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CHEMICAL EVOLUTION OF GALAXIES. III. THE N/O VERSUS O/H RELATIONSHIP

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RESUMEN

Se encuentra que el diagrama N/O, O/H construído a partir de observaciones de regiones H II galácticas y extragalácticas, puede ser explicado si se adoptan las siguientes hipótesis: a) la mayor parte del nitrógeno es de origen secundario, b) la mayor parte del nitrógeno ha sido producido por estrellas de masa intermedia ($1 \lesssim M/M_{\odot} \lesssim 5$), y c) la acreción juega un papel importante en la evolución química de las galaxias. Se presentan argumentos teóricos y observacionales en favor de estas hipótesis. Dado que los modelos de rendimiento constante nunca alcanzan 12 + log O/H $\gtrsim$ 9.0 porque el reciclaje instantáneo falla, se infiere del modelo aquí presentado que el rendimiento aumenta con la metalicidad.

ABSTRACT

The N/O, O/H diagram derived from galactic and extragalactic H II regions can be explained under the assumptions that most of the N is of secondary origin, that most of the N is produced by intermediate mass stars ($1 \lesssim M/M_{\odot} \lesssim 5$), and that accretion plays an important role in the chemical evolution of galaxies. Several observational and theoretical arguments are presented in favor of these hypotheses. Given that constant-yield models can never reach 12 + log O/H $\gtrsim$ 9.0 because the instantaneous recycling for O, breaks down it is an essential component of the new model that the yield increases with metallicity.

Key words: GALAXIES-EVOLUTION – GALAXIES-STELLAR CONTENT – H II REGIONS – INTERSTELLAR-ABUNDANCES – STARS-MASS LOSS

I. INTRODUCTION

A relationship of the type

$$\log (N/O) = a \log (O/H) + b \quad , \qquad (1)$$

with $a = 1$ is predicted by simple models with instant recycling approximation for the abundances of the interstellar medium assuming N to be of secondary origin and O to be of primary origin (e. g., Talbot and Arnett 1973).

From H II regions it is possible to determine N, O and H abundances that are representative of the general gaseous content of the interstellar medium. Recent determinations of a yielded values considerably smaller than unity for several spiral galaxies and for a set of irregulars and blue compact galaxies. These results led several authors to suggest that a significant fraction of N is of primary origin (e. g. Smith 1975; Peimbert 1979*a*; Alloin *et al.* 1979; Edmunds and Pagel 1978; Lequeux *et al.* 1979).

Edmunds and Pagel (1978) and Peimbert (1979*b*) noticed that H II regions in irregular and dwarf blue galaxies have a smaller N/O ratio for a given O/H ratio than galactic H II regions; possible explanations are that: a) the galactic initial mass function is different to those of irregular and dwarf blue galaxies, b) infall in the Galaxy of material with pregalactic abundances has reduced the O/H ratio of regions with substantial amounts of N produced by secondary mechanisms (Peimbert 1979*b*), c) most of the N is of primary origin produced by stars in the 1-2.5 $M_{\odot}$ range and the differences in N/O for a given O/H are due to age effects (Edmunds and Pagel 1978).

Maybe a combination of the three explanations suggested above is needed to explain the location of all the objects in the N/O, O/H diagram. Nevertheless there are substantial differences among these interpretations and perhaps only one of them is significant. It is the purpose of this paper to study the implications of the different explanations and to test these implications with observational data.

In § II we will discuss direct observational evidence in favor of primary and secondary production of N, as well as the best observations available to construct the N/O, O/H diagram; in §III we will discuss the effects of star formation rates varying with time and of models with different values of γ, the ratio of accretion rate to star formation rate, and in §IV we will present the discussion and the conclusions.

II. OBSERVATIONAL DATA

To study the galactic enrichment of N and O it is necessary to know which stars are responsible for it and if the production is due to primary or secondary mechanisms. It is thought that O is of primary origin and that most of it is produced by very massive stars; for N the situation is not as clear. To be certain that there has been primary production of N in objects of the solar vicinity it is not enough that $N/H > (CN/H)_{\odot}$ but that $X_{NO} > (X_{CNO})_{\odot}$ because some H might have been transformed into He and some O into N.

There are direct observations of N production by very massive stars: ring H II regions like NGC 2359 and NGC 6888 ionized by WN stars which are losing mass (Peimbert 1979*a, b*; Talent and Dufour 1979; Parker 1978; Kwitter 1981), the shells around WN9 stars in the LMC (Walborn 1982), and the quasi-stationary flocculi of the SNR Cas A (Peimbert and van den Bergh 1971; Chevalier and Kirshner 1978). It seems that the production of N is mostly secondary, however without accurate H, He, C, N and O abundance determinations it is very difficult to assess if the production of N is only secondary; for example HD 192163, the ionizing star of NGC 6888 has $X_N = 0.035$ and $X_C = 0.001$ (Willis and Wilson 1978) which implies not only that it has transformed most of its C into N but since $X_N \gtrsim 2(X_{CNO})_{\odot}$ that there might have been some primary production of N.

There are also direct observations of N production by intermediate mass stars, $1 \lesssim M^*/M_{\odot} \lesssim 6$, which are progenitors of planetary nebulae and novae (e.g., Peimbert 1978; Stickland *et al.* 1981; Williams *et al.* 1981; Williams 1982).

Peimbert and Serrano (1980) based on Type II and Type III PN found that

$$\log N/O = (1.24 \pm 0.22) \log (O/H) + 3.29$$

which implies that most of the excess N is of secondary origin probably produced during the red giant phase (e.g. Peimbert and Torres-Peimbert 1971). For some PN of Types II and III $X_{CNO} > (X_{CNO})_{\odot}$ (e.g., Peimbert 1980; Aller and Czyzak 1983) indicating some primary production of CNO; from the previous argument we think that most of the primary production is in the form of C.

In those PN of Type I with C abundance determinations and in Nova U Scorpii the He/H and N/CNO ratios are considerably higher than solar, which is indicative of material that has experienced substantial CNO burning (Peimbert and Torres-Peimbert 1983*a*; Aller and Czyzak 1983; Williams *et al.* 1981); moreover for these objects $X_{CNO} \sim (X_{CNO})_{\odot}$, indicating that primary production if present is not very important. The most extreme case of an N-rich PN is the object in NGC 6822 observed by Dufour and Talent (1980), for which they derived $\log X_N = -2.32 \pm 0.3$ dex a value only marginally higher than the oxygen $\log X_O = -2.68 \pm 0.1$ dex (Lequeux *et al.* 1979, for $t^2 = 0.00$); consequently the evidence for a substantial primary production of N by Type I PN is not very strong.

At present it is not possible to rule out some primary production of N by PN progenitors but it seems that most of the N produced by these objects is of secondary origin.

For most novae the situation is different; the large overabundances of N clearly imply that it has been produced as a primary synthesis element (Stickland *et al.* 1981; Williams 1982 and references therein). Moreover, from a very crude estimate Williams (1982) has found that novae inject more nitrogen than PN into the interstellar medium of the Magellanic Clouds and M31; this estimate, if correct, would support the suggestion given by Edmunds and Pagel (1978) to explain the N/O versus O/H diagram.

In Figure 1 we present some of the best abundance determinations of galactic and extragalactic H II regions where the abundances have been derived from direct measurements of the electron temperature and under the assumption that the mean square temperature variation over the observed volume, t^2, is equal to zero (for results derived with different values of t^2 see Lequeux *et al.* 1979 and Torres-Peimbert, Peimbert, and Daltabuit 1980). The data in Figure 1 include irregular and blue compact galaxies (Lequeux *et al.* 1979), M101 (Rayo, Peimbert, and Torres-Peimbert 1982), M33 (Kwitter and Aller 1981), and the Galaxy (Peimbert, Torres-Peimbert, and Rayo 1978; Peimbert and Torres-Peimbert 1977 1983*b*). The cross in the lower right hand corner indicates typical standard errors related to the accuracy of the line intensities that intervene in the electron temperature determination; the vertical error is smaller because the

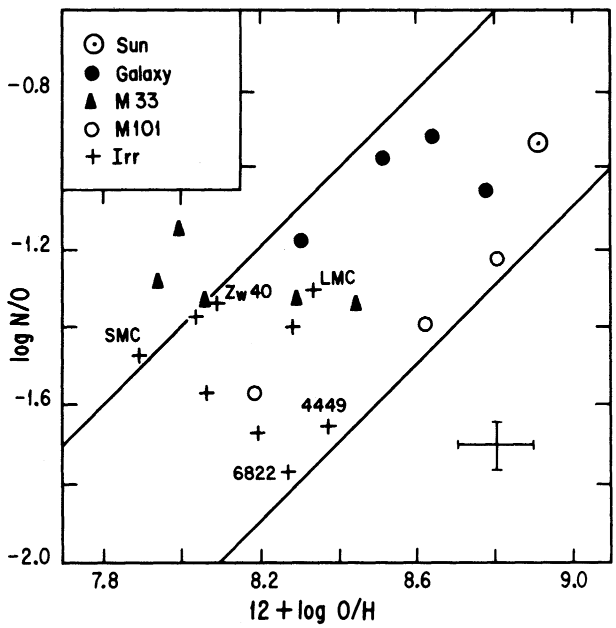

Fig. 1. N/O vs. O/H plot for galactic and extragalactic H II regions, crosses denote irregular and compact galaxies, the error bar in the lower right hand corner corresponds to a typical standard deviation for the plotted data. The straight lines are 45° lines for $a = 1$ in equation (1).

N/O ratio is less temperature sensitive than the O/H ratio. For comparison we show in Figure 1 the solar values obtained by Lambert (1978).

It is very difficult to measure the auroral and trans-auroral lines needed to determine the electron temperature because they are typically about two orders of magnitude fainter than the corresponding nebular lines, therefore several authors have suggested empirical methods to estimate the electron temperature based on nebular lines alone. Pagel *et al.* (1979) have proposed an empirical relationship between the ([O II]+ [O III])/Hβ ratio and T_e, while Alloin *et al.* (1979) have proposed an empirical relationship between the [O III]/[N II] ratio and T_e. These methods are uncertain due to several reasons (e.g., Stasinska *et al.* 1981); moreover for 12 + log O/H > 9.0 the relationships have been calibrated with models and not with observed T_e values (Pagel *et al.* 1979, 1980; Shields and Searle 1978; Dufour *et al.* 1980). It has been suggested that for 12 + log O/H > 9.0 the calibration underestimates T_e and consequently overestimates the O/H ratio (Rayo *et al.* 1982).

The empirical methods permit to extend the abundance determinations to H II regions with emission measures about two orders of magnitude fainter than those where direct measurements of T_e have been made. In Figure 2 we present the values of Figure 1 and additional data for: M33 (Kwitter and Aller 1981), NGC 1365 (Alloin *et al.* 1981) and M83 where the nuclear region has not been included (Dufour *et al.* 1980). Again the solar values are presented in Figure 2 and the cross on the lower right hand corner gives an estimate of the expected standard errors for 12 + log O/H > 9, which from the previous discussion are considerably larger than those at lower values of O/H. Other determinations based on the ([O II] + [O III]/Hβ) method have been presented in the excellent review by Pagel and Edmunds (1981).

What we mainly want to show in Figures 1 and 2 is that: a) for values of 12 + log O/H < 9 most of the objects are located between the two 45° lines; b) each spiral galaxy only covers a small fraction of the N/O, O/H diagram; c) the spiral galaxies show a slope considerably smaller than unity; and d) for a given O/H value there are galaxies with very different N/O values and, similarly, for a given N/O value there are galaxies with very diffent O/H values.

III. THE MODELS

If nitrogen is mainly produced by stars that live longer than those that produce oxygen, then the arrival of nitrogen into the interstellar medium may be delayed with respect to oxygen. To take this delay into account, Edmunds and Pagel (1978) have suggested that the coefficient *b* in equation (1) is to be considered time dependent. Its initial value is $-\infty$, corresponding to no nitrogen production, and it tends to a constant after a relatively long time, of the order of the delay.

For open systems equation (1) is no longer valid, even in the instantaneous recycling approximation. For this reason, we have constructed simplified models of chemical evolution of oxygen and nitrogen that incorporate the delay in the production of nitrogen in a more general and consistent manner than that proposed by Edmunds and Pagel. In these models, the possibilities of variable yields, as discussed by Peimbert and Serrano (1982), and of accretion are also considered.

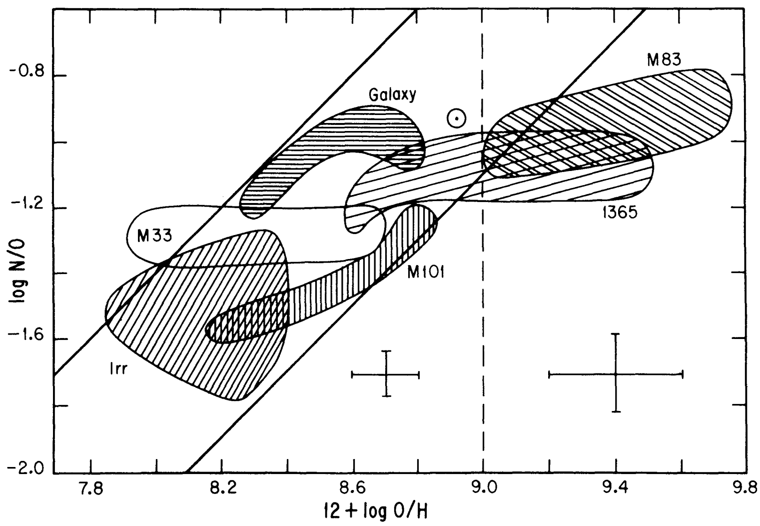

Fig. 2. Schematic representation of Figure 1 where two more spiral galaxies have been added. For 12 + log O/H > 9 the errors in the abundance determinations are larger and an estimate of the standard deviation for an abundance determination in that region is presented in the lower right hand corner.

a) *Delay and Variable Yields*

For any quantity Q(m) that depends on the initial mass of a star, m, let us define its average over a stellar generation

$$\langle Q \rangle = \int_{m(t)}^{m_u} Q(m)\, \phi(m)\, dm \quad , \qquad (2)$$

where ϕ is the initial mass function by mass (IMF), m(t) is the turn-off mass at time t and m_u is the upper stellar mass limit. In this way, the ejection rate of material originally present in stars as species j and eventually ejected as species i, $\dot{e}_{ij}$, can be written as

$$\dot{e}_{ij} = \langle (\dot{S}\, X_j)_R\, f_{ij} \rangle \quad , \qquad (3)$$

where $\dot{S}$ is the stellar formation rate by mass, X_j is the abundance-by-mass of species j in the star, f_{ij} is the fraction of a star where material is in form of species j at birth and is eventually ejected as species i, and quantities with subindex R are evaluated at a retarded time $t_R = t - \tau(m)$ where $\tau(m)$ is the lifetime of a star of mass m.

In the instantaneous recycling approximation (*IRA*), m(t) in equation (2) is assumed to vanish and retarded quantities have their instantaneous values, i.e., $Q_R = Q(t)$.

Assuming that for all species the material not processed is returned instantaneously, i.e.,

$$\dot{e}_{ii} = \dot{S}\, X_i \langle f_{ii} \rangle \quad , \qquad (4)$$

and neglecting the fraction of stellar mass where species i is burned into heavier species and ejected, the change in abundance of species i is given by

$$M_g\, dX_i = \sum_{i \neq i} \langle f_{ij}\, (X_j\, dS)_R \rangle - X_i\, dM \quad , \qquad (5)$$

where M is the total mass of the system, M_g is the mass in the form of gas, and where it has been assumed that species i is not present in the accreted material dM. Equation (5) is a good approximation for species heavier than carbon. It obviously fails for He where the fraction of the star in which He is destroyed must be considered.

For primary species, results of synthesis from H and He, the only non-vanishing chemical coefficient is

$$f_i = f_{iH} = f_{iHe} \qquad (6)$$

i.e., that fraction of a star where He burning takes place and which is ejected as species i. For these elements, assuming IRA

$$M_g\, dX_i = p_i\, (1 - Z)\, (dM - dM_g) - X_i\, dM \quad , \qquad (7)$$

where p_i is the yield of species i.

Following Peimbert and Serrano (1982) let us assume that the yield is increasing with metallicity:

$$p_Z = p_0 + a\, Z \quad , \qquad (8)$$

and apply equation (7) to the heavy element abundance, Z, to obtain

$$M_g\, dZ = -Z\, \{ (1 - b)\, dM + b\, dM_g \} + p_0\, (dM - dM_g) \quad , \qquad (9)$$

where

$$b = a - p_0 \qquad (10)$$

and it was assumed that $Z \ll (p_0/a)^{1/2}$.

If ξ is defined by

$$d\xi = (1 - b)\, dM/M_g \quad , \qquad (11)$$

and subindex 0 denotes the initial value of a given quantity at $t = t_0$, then the formal solution of equation (9) is

$$Z/p_0 = \chi_b + (Z_0/p_0 - \chi_{b0})\, \mu^{-b}\, e^{\xi_0 - \xi} \quad , \qquad (12)$$

where

$$\mu = M_g/M_{g0} \quad , \qquad (13)$$

and

$$\chi_b = \frac{1}{b(1 - b)} \left\{ e^{-\xi}\, \mu^{-b} \int^{\xi} e^{\xi}\, \mu^{b}\, d\xi \right\} - \frac{1}{b} \quad . \qquad (14)$$

For closed models (dM = 0) we have ξ = constant and $\chi = -1/b$ so that if $Z_0 = 0$,

$$Z = \frac{p_0}{b} (\mu^{-b} - 1) \quad ,$$

while for models with infall balancing star formation, M_g is constant, $\xi = M/M_0$ and so $\chi = 1/(1 - b)$ and

$$Z = \frac{p_0}{1 - b} \{ 1 - \exp[(1 - b)(1 - M/M_0)] \} \quad .$$

In the case of a constant yield (a = 0) the expressions given above reduce to the well known equations

$$Z = -p_0 \ln \mu \quad ,$$

for the closed models, and

$$Z = p_0 [1 - \exp(1 - M/M_0)]$$

for the extreme infall models.

Since oxygen is produced by massive stars we will assume that it is always 0.454 of Z and hence that it is described by equation (12).

For nitrogen, we have from equation (5) that

$$M_g \dot{X}_N + X_N \dot{M} = \dot{A} \quad , \qquad (15)$$

with

$$\dot{A}_p = \langle r f_N (p_Z (1 - Z) \alpha \dot{S})_R \rangle \qquad (16)$$

for primary nitrogen, and

$$\dot{A}_S = \langle r c_O h_N (p_Z Z \alpha \dot{S})_R \rangle \qquad (17)$$

for secondary nitrogen. Here, α is the locked-up fraction, c_O is the fraction of Z in the form of oxygen,

$$r = 1/\langle f_Z \rangle \quad , \qquad (18)$$

f_N is the fraction of a star where H and He have been converted to N, and ejected, and h_N is the fraction of a star where the initial CO has been converted to N and ejected.

In equation (15) it has been assumed that p_Z changes with Z due to variations in α (Peimbert and Serrano 1982) and hence that all yields vary in the same way.

The solution of equation (15) is

$$X_N(t) = X_N(t_0)\, e^{\theta_0 - \theta} + e^{-\theta} \int_{t_0}^{t} e^{\theta'} (\dot{A}/M_g)_{t'}\, dt' \quad , \qquad (19)$$

with θ defined by

$$d\theta = dM/M_g \quad . \qquad (20)$$

It must be stressed that equations (12) and (19) can be applied to other species, and that the difference between them comes from different assumptions about the ejection process. In equation (19) we have allowed for a delay in the ejection of newly processed material, while in equation (12) instantaneous recycling has been assumed. In both cases, however, it has been considered that the material not processed is returned instantaneously as stated in equation (4). The results of our models will be valid, then, as long as the latter condition is fulfilled. For a very small gas fraction, ejection of H rich material by old stars becomes important, and equations (12) and (19) are no longer valid (Talbot and Arnet 1973).

In order to construct a theoretical N/O versus O/H diagram it is necessary to know the accretion law $\dot{M}$ and the effective star formation rate $\alpha\dot{S}$ as a function of time.

b) *Models with a Constant Ratio of Accretion to Star Formation*

Let us define

$$\gamma = dM/\alpha\, dS \quad , \qquad (21)$$

and consider models of constant γ, similar to those studied by Twarog (1980) and by Tosi (1982). Since in this case, if $\gamma \neq 1$,

$$dM_g = -(1 - \gamma)/\gamma\, dM \quad , \qquad (22)$$

we obtain from equations (11) and (14)

$$\chi_b = -1/(b - \gamma) \quad , \qquad (23)$$

and so equation (12) becomes

$$Z = Z_0 + \{p_0/(b - \gamma) + Z_0\} (\mu^{-u} - 1) \qquad (24)$$

where

$$u = (b - \gamma)/(1 - \gamma) \quad , \qquad (25)$$

and it has been assumed that $b \neq \gamma$.

On the other hand, if $\gamma = 1$ then

$$d\xi = (1 - b)\, dM/M_0 \quad , \qquad (26)$$

so that from equation (14)

$$\chi_b = 1/(1 - b) \qquad (27)$$

and hence equation (15) becomes

$$Z = Z_0 + \{p_0/(1 - b) - Z_0\} (1 - e^{-(1-b)(\mu-1)}) \qquad (28)$$

for $\gamma = 1$. This equation and equation (24) for a closed system ($\gamma = 0$) have already been considered by Peimbert and Serrano (1982).

From equations (8) and (10) and neglecting terms in Z^2

$$p_Z (1 - Z) \cong p_0 (-\gamma + b\,\zeta)/(b - \gamma) \tag{29}$$

and

$$Z\,p_Z \simeq p_0^2 \{(\gamma + p_0) - (b + 2\,p_0 + \gamma)\,\zeta + (b + p_0)\,\zeta^2\}/(b - \gamma)^2 \tag{30}$$

where

$$\zeta = 1 + (b - \gamma)\,Z/p_0 \quad . \tag{31}$$

In this way, if s_N denotes f_N in the case of primary N and h_N for secondary N, equations (16) and (17) can be rewritten as

$$\dot{A}_p = \frac{r\,p_0}{b - \gamma} \{-\gamma \langle s_N\,\alpha\,\dot{S}_R\rangle + b\,\langle s_N(\zeta\alpha\,\dot{S})_R\rangle\} \tag{32}$$

and

$$\dot{A}_S = r\,c_O\,(p_0/(b - \gamma))^2 \{(\gamma + p_0)\,\langle s_N\,\alpha\,\dot{S}_R\rangle - (b + 2\,p_0 + \gamma)\,\langle s_N\,(\zeta\,\alpha\,\dot{S})_R\rangle + (b + p_0)\,\langle s_N\,(\zeta^2\,\alpha\,\dot{S}_R)\rangle\} \tag{33}$$

respectively.

Using now the total N production

$$q = \int_0^\infty s_N\,\phi\,dm \quad , \tag{34}$$

and for ζ of the form

$$\zeta = \zeta_0\,C^{-y} \quad , \tag{35}$$

with $C = \mu$, $y = u$ if $\gamma \neq 1$ (equation 24) and $C = e^{1-\mu}$, $y = 1 - b$ if $\gamma = 1$ (equation 28), then the evolution of N given in equation (19) becomes

$$X_N(t) = X_N(t_0)\,e^{\theta_0 - \theta} + \frac{r\,q_P\,p_0}{b - \gamma} (-\gamma\,I_0 + b\,I_y) \quad , \tag{36}$$

for primary N, and

$$X_N(t) = X_N(t_0)\,e^{\theta_0 - \theta} + r\,q_S\,c_0\,(p_0/(b - \gamma))^2 \{(\gamma + p_0)\,I_0 - (b + 2\,p_0 + \gamma)\,\zeta_0\,I_y + (b + p_0)\,\zeta_0{}^2 I_{2y}\} \quad , \tag{37}$$

for secondary N, with

$$I_y = \frac{e^{-\theta}}{q} \int_{t_0}^{t} \frac{e^{\theta}}{M_g} \langle s_N\,C_R{}^{-y}\,\alpha\,\dot{S}_R\rangle\,dt \quad . \tag{38}$$

When $\gamma < 1$, let us define

$$\kappa = \gamma/(1 - \gamma) \quad , \tag{39}$$

so that from equation (20)

$$\theta = -\kappa \ln M_g \quad . \tag{40}$$

Using then from equation (24) $C = \mu$ and $y = u$ in equation (35), it is obtained that

$$X_N(t) = X_N(t_0)\,(\mu/\mu_0)^\kappa + r\,q_p \frac{p_0}{b - \gamma} \times (-\gamma\,I_0 + b\,I_u) \quad , \tag{41}$$

for primary N, and

$$X_N(t) = X_X(t_0)\,(\mu/\mu_0)^\kappa + r\,q_S\,c_0 \left[\frac{p_0}{b - \gamma}\right]^2 \{\gamma + p_0)\,I_0 + - (b + 2\,p_0 + \gamma)\,\zeta_0\,I_u + (b + \zeta_0)^2\,I_{2u}\} \quad , \tag{42}$$

for secondary N. In this case ($\gamma < 1$), equation (38) becomes

$$I_X = -\frac{1}{q(1 - \gamma)} \mu^\kappa \int_{t_0}^{t} \mu^{-\kappa - 1} \langle s_N\,\mu_R{}^{-x}\,\dot{\mu}_R\rangle\,dt \quad . \tag{43}$$

In a similar way, when $\gamma = 1$ it is obtained from equation (36) that

$$X_N(t) = X_N(t_0)\,e^{\mu_0 - \mu} + r\,q_p \frac{p_0}{b - \gamma} (-\gamma\,I_0 + b\,\zeta_0\,I_u) \quad , \tag{44}$$

for primary N, and

$$X_N(t) = X_N(t_0)\, e^{\mu_0 - \mu} + r\, q_S \left[\frac{p_0}{b - \gamma}\right]^2 \{(\gamma + p_0)\; I_0$$

$$-(b + 2\,p_0 + \gamma)\,\zeta_0\, I_u +$$

$$+ (b + p_0)\,\zeta_0^2\, I_2\, u\} \quad , \qquad (45)$$

for secondary N where now

$$u = 1 - b \qquad (46)$$

and

$$I_X = \frac{e^{-\mu}}{q} \int_{t_0}^{t} e^{\mu} \langle s_N\, e^{-X(\mu_R - 1)} \dot{\mu}_R \rangle\, dt \quad . \qquad (47)$$

Evolution of N is described by equations (41), (42) or (44), (45). In order to calculate I_X given by equations (43) or (47), a knowledge is needed of both the sources of N, $s_N(m)$, and of the explicit time dependence of $\mu(t)$.

For reasons of simplicity, let us adopt

$$- s_N\, \phi\;\, dm/d\tau \;= q/\tau_N = \text{constant} \qquad (48)$$

for stars with $m > m_N$. Here m_N is the mass of the lightest star that produces N and τ_N is its lifetime. Equation (48) corresponds to the case when all stars contribute, per unit time, the same amount to the production of N. The validity of equation (48) is discussed in §IV. Using equation (48) we get

$$[s_N\, \dot{Q}_R] = \frac{q}{\tau_N} \begin{cases} Q(t) - Q(0) & \text{if } t < \tau_N \\ Q(t) - Q_R & \text{if } t > \tau_N \end{cases} \qquad (49)$$

for any quantity Q. Hence, equation (43) becomes

$$I_X = -\frac{\mu^{\kappa}}{\tau_N\,(1-\gamma)} \int_0^t \mu^{-\mu-1}\, \frac{\mu^{-X+1} - 1}{-X+1}\, dt \quad , \qquad (50)$$

$$I_1 = -\frac{\mu^{\kappa}}{\tau_N\,(1-\gamma)} \int_0^t \mu^{-\kappa-1}\; \ln \mu \; dt \quad ,$$

if $t < \tau_N$, and

$$I_X = -\frac{\mu^{\kappa}}{\tau_N\,(1-\gamma)} \int_{\tau_N}^{t} \mu^{-\mu-1}\, \frac{\mu^{-X+1} - \mu_R^{\;-X+1}}{-X+1}\; dt \quad ,$$

$$I_1 = -\frac{\mu^{\kappa}}{\tau_N\,(1-\gamma)} \int_{\tau_N}^{t} \mu^{-\kappa-1}\; \ln \mu/\mu_R \; dt \qquad (51)$$

if $t > \tau_N$. Similar expressions can be obtained for the case $\gamma = 1$.

c) *Time Evolution for Constant and Exponential SFR*

In order to calculate integrals (50) and (51) and hence $X_N(t)$ from equations (41) and (42), let us adopt two idealized cases of gas consumption,

$$\mu = 1 - \omega'\, t \quad , \qquad (52a)$$

$$\mu = e^{-\omega' t} \quad , \qquad (52b)$$

which correspond to a SFR of the form:

$$\alpha \dot{S} / M_0 = \begin{cases} \omega & (53a) \\ \omega\, e^{-\omega' t} & (53b) \end{cases}$$

i.e., either a constant SFR (53a), or an exponentially decreasing SFR (53b). Here

$$\omega = \omega'/(1 - \gamma) \qquad (54)$$

is the effective star formation rate at $t = 0$. Notice that in these models

$$M_{tot}/M_0 = (1 - \gamma\, \mu)/(1 - \gamma) \quad , \qquad (55)$$

which tends to $1/(1 - \gamma)$ as $\mu \to 0$.

The retarded mass of gas, μ_R, is given by

$$\mu_R = \mu + \omega'\, \tau_N \quad , \qquad (56a)$$

or

$$\mu_R = \mu \exp\,(\omega'\, \tau_N) \qquad (56b)$$

Using equation (56) into integrals (50) and (51) one can obtain I_u either as an elementary function, or as an incomplete Beta function.

Models do not depend on τ_N directly, but only on the product $\omega' \tau_N$. In order to compare models with a different star formation rate (i.e. ω'), we have normalized models to a standard time, $\tau_{7.5}$, the time at which $12 + \log O/H = 7.5$. In this form the dependence on τ_N is through the parameter

$$\eta = \tau_N / \tau_{7.5} \quad . \qquad (57)$$

IV. DISCUSSION

One of the basic features of the observational N/O versus O/H diagram (Figure 2) is that for a given O/H value there are galaxies with values of N/O that differ by up to a factor of 4. Since the oxygen content is determined mainly by the gas fraction, this means that there must be another physical parameter whose variation from galaxy to galaxy explains the spread in N/O at a given O/H. Such a second physical parameter is also needed to explain the variation of O/H, at N/O nearly constant, in disks of spiral galaxies. In this case, it is needed that in a given galaxy, the second parameter varies from place to place across the disk.

We have identified three possible factors (in addition to a varying IMF, see §IV*b*) which can produce a spread in the N/O versus O/H diagram:

i) Differences in the value of the SFR.

ii) Differences in the history of star formation and,

iii) Differences in the accretion rates. In the models we have developed in §III these factors correspond, respectively to i) differences in η, at a fixed τ_N; ii) an exponential versus a constant stellar formation rate, and iii) differences in γ, the ratio of the accretion rate to the effective SFR.

The primary or secondary character of N production is one of the basic questions to answer. The other question is to know which of these factors, or which combination of them, could be physically responsible for the spread in the N/O versus O/H diagram. It is then necessary to construct theoretical diagrams for N primary and for N secondary, varying one of the factors at a time while keeping the other two constant.

Additionally to these basic parameters we must give, as an input to each model, a prescription for the yields (§IV*a*) and for the sources of nitrogen (§IV*b*). As an output, one obtains Np_O/Op_N so that a comparison with observations yields both a set of selected physical parameters (§IV,*e*,*f*) and the value of p_N/p_O (§IV*c*)

a) *IRA and Constant versus Variable Yields*

In simple models of chemical evolution, when *IRA* is not assumed and the yields are constant, it is found that abundances saturate as the galaxy evolves (Talbot and Arnett 1973). When dilution effects due to the ejection of unprocessed material by low mass stars become important, abundances cannot increase with time and tend to their yields. To reach this asymptotic value there can even be a decrease in the abundance of heavy elements after a "super metal rich phase". According to Talbot and Arnett, dilution effects become important when the gas fraction is about 0.1 and dominate completely when it is of the order of 0.01. Since oxygen abundance depends mainly on the gas fraction, this implies that during the evolution of a galaxy there is an upper limit to O/H, of the order of the yield. After this value has been attained, O/H *decreases* with further evolution of the galaxy. This is the reason why in Figure 3 of Talbot and Arnett (1974), once the upper limit to O/H has been reached, N/O still increases while O/H decreases with evolution.

When *IRA* is assumed and the yields remain constant, there is a substantial disagreement with non-*IRA* models, because *IRA* models predict that O/H always increases as the gas fraction decreases, even when oxygen abundance is larger than the upper-limit obtained in the non-*IRA* case.

However, the very fact that an upper limit to O/H, of the order of the yield, is not present in the O/H versus Mg/M_{tot} diagram questions the validity of the non-*IRA* constant yield models. In fact, Peimbert and Serrano (1982) have suggested that the heavy elements yield, p_Z, is not a constant. To explain the distribution of galactic and extragalactic H II regions in the O/H versus M_g/M_{tot} diagram, they need p_Z to increase with Z as indicated by equation (8). Such a change of p_Z with Z is explained, according to Peimbert and Serrano, by an increase with Z of the fraction of low mass stars in a stellar generation. If this is the case, all yields, including that of N, must increase in the same way, as assumed in equation (16). Hence, the main difference between a model with a constant yield and one with a variable yield, is that in the latter, a given abundance is attained at a much larger gas fraction than in the former. The consequence is that *IRA* is valid, in variable yield models up to O/H values much higher than for constant yield models.

To be definite about the effect of the *IRA* assumption for oxygen, let us consider as not physically significant those values of O/H corresponding to gas fractions which are less than .003. On the other hand, let us consider as marginally significant values of O/H corresponding to gas fractions between .01 and .003. For closed models and constant yields (Figures 3, 5 and 7), the region of marginal significance is between 9 and 9.1 in 12+ log O/H. On the contrary, for variable yield models (Figures 4, 6 and 8), it is between 9.4 and 9.9. For models with accretion, the same gas fraction is attained at lower O/H, but the non-*IRA* effects are also lower than for the closed models, being negligible in the extreme infall models ($\gamma = 1$).

We have calculated N/O versus O/H diagrams for models with constant yields p_Z between 0.0035 and 0.013 and for models with a variable yield $p_Z = 0.002 + 0.6\ Z$ as in Peimbert and Serrano (1982).

b) *The Sources for N and the IMF*

It must be stressed that we have not assumed any particular IMF to model N production. In equation (48) we have adopted an idealized form of the effective sources of N that involves the product $S_N \phi dm/d\tau$. If we assume a mass-luminosity relationship $L \propto m^y$ and an IMF $\phi \propto m^{-x}$, then we have that $dm/d\tau \propto m^y$ and so that assumption (48) is equivalent to

$$S_N \propto m^{x-y} \quad . \qquad (58)$$

If we take, for example, x = 2 (Serrano 1978) and y = 3.5, then $S_N \propto m^{-1.5}$. This form of S_N gives more weight to lower mass stars than S_N calculated from stellar evolutionary models, which is almost independent of mass (e.g., Iben and Truran 1978; Renzini and Voli 1981). Given our poor present knowledge of mixing in stars, the discrepancy between S_N from stellar models and equation (48) does not seem to be serious. On the other hand, adopting equation (48), it has been possible to obtain an analytic solution for N production. In any case, we have carried out similar calculations for an assumed

$$S_N \propto \delta\,(m - m_N) \quad , \tag{59}$$

where only stars of a single mass contribute to the N production, and found that the general features of the N/O vs O/H diagram are the same. Thus the results do not seem to be very sensitive to the specific form of $S_N(m)$.

c) *The Ratio of the Yields*

Absolute values of N/O in the models depend on the unknown ratio p_N/p_O. In order to fit the theoretical to the observed diagram it is necessary to shift the theoretical values of N/O. From this shift, and independently of all other assumptions, it is found that

$$p_N/p_O \sim 0.1$$

if N is primary, while

$$p_N/p_O \sim 67$$

if N is secondary. Since p_O is of the order of 0.5%, then p_N, and hence $q = [s_N]$ must be of the order of a third of an average stellar mass. Such a situation of a significant fraction of the ejecta having converted the original C into N is preciselv what stellar evolutionary models predict (e.g., Iben and Truran 1978; Renzini and Voli 1981).

d) *The Delay in the Production of N*

The evolution of nitrogen in a galaxy is described, in our models, by equations (41) to (45). We have constructed a number of theoretical N/O versus O/H diagrams using these equations for the production of N and equation (12) for that of O, assumed to be a constant fraction of Z. The main reason for modifying the instantaneous recycling approximation, given by equation (12) into equations (41) to (45) is to take into account the delay in the ejection of nitrogen that follows if it is assumed that stars that produce nitrogen are, on the average, of lower mass than those that produce oxygen.

The effects of the delay for the closed models can be seen in Figures 3 and 5. In these figures the model with

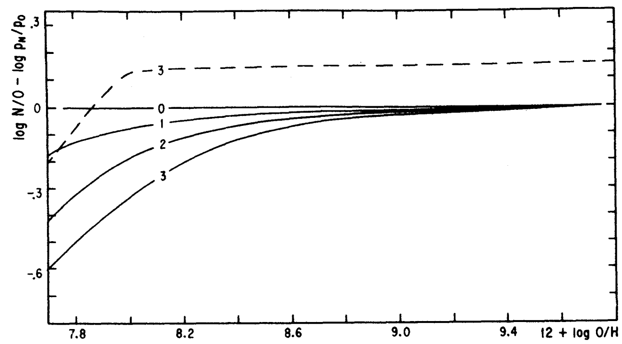

Fig. 3. Theoretical N/O vs. O/H diagram assuming primary production of N in closed models. The vertical axis represents log N/O for an assumed ratio of the yields $p_N/p_O = 1$. A constant vertical shift of $-\log p_N/p_O$ with the actual value of p_N/p_O, must be applied to the vertical axis in order to compare it with the observational values. Full curves correspond to a constant SFR, dashed curve to an exponential SFR. Lines are labelled by the value of η, which is proportional to the SFR. Models are calculated for a constant yield $p_Z = 0.0035$.

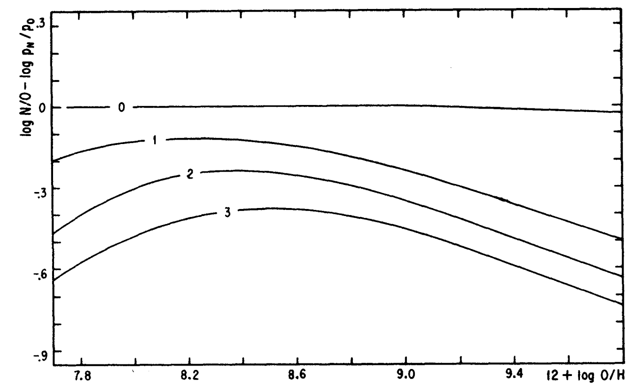

Fig. 4. As Figure 3 but for $p_Z = 0.002 + 0.6$ Z.

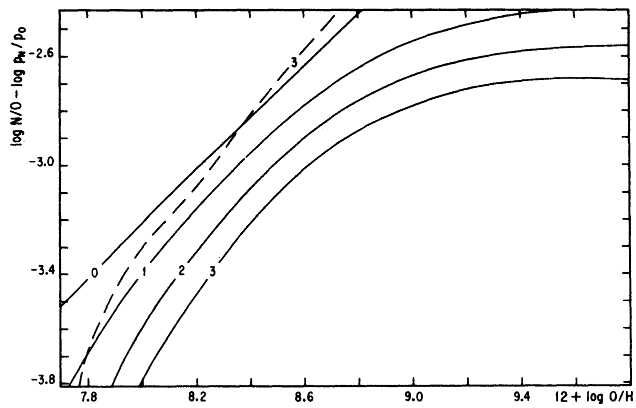

Fig. 5. As Figure 3 but for secondary production of N. As in Figure 3, a constant shift of $-\log p_N/p_O$, but now for the secondary N yield, must be applied to the vertical axis in order to compare it with the observational values.

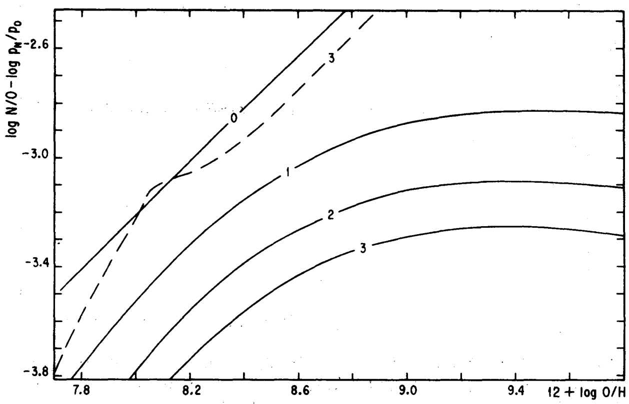

Fig. 6. Similar to Figure 3 but for $p_Z = 0.002 + 0.6$ Z and secondary production of N.

$\eta = 0$ corresponds to the instantaneous recycling approximation. Assuming for a moment an average and fixed star formation rate for galaxies, higher values of η correspond to higher values of τ_N and hence to longer delays. Figure 3 shows the case of primary production of nitrogen and Figure 5 corresponds to secondary production. In both cases, the main effect is to decrease N production, at any given O/H as the delay increases.

It must also be noticed that the condition that all stars capable of producing N do so before the gas in the galaxy is exhausted, delivers a critical value for η,

$$\eta_{crit} = \frac{1}{1 - \mu_{7.5}}$$

for each model of constant *SFR*. For galaxies with η greater than η_{crit} the *SFR* is so high, and so the lifetime of the galaxy is so short that when the gas in it is exhausted there are still low mass stars of the first stellar generation which have had no time to eject their nitrogen. On the other hand, for galaxies with η less than η_{crit}, all stars of the first stellar generation eject the N they have produced while star formation still continues. The values of η_{crit}, for closed models are shown in Table 1.

TABLE 1

DEPENDENCE OF η_{crit} ON YIELD

p_Z	η_{crit}
.002 + 0.6Z	2.88
.0035	4.62
.0065	8.12
.013	15.72

Although there can be galaxies with η greater than η_{crit}, i.e., those under strong bursts of star formation, and which would populate the lower right hand region of Figures 3 to 9, we shall assume that the majority of galaxies shown in Figure 1 and 2 have a *SFR* small enough so that η is less than η_{crit} for them.

e) *Effects of Variation of the SFR*

The procedure to compare the theoretical and the observational N/O versus O/H diagrams is to shift the theoretical values of N/O until a best fit is obtained, i.e., until a maximum overlap is attained between the regions occupied in this diagram by the models and by the observations. This determines a value for p_N/p_O. Then, the detailed form of the respective regions, i.e., the spread in N/O at each O/H, must be compared. From this comparison, we shall select the second parameter which is in a principal way physically responsible for the spread in the diagram.

If we assume a given τ_N, i.e., a given minimum mass for N production, models with different values of η have different values of $\tau_{7.5}$ and correspond then to models with different *SFR*. Figures 3 and 5 show then N/O versus O/H diagrams for galaxies with different values of the *SFR*.

In Figure 3 the N/O versus O/H diagram is shown assuming a primary production of N. Galaxies with higher values of the *SFR* have lower N/O ratios. In Figure 3 the lines of constant time have almost constant N/O, as Edmunds and Pagel (1978) suggested, furthermore, outer parts of galaxies with smaller O/H, correspond to smaller η and so to slower star formation in the outer parts. This implies a negative gradient of star formation across disks of galaxies, in agreement with observations (e.g., Serrano 1983). However, the general form of the observational diagram (Figures 1 and 2) is not reproduced because all models tend to the same N/O as O/H increases. Figure 3 corresponds to $p_Z = .0035$, and other values of the yield give nearly the same result because higher p_Z give narrower diagrams but larger values of η_{crit}. Differences in the history of the *SFR*, on the other hand, produce N/O values that differ by a constant over most of the O/H range. Even in this most favourable case the form of the permitted region in the N/O versus O/H diagram is not reproduced. Moreover, if the spread in the diagram is due to differences in the history of the *SFR*, the conclusion would be that disks in galaxies are formed first in the outer parts and later at the center. This is because with an exponential *SFR*, the average age of stars is larger than that for a constant *SFR*. This conclusion does not agree with current ideas on disk formation (e.g. Larson 1976). Notice that the argument used here about the *history* of the formation of the disk, is different from that given above about the variation of the *value* of the *SFR* with galactocentric distance. As it is shown in Figure 4, a variable yield and primary production give even worse results because galaxies are allowed to exist with high O/H and low N/O and they are not observed.

Alternatively, it is shown in Figures 5 to 8 how models

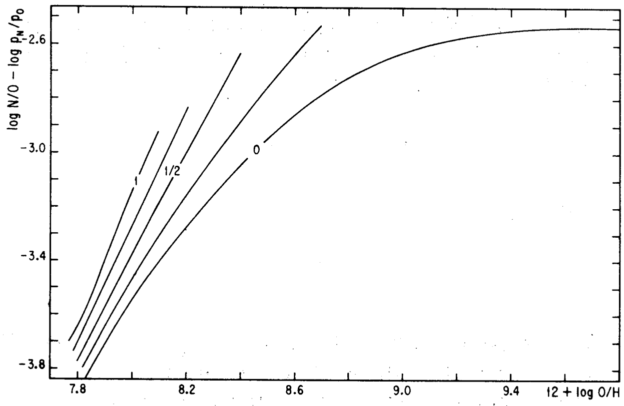

Fig. 7. Theoretical N/O vs. O/H diagram assuming secondary production of N, $p_Z = 0.0035$, $\eta = 1.75$ and constant SFR. Curves are labelled by different values of γ, the ratio of accretion to star formation.

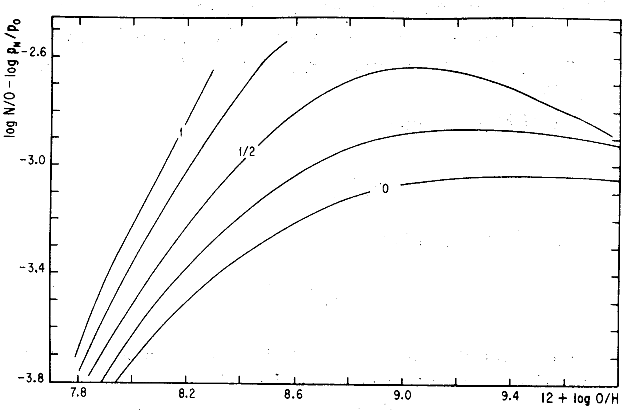

Fig. 8. Similar to Figure 7 but for a variable yield $p_Z = 0.002 + 0.6$ Z.

with a secondary production of N give rise to N/O versus O/H diagrams that reproduce the general form of the observed diagram. However, differences in the value and in the history of the *SFR* (Figure 5) produce a much too narrow diagram.Better agreement between models and observations is obtained for $\eta \sim 2$ and varying values of γ (the ratio of the accretion to the effective star formation rates) as it is shown in Figures 7 and 8. Evidence for infall of gas in our galaxy has been presented by Mirabel (1982), and by Mirabel (1983). If one accepts the high O/H values that have been derived by the method by Pagel *et al.* (1979, 1980), then models with a variable yield (Figure 8) seem to agree with observations while models with a constant yield predict higher N/O values for high O/H. Notice that in Figure 8 the Sun would correspond to $\gamma \sim 0.5$ in excellent agreement to the values derived by Twarog (1980) from the age metallicity relationship, by Tinsley (1981) from a model of the solar neighborhood and by Tosi (1982) from isotopic CNO ratios.

In Figure 9 we show the lines of constant M_g/M_{tot} and of constant time for models with variable yield. In this figure it can be seen how the galactic disk lies, roughly, on a constant age line. In these isochrones, the variation of N/O with O/H is small such that most of the disks shown in Figure 2 will correspond to a line of a given age in Figure 9. Moreover, in models of variable yield the gas is not used as rapidly as in models of constant yield. For example, in Figure 9, the galactic M_g/M_{tot} varies between 0.04 in the inner galaxy and 0.16 in the outer regions; in models with $p_Z = 0.0035$, on the other hand, this variation goes from M_g/M_{tot} of 0.002 to 0.08. Notice also that for the LMC and SMC we obtain from Figure 9, $M_g/M_{tot} = 0.17$ and 0.38, respectively, in good agreement with the observed values of 0.12 and 0.42 (Lequeux *et al.* 1979).

The large scatter in the N/O values presented in Figures 1 and 2 can be explained by making use of Figure 9 as follows: O/H is fixed primarily by the gas fraction, though at small gas fractions it is reduced by inflow when present. N/O is time dependent, and increased by inflow because of the longer time then needed to get down to a given gas fraction. For any given oxygen abundance, there is a relationship between inflow rate and time expressed by the isochrones in Figure 9. In terms of this figure, one can classify the different irregular galaxies; e.g., NGC 4449 is unaffected by inflow, SMC is very strongly affected, NGC 6822 is one of the youngest galaxies around and LMC one of the oldest. The age classification is fairly similar to that of Edmunds and Pagel (1979), but in the present work it is based on secondary as opposed to primary nitrogen and it involves inflow as an essential ingredient.

There are no objects with log O/N > 1.8; at first sight

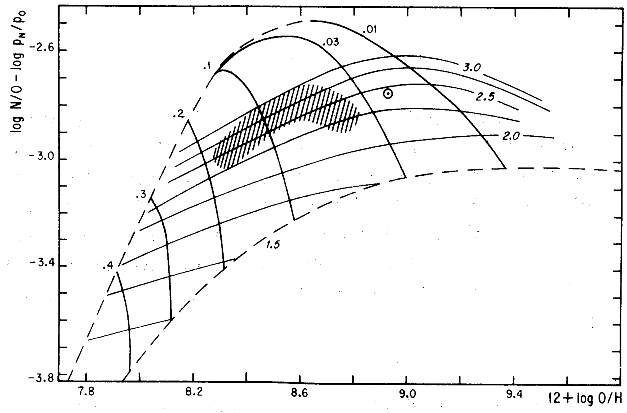

Fig. 9. Isochrones and lines of constant M_g/M_{tot} for the models in Figure 8. The nearly horizontal lines are isochrones, and are labelled by the value of t/τ_N. The nearly vertical lines are labelled by the value of M_g/M_{tot}, which is constant along each line. The position of the galactic H II regions and of the Sun in Figure 8 are also reproduced in this diagram.

this seems to indicate that some primary production of N is needed, nevertheless by looking at Figures 1 and 9 (see also the equivalent figures by Pagel and Edmunds 1981) it can be seen that more likely it is due to the lack of galaxies with gas mass fractions larger than 40%. The lack of galaxies with $M_g/M_{tot} > 40\%$ probably indicates that there are very few young galaxies or that these galaxies do not have bright H II regions and therefore have not been placed in the N/O versus O/H diagram.

Although our choice for the sources of N is arbitrary, and given by equation (48), results seem to be independent of this prescription. Models based on the alternative prescription (59) give rise to N/O versus O/H diagrams similar, but narrower, than those based on equation (48).

g) *The N Producers*

Isochrones in Figure 9 are calculated in units of τ_N, the maximum lifetime of N producers. From a known age, we can calculate τ_N, and so put the times of evolution on an absolute basis. Since we have an estimate for the ages of the sun and of the solar neighborhood, we shall choose the former to date Figure 9.

The Sun seems to be metal-rich relative to H II regions at similar galactocentric distances. Based on the presence of ^{26}Al and ^{107}Pb in meteorites, Schramm and Olive (1982) have suggested that the solar system formed very near a supernova explosion, and this could account for a metallicity of the Sun being a factor of 2 higher than that time. In any case, the Sun corresponds in Figure 9 to $t_\odot/\tau_N \sim 2.5$, where $t_\odot$ is the age of the galactic disk when the Sun was formed, i.e., $t_\odot \sim 7\text{-}8$ Gy.

Hence

$$\tau_N \sim 3 \text{ Gy}$$

and so

$$m_N \sim 1.2 - 1.3 \, m_\odot \quad .$$

It is reasonable to expect such a lower stellar mass limit for the production of N since stars of mass less than $\sim 1.2\, m_\odot$ are not effective in producing nitrogen (Iben and Truran 1978). Notice also that with this calibration of τ_N, the Magellanic Clouds have an average stellar age about 2/3 that of the solar vicinity in good agreement with the results of Cohen (1982) and Mould and Aaronson (1982).

V. CONCLUSIONS

1) It seems unlikely that N is mainly a primary product. The best fit between observations and theory is for secondary production of N. A small primary contribution, is not excluded.

2) Models with accretion, secondary N and yield increasing with metallicity explain the N/O versus O/H diagram. From the best fit model we derive: *i)* An infall rate of 1/2 the star formation rate for the solar N/O. *ii)* Stars relevant for the production of nitrogen have masses.

$$m \gtrsim 1.2\text{-}1.3 \, m_\odot \quad ,$$

and ages

$$\tau \lesssim 3 \text{ Gy} \quad .$$

iii) The average age of the stellar component of the MC is $\sim 2/3$ that of the solar vicinity. *iv)* M_{gas}/M_{tot} values vary from 0.04 in the inner regions to 0.16 in the outer regions of the galaxy. Constant yield models, on the other hand, would need a higher variation of M_g/M_{Tot}.

3) Disks of galaxies are of approximately constant age and have small variations of M_g/M_{tot} across them. They have O/H gradients and much smaller N/O gradients because there is a gradient of γ, the ratio of accretion to the effective star formation rate. Accretion is more important, *relative* to star formation in the outer parts of the disk.

4) If N is primary

$$p_N/p_O \sim 0.1$$

while if it is secondary

$$p_N/p_O \sim 67$$

which implies that in about 1/3 of an average stellar mass the original C is converted to N.

5) The main difference between models of constant and variable yield is for log O/H $\gtrsim -3$. It is important to elucidate whether or not the values of O/H derived with the method by Pagel *et al.* (1979) and Pagel, Edmunds and Smith (1980) for metal-rich H II regions are correct.

6) If our models are valid then most of the N in the interstellar medium has been ejected by PN progenitors and not by novae, contrary to the suggestion by Williams (1982).

7) From our models, it is predicted that some irregular galaxies have large infall to star formation rates while others do not, this prediction might be tested by a combination of 21-cm observations (infall) and Hα observations (star formation).

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