davidson 1979).

Consequently, the evolution of Hα/N II indicates that nitrogen rich material has been deposited in the interstellar medium either at the time of the explosion or, prior to it, by the activity of the pre-supernova star. Research on the evolutionary behavior of emission lines from other elements would have a twofold interest. On an immediate level it would test the likelihood of our hypothesis. For instance, the enriched presence of elements arising from helium burning, such as carbon and oxygen, would cast some doubts on the simultaneous existence of nitrogen enrichment. But more basically, such an information would be of considerable interest in relation to stellar evolution and, in particular, to such difficult topics as stellar winds and mass convection in stellar interiors. With this in mind we explored the behavior of Hα/S II and were unable to discover any correlation with the radius, the velocity or 6717/6731. The line ratio is confined, with a few exceptions, to the interval [0.9, 1.4], suggesting that sulphur enrichment is not as generalized as nitrogen enrichment seems to be. Similarly, we explored the behavior of ([O II] = 3727 A)/Hβ and found it to be extremely erratic. This may also indicate that oxygen enrichment is not commonplace, but it may also be a consequence of the complications involved in the behavior of the ions of oxygen in the remnant's cooling region, as has been already suggested by Blair *et al.* (1982), and which is probably due to the fact that oxygen is one of the most important coolants in the relaxation region. There are only a few other interesting emission lines in the optical region, such as [Ne III] at 3869 A, which seems to be more intense relative to Hβ in younger objects. It would be unwise to derive any immediate conclusion concerning the abundance of Ne from this statement because there is very little information supporting it and because species more than singly ionized are quite suspect in relation to the question of determining abundances. Thus, beyond the clear necessity on deepening our knowledge of the optical spectrum of SNRs, it seems advisable to investigate in detail those regions of the spectrum where other singly ionized species are active, and recover in this way information which can be extremely useful in relation to the more fundamental problems of stellar evolution, SN explosions and chemical evolution of galaxies.

A recent work of Dennefeld (1982) shows that nitrogen may be a factor of 4 overabundant in Kepler. This is similar to our predictions.

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