Rev. Mexicana Astron. Astrof., 8, 109-114 (1983)

ARE THE SECONDARY STARS IN CATACLYSMIC VARIABLES MAIN SEQUENCE STARS?

Juan Echevarría

Royal Greenwich Observatory
and Astronomy Centre University of Sussex
Received 1983 February 23

RESUMEN

Se presenta una relación tipo espectral-período orbital para variables cataclísmicas con secundarias de tipo secuencia principal. Los tipos espectrales de 17 sistemas con períodos orbitales conocidos son derivados y comparados con los observados. El análisis muestra que estos sistemas tienen en general tipos espectrales más tardíos que estrellas de secuencia principal con la misma masa.

ABSTRACT

A spectral type-orbital period relation for cataclysmic variables with main sequence companions is presented. The spectral types of seventeen systems with known orbital periods are derived and compared with the observed ones. The analysis shows that they have in general later spectral types than main sequence stars of the same mass.

Key words: STARS-BINARIES – STARS-CATACLYSMIC – STARS-VARIABLES

I. INTRODUCTION

Cataclysmic variables are semi-detached binaries. The accreting primary is surrounded by a ring or a disc, and the secondary is a late type star filling its Roche Lobe (Warner and Nather 1971; Smak 1971). In the AM Her subgroup, a strong magnetic field, present in the primaries, inhibits the formation of the disc and the matter is instead funneled through one or both magnetic poles (see the review by Chiappetti, Tanzi and Treves 1980).

To understand the role of the secondary stars, it is essential to know whether they are on the main sequence or evolved. Several authors have argued that some secondaries may have radii larger than that of a main sequence star with the same mass (U Gem: Wade 1981; DQ Her: Smak 1980; LX Ser: Young, Schneider and Shectman 1981; AE Aqr: Patterson 1979; Z Cha: Bailey *et al.* 1981). Most of these results are based on mass determinations, which are, in general, difficult to obtain for cataclysmic variables (Robinson 1976). Hence, a systematic analysis based on radial velocity measurements may not be very promising. The spectral types of the secondaries, however, may be compared with the spectral types expected from main sequence semi-detached stars. In this paper we obtain a spectral type-period relation based on the Roche-Lobe condition (discussed in Section II) and on a mass-radius relation from well known main-sequence stars (discussed in Section III) and the results are applied to a number of systems with known spectral types and orbital periods.

II. THE ROCHE-LOBE CONDITION

Any close binary, whatever its mass ratio or its proximity, can be described by the Roche model (Kopal 1978). When a component fills its critical Roche lobe then it is said to be a contact star. Kopal (1959) has calculated various parameters of this configuration, including the equivalent radii, r*, of spheres having the same volume as the contact components. Plavec (1968), using the tabulations given by Kopal (1954), derived the radius of the secondary, R_2 as a function of $q(\equiv M_2/M_1)$. Paczynski (1971) has done similar calculations to derive an expression (accurate to 2 per cent)

$$R_1/a = 0.38 + 0.2 \log(M_1/M_2) \qquad 0.3 < M_1/M_2 < 2.0 \ , \qquad (1)$$

$$R_1/a = 0.46224\,[M_1/(M_1 + M_2)]^{1/3} \qquad 0 < M_1/M_2 < 0.8 \ . \qquad (2)$$

where R_1 is the radius of the contact star and a is the separation of the binary. The symmetry of the problem allows an interchange of indices and permits the use of (1) and (2) to describe the radius of the secondary.

For the cataclysmic variable problem, however, we may use only one modified equation since the masses of these objects are usually less than two solar masses. We find that the analytical expression

$$R_2/a = 0.47469\,[q/(1+q)]^{1/3} \qquad (3)$$

obtained from the tabulations by Kopal (1959) is accurate to 2 per cent for $0.6 < q < 1.25$ and to 5 percent for $0.2 < q < 2$. The advantage of using a power law with the fixed value of 1/3 was pointed out by Faulkner, Flannery and Warner (1972). Combined with Kepler's third law equation (3) gives

$$M_2/M_\odot = 72.38\,(R_2/R_\odot)^3\, P(h)^{-2} \quad , \qquad (4)$$

or in terms of the mean density of the secondary

$$\rho = 101.92\, P(h)^{-2} \ \ g\, cm^{-3} \quad , \qquad (5)$$

where P(h) is the orbital period in hours.

III. THE MASS-RADIUS RELATION

The lower main-sequence mass-radius relation can be approximated by a single power law

$$R/R_\odot = b\,(M/M_\odot)^x \qquad (6)$$

(Faulkner 1971). The values of b and x calculated by several authors, using a wide variety of methods, are shown in Table 1. Most of these values rely on theoretical models since accurate masses and especially radii are very difficult to obtain. A purely empirical relation for the lower main sequence was first derived by Lacy (1977) from spectroscopic eclipsing binaries. For masses less than 0.60 $M_\odot$ his relation depends only on CM Dra and YY Gem. Lacy also calculates of visual radii binaries from the Barnes-Evans relation (Barnes and Evans 1976) and finds that they are systematically larger when compared with CM Dra. He notes the latter could be an atypical star and its radius may not be representative of the lower main sequence. It seems then appropriate to provide an alternative relation

TABLE 1

MASS-RADIUS RELATION PARAMETERS

b	x	Method	Mass Range ($M_\odot$)	References
0.87 - 1.0	1	Stellar models	0.1 - 1.0	Faulkner (1971)
0.93	1	Stellar models	1	Robinson (1973, 1976)
0.959	1	3 Visual binaries + Stellar models	0.2 - 1.5	Warner (1973, 1976)
0.955	0.917	10 Spectroscopic binaries	0.10 - 1.32	Lacy (1977)
0.994	1.04	8 Visual and spectroscopic binaries	0.07 - 1.15	Hutchings, Cowley, and Crampton (1979)
1.057	0.906	25 Visual and spectroscopic binaries	0.01 -1.32	This paper
1.085	1.00	13 Spectroscopic binaries	0.21 - 1.32	This paper
0.987	0.838	12 Visual binaries	0.11 - 1.15	This paper

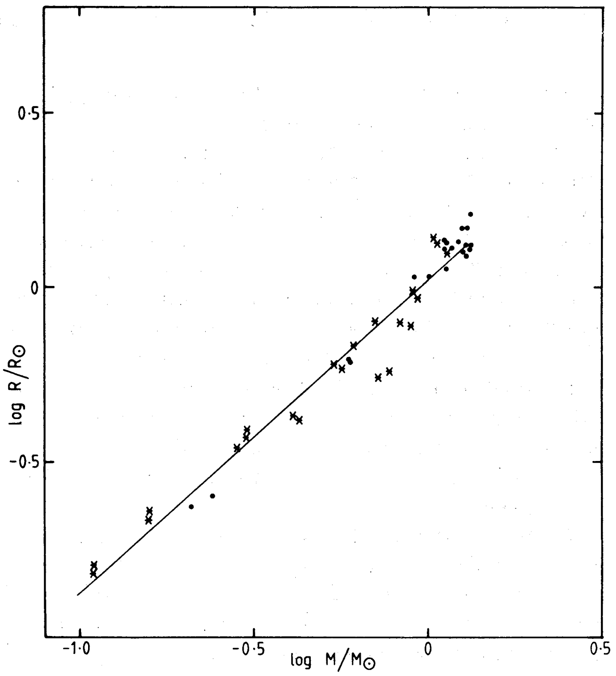

Fig. 1. Mass-radius relation for the lower main sequence (equation 7). Asterisks are visual binaries, and filled circles are spectroscopic binaries.

also based on visual binaries. We use a recent compilation by Popper (1980) of well determined visual and spectroscopic binaries with main sequence components. We took detached main sequence eclipsing binaries with $M \leqslant 1.32\ M_{\odot}$ from his Table 2 and visual binaries from his Table 8, whose radii are based on a flux scale by Hayes (1978). A least squares fit from all spectroscopic and visual binaries is shown in Figure 1. The expression of the fit is

$$R/R_{\odot} = 1.057(\pm 0.024)\,(M/M_{\odot})^{0.906\,(\pm 0.027)} \quad . \quad (7)$$

We have also calculated separate values of b and x based on spectroscopic binaries and on visual binaries. The results are shown in Table 1 and discussed in Section IV.

From (4) and (7) we obtain the mass-period relation

$$M_2/M_{\odot} = 7.51 \times 10^{-2}\ P(h)^{1.16} \quad 1.4 < P(h) < 12 \quad , \quad (8)$$

and the radius-period relation

$$R_2/R_{\odot} = 0.101\ P(h)^{1.05} \quad 1.4 < P(h) < 12 \quad . \quad (9)$$

We expect to observe these relations if the contact components are main sequence stars.

The absolute visual magnitude-mass relation for the lower main-sequence obtained by Serrano (1978)

$$M_V = -10.50 \log(M/M_{\odot}) + 4.98 \qquad (10)$$

can be used to obtain the M_V - P relation

$$M_V = 16.80 - 12.19 \log P(h) \quad . \qquad (11)$$

This is the relation expected between the absolute visual magnitude and period for main-sequence contact components.

IV. DISCUSSION

The position in the M - R diagram of a semi-detached binary of given orbital period lies along one of the lines of constant period labelled in Figure 2. Also shown is the M - R relation (7), Lacy's and Warner's empirical relations and theoretical calculations by Grossmann, Hays and Graboske (1974) and Copeland, Jensen and Jørgensen (1978). At any given mass our main sequence rela-

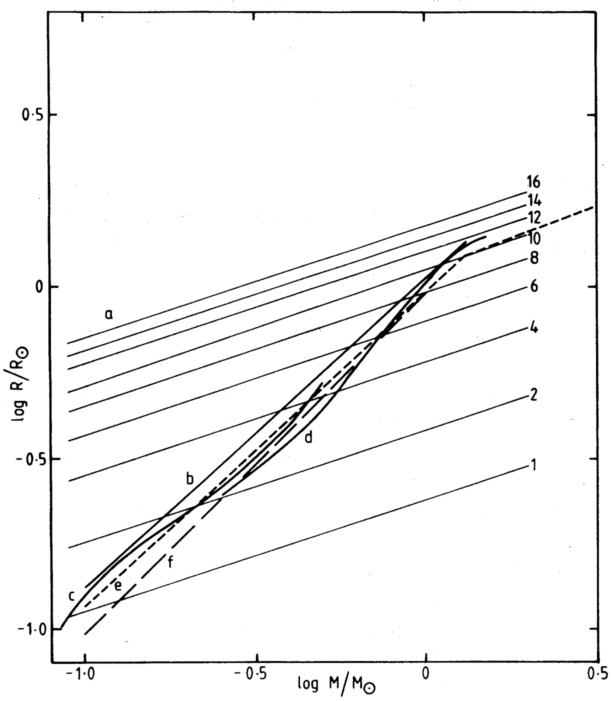

Fig. 2. Mass-radius diagram. a) Semi-detached binaries; lines are drawn for different orbital periods (in hours). b) as in Figure 1. c) Theoretical main sequence for X = 0.68, Y = 0.29, Z = 0.03 and $\ell/H_p = 1$ from Grossman, Hays and Graboske (1974). d) Theoretical main sequence for X = 0.70, Y = 0.27, Z = 0.03 and $\ell/H_p = 1$ from Copeland, Jensen and Jørgensen (1970). e) Empirical main sequence for spectroscopic binaries (Lacy 1977). f) Semi-empirical relation from Warner (1976).

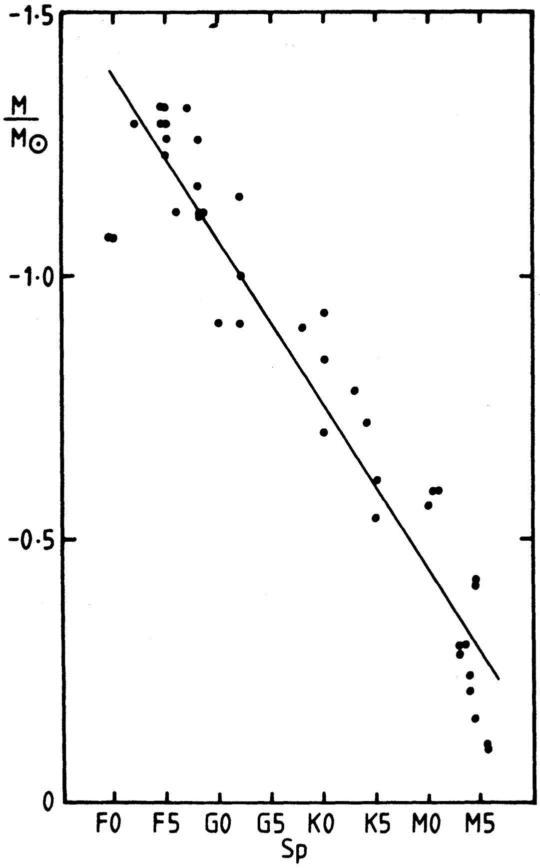

Fig. 3. Mass-spectral type relation for the lower main sequence from visual and spectroscopic binaries.

tion yield larger radii than previous work, and agrees well with theoretical calculations around 0.1 $M_\odot$ and 1.2 $M_\odot$.

Patterson (1979) derived the mass of the secondary star in AE Aqr from the observed spectral type assuming main sequence membership and found its radius greater than the Roche Lobe radius. We can generalize this argument by deriving a spectral type from the orbital period of semi-detached binaries with main sequence companions. We obtain a relation between spectral type and mass from main-sequence stars and then combine the results with equation (8) to obtain a relation between M and the spectral type of the binaries used to derive equation (7). A sequential number I has been given to the spectral types of the components beginning with F0 = 0. Thus the notation I = 10, for instance, implies a spectral type G0. The solid line in Figure 3 represents our least squares estimate of the lower main-sequence mass-spectral type relation. The equation of the fit is

$$M/M_\odot = 1.38(\pm 0.03) - 0.0311(\pm 0.0016)\, I \quad , \qquad (12)$$

which is shown in Figure 3. From equations (8) and (12) we obtain

$$I = 44.37(\pm 1.32) - 2.41\,(\pm 0.15)\, P(h)^{1.16\,(\pm 0.006)} . \qquad (13)$$

This is the relation expected between spectral type and period for main-sequence contact components.

The spectral types of seventeen cataclysmic variables were taken from the catalogue by Ritter (1983). These are systems for which direct spectral analysis is available. Spectral types obtained from infrared light curves or by other methods are not included here. The spectral indices defined by equation (13) are shown in Table 2 and plotted against the observed spectral type in Figure 4. We have also calculated a period-spectral type relation from equation (2) (see Section II) and from the mass-radius relation derived by Lacy (see Section III). This produces earlier types than those calculated before. BV Cen and SS Cyg, for example would have B3 and K0 spectral types.

We find that there is a general trend for systems to show spectral types later than predicted for main sequence of the same period. This difference increases with orbital period. CW1103+254, PG1550+191 and Lanning 10 are the only systems with spectral types earlier than predicted for main sequence stars.

The uncertainties in the parameter I given in equation (13) are small for the short period systems. However, we have implicitly assumed that the scatter in Figure 3 is due to observational errors. If, on the other hand, we believe that the scatter is instrinsic then I should have an uncertainty of ± 3-5 for a given mass, and therefore, some of the short period systems will be

TABLE 2

SPECTRA OF CATACLYSMIC VARIABLES

Name	Period (hr)	Sp Observed	Sp Inferred from Period
GK Per	16.44	K2 IV-V	B2
BV Cen	14.63	G5 - 8 V	A0
V Sge	12.34	F6 - G0 V	F0
AE Aqr	9.88	K5 V	G0
RU Peg	8.99	K0 IV	G3
Lanning 10	7.71	F5 - G	G8
AC Cnc	7.21	K5 V	K0
EM Cyg	6.98	K5 V	K1
Z Cam	6.95	K7 V	K1
SS Cyg	6.60	K5 V	K3
RW Tri	5.56	M0 V	K6
DQ Her	4.65	M3 V	M0
U Gem	4.24	M4.5 V	M1
MV Lyr	3.21	M5 V	M5
AM Her	3.09	M4.5 V	M5
CW 1103+254	1.90	M3 V	M9
PG 1550+191	1.89	M3 V	M9

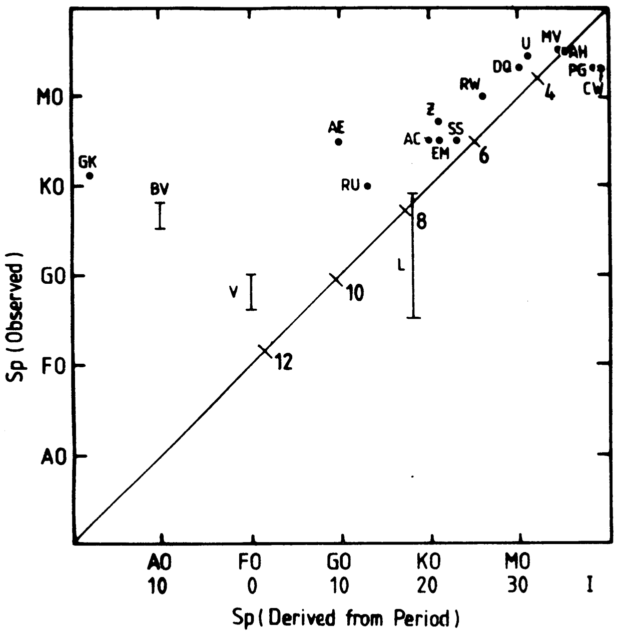

Fig. 4. Spectral index (abscissa) versus observed spectral type (ordinate) of cataclysmic variables (see text). Numbers on diagonal line denote period in hours. BV = BV Cen; GK = GK Per; V = V Sge; L = Lanning 10; RU = RU Peg; AE = AE Aqr; AC = AC Cnc; EM = EM Cyg; SS = SS Cyg; Z = Z Cam; RW = RW Tri; DQ = DQ Her; U = U Cem; MV = MV Lyr; AM = AM Her; CW = CW 1103 + 254; PG = PG1150 + 91.

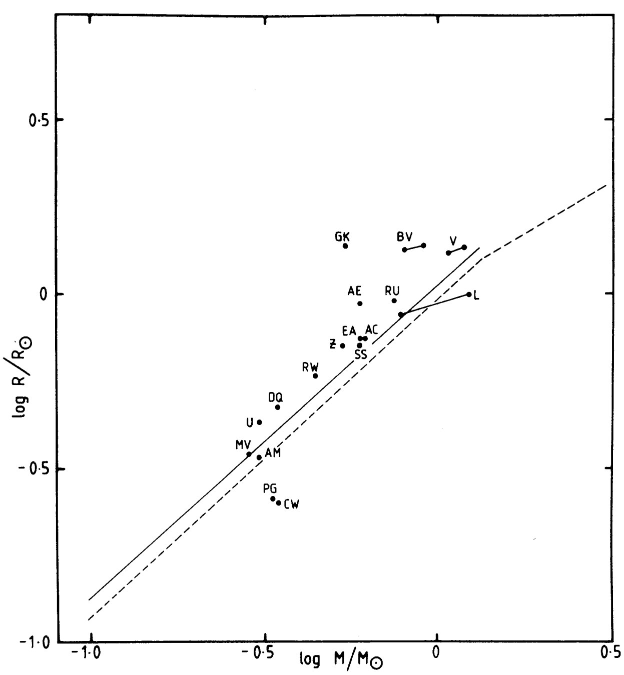

Fig. 5. Mass-radius diagram of cataclysmic variables with known spectral type. Masses have been derived from the spectral type. The M-R relations are as in Figure 2. The lines drawn for some cataclysmic variables represent the uncertainties in the spectral types. The labels have the same meaning as in Figure 4.

consistent with main sequence membership. Another source of error is concerned with the accuracy of spectral classification of the secondary stars. Although there are other estimates of the spectral types in addition to those tabulated in Ritter's catalogue, they are in general the best types available. Spectral classification in cataclysmic variables is difficult due to the presence of a strong blue continuum and is not very accurate. However, it is difficult to think how such an effect can produce the trend in Figure 4.

The masses and radii derived from the observed spectral types and the orbital periods using equations (12) and (4) are plotted in the M-R diagram in Figure 5. As expected there is a general trend for systems to have larger radii than main sequence stars of the same mass, the trend increasing with orbital period. AE Aqr shows a radius 40 per cent greater than a main-sequence star of the same mass, compared with the 50 per cent calculated by Patterson (1979). CW1103+254 and PG1550+191 must be very cool stars to fit the main sequence. Earlier types will make their radius even smaller compared with a main sequence of the same spectral type!

V. CONCLUSIONS

We have presented new analytical expressions for the radius of a contact secondary and for a mass-radius relation of main sequence stars, from basic data on visual and spectroscopic binaries. Because mass determinations are difficult to obtain, a systematic study based on radial velocity measurements, to decide whether or not the secondaries are main sequence stars is not very promising. However, by studying the spectral types one may show that the secondaries are not normal main sequence stars, certainly not so for the longer periods.

I thank Drs. R.C. Smith, D.H.P. Jones and M. Tapia for their critical reading of the manuscript and their multiple suggestions. This work was done under a scholarship by CONACYT.

REFERENCES

Bailey, J., Sherrington, M.R., Giles, A.R., and Jameson, R.F. 1981, *M.N.R.A.S.*, **196**, 121.
Barnes, T.G. and Evans, D.S.1976, *M.N.R.A.S.*, **174**, 489.
Chiappetti, L., Tanzi, E.G., and Treves, A. 1980, *Space Sci. Rev.*, **27**, 3.,
Copeland, R., Jensen, J.O., and Jørgensen, H.E. 1970, *Astr. and Ap.*, **5**, 12.
Faulkner, J. 1971, *Ap. J. (Letters)*, **197**, L99.
Faulkner, J., Flannery, B.P., and Warner, B. 1972, *Ap. J. (Letters)*, **179**, L79.
Grossman, A.S., Hays, D., and Graboske, H.C. 1974, *Astr. and Ap.*, **30**, 95.
Hayes, D.S. 1978, in *IAU Symposium No. 80, The HR Diagram*, eds. A. G. Davis Philip and D.S. Hayes (Dordrecht: D. Reidel p. 65.
Hutchings, J.B., Cowley, A.P., and Crampton, D. 1979, *Ap. J.*, **232**, 500.

Kopal, Z. 1954, *Jodrell Bank Ann.*, **1**, 37.
Kopal, Z. 1959, *Close Binary Systems* (London: Chapman and Hall).
Kopal, Z. 1978, *Dynamics of Close Binary Systems* (Dordrecht: D. Reidel).
Lacy, C.H. 1977, *Ap. J. Suppl.*, **34**, 479.
Paczynski, B. 1971, *Ann. Rev. Astr. and Ap.*, 9, 183.
Patterson, J. 1979, *Ap. J.*, **234**, 978.
Plavec, M. 1968, *Adv. Astr. and Ap.*, **6**, 202.
Popper, D. 1980, *Ann. Rev. Astr. and Ap.*, **18**, 115.
Ritter, H. 1983, *Catalogue of Cataclysmic Variables, Low-Mass X-Ray Binaries and Related Objects*, Second Edition (Garching: Max-Planck-Institut für Physik und Astrophysik).
Robinson, E.L. 1973, *Ap. J.*, **180**, 121.
Robinson, E.L. 1976, *Ann. Rev. Astr. and Ap.*, **14**, 119.
Serrano, A. 1978, Ph. D. Thesis, University of Sussex.
Smak, J. 1971, *Acta Astr.*, **21**, 15.
Smak, J. 1980, *Acta Astr.*, **30**, 267.
Wade, R.A. 1981, *Ap. J.*, **246**, 215.
Warner, B. 1973, *M.N.R.A.S.*, **162**, 189.
Warner, B. 1976, in *IAU Symposium No. 73, Structure and Evolution of Close Binary Systems*, eds. P. Eggleton, S. Mitton, and J. Whelan (Dordrecht: D. Reidel), p. 85.
Warner, B. and Nather, R.E. 1971, *M.N.R.A.S.*, **152**, 219.
Young, P., Schneider, D.P., and Shectman, S.A. 1981, *Ap. J.*, **244**, 259.

Juan Echevarría : Observatorio Astronómico Nacional, Instituto de Astronomía, UNAM, Apartado Postal 877, 22860 Ensenada, B.C., México.