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ON THE MASS EJECTED BY SUPERNOVA EXPLOSIONS

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RESUMEN

Se presenta un modelo sencillo para poder calcular la masa que eyectan las explosiones de supernova. Se encuentra que las supernovas ocurridas en los años 185, 1006, 1576 y 1604 D.C., todas ellas clasificadas como probables ó posibles supernovas de tipo I, eyectaron una masa de entre 0.1 y 0.4 $M_\odot$ a una velocidad aproximada de 10 000 km s^{-1}. Este rango de masas sugiere que un objeto colapsado se encuentra en la parte central de los remanentes producidos por estas explosiones si la estrella precursora fue una enana blanca de masa cercana al límite de Chandrasekhar. Para el Cangrejo encontramos una masa eyectada de 0.45 $M_\odot$ y señalamos que este valor es favorable a la proposición de que estrellas de helio moderado son las progenitoras de este tipo de supernovas. Finalmente encontramos una masa eyectada igual a 3.1 $M_\odot$ para Cas A, lo cual indica que una supernova de tipo II produjo este remanente. Esta masa eyectada es similar a lo que se esperaría si el progenitor fuera una estrella tipo OBN.

ABSTRACT

A simple model is developed in order to calculate the mass ejected by supernovae. We find that the 185, 1006, 1572 and 1604 AD events, all of them classified as either probable or possible type I supernovae, ejected between 0.1 and 0.4 solar masses with an expansion velocity of roughly 10 000 km s^{-1}. This range of masses suggests that a collapsed object is at the center of the remnants produced by these supernovae if the precursor was a white dwarf whose mass was close to the Chandrasekhar limit. For the Crab we obtain an ejected mass of 0.45 $M_\odot$ and point out that this value is not in contradiction with a proposal in which the moderate helium stars are good candidates for producing this kind of supernovae. Finally we obtain an ejected mass of 3.1 $M_\odot$ for Cas A, indicating that a type II event produced this remnant. This ejected mass is close to what would be expected for a progenitor like an OBN star.

Key words: SUPERNOVAE – SUPERNOVA REMNANTS

I. INTRODUCTION

Practically every stage of evolution of supernova remnants (SRNs) has been modelled with detailed hydrodynamic codes, and in a variety of physical situations (Chevalier 1977 and references thereafter). Furthermore, some models have also considered the problematic question of the transition from one stage to the next. Thus, it would seem superfluous to present a very simple model in which the structure of the remnant is not taken into account and the interstellar medium is assumed to be uniform. Nevertheless this is what we do in this paper by proposing a simple formula to describe the evolution of a SNR during the free expansion and self-similar expansion phases. Obviously we do not pretend to extend our knowledge of the physical structure of the remnant during these two stages. Our object is to present a simple and manageable model to calculate the mass ejected by supernova (SN) explosions, which, in most models, is taken as a free parameter. The ejected mass is a quantity of some importance for the structure of a supernova, the mechanism producing it and the type of star that became one. It is also relevant for the chemical evolution of galaxies since, if we know the SN frequency and the chemical composition of the ejecta, we can determine the mass of different elements that SN explosions deposit in the interstellar medium. Of course, this is not the first effort made in order to estimate the ejected mass. Based on light curve models and on the sophisticated models for SN explosions it has been estimated that type I supernovae eject approximately 0.5 $M_\odot$ whereas type II seem to eject as much as 10 times more (Chevalier 1977). On the other hand Utrobin (1978) developed a model in which energy is slowly pumped into the remnant and found an ejected mass of approximately 0.25 $M_\odot$ for SN 1006, Tycho and Kepler and 0.73$M_\odot$ for the Crab. As we will show, our results are very similar to these estimates.

II. THE MODEL

The earliest descriptions of the dynamical evolution of SNRs assumed an spherically symmetric explosion in an homogeneous medium. Both assumptions clearly are in opposition with reality, as is evident from the morphology of the Crab, the quasi stationary flocculi in Cas A and the interaction of IC 443 with ambient molecular clouds. Later descriptions have taken into account either the presence of primeval inhomogeneities –in the form of circumstellar material (Chevalier 1982*a*), pre-ejected clumps, or diffuse and molecular clouds (McKee and Cowie 1975)– or the formation of such inhomogeneities by the excitation of different kinds of instabilities within the remnant (Gull 1973). The effect of these inhomogeneities becomes increasingly important as the remnant evolves (Shaver 1982). Yet, radio maps of young SNRs show an overall structure that is almost spherically symmetric (see Bell 1977, for Cas A; Henbest 1980, for Tycho), indicating that these inhomogeneities have only a localized effect in young objects and that their evolution is not profoundly altered by them. Furthermore, self-similar solutions of the interaction of the ejecta with the surrounding medium indicate that the latter is probably homogeneous around type I supernovae, as expected from the kind of progenitor star associated with these events, whereas the density profile of the circumstellar gas around type II supernovae will be of some importance only in the earlier stages (Chevalier 1982*b*). Thus, though the presence of inhomogeneities must be taken into account when interpreting the observations, it is probable that the first models, naive as they might be, are essentially correct when the overall properties of a young remnant are considered.

In this simple scenario the evolution of the SNR is described in four stages (Woltjer 1972). As long as the mass ejected by the SN explosion (M_e) is much larger than the mass accreted by the shock wave (M_{SW}), the SNR expands with a constant velocity and almost all its energy is kinetic. The remnant can be described by a self-similar solution when $M_{SW} \gg M_e$ and radiative losses are not important. During this phase the kinetic energy is a constant fraction of the total energy. When radiative losses are important the evolution of the SNR is more appropriately described by the condition of momentum conservation. Finally, when the expansion velocity is equal or less than the sound speed, the remnant disappears as such. The inhomogeneity of the interstellar medium will become important only when the radius of the SNR is larger than 15 pc, since from here on collisions with diffuse clouds become more and more important. At this point the shock velocity is approximately equal to 300 km s^{-1} and the remnant is about to enter into the radiative phase. Consequently it follows that the idealized circumstances of a symmetric explosion in an homogeneous medium are practically verified during the first two stages of evolution of a SNR. We will assume that these two stages can be approximately described with

$$1/2(M_e + M_{SW}/\alpha)\, U^2 = E \qquad (1)$$

where E is the energy deposited in the remnant by the SN explosion, U is a characteristic velocity and α is a parameter that is proportional to the ratio of the kinetic to the total energy during the self-similar expansion phase. This equation can be easily integrated, the result being

$$U\, t/R = {}_1F_2\, [-1/2, 1; 4/3; Z/(Z+1)] \qquad (2)$$

where

$$Z = M_{SW}/\alpha\, M_e\ ; \qquad (3)$$

t is the age of the remnant, R its radius and ${}_1F_2$ represents a Gaussian hypergeometric function, which is a universal function of the parameter Z and is plotted in Figure 1. As expected, when $Z \simeq 0$ ($M_{SW} \ll M_e$), ${}_1F_2 \simeq 1$, and the remnant moves at constant velocity. During the self-similar expansion phase $M_{SW} \gg M_e$ ($Z \gg 1$) and ${}_1F_2 = U\, t/R \to 0.4$. This is identical to Sedov's (1959) adiabatic solution if we interpret U as being equal to the shock velocity. Our result is somewhat different from what Solinger, Rappaport and Buff (1975) obtained from an isothermal model, where $U\, t/R \simeq 0.29$. Taking into account that this simple approach can only match Sedov's solution for the self-similar expansion phase, we must then conclude that U has to be regarded as the shock velocity so that α must be equal to 0.85 (from equation 1).

The procedure to estimate the ejected mass and the total energy is extremely simple. Given U, t and R we can evaluate ${}_1F_2$ which, from Figure 1, leads to a value for Z. Given the value of Z we can determine the ejected mass provided that the mean interstellar density is known so that the swept-up mass can be calculated. Finally, the total energy can be determined from equation (1). Thus, the main problem is to find a way to calculate the value of the hypergeometric function and the mean interstellar density. To do the former we can use angular quantities, which leads to

$${}_1F_2[-1/2, 1; 4/3; Z/(Z+1)] = 2 \times 10^3\, U_T\, t_3/\beta \qquad (4)$$

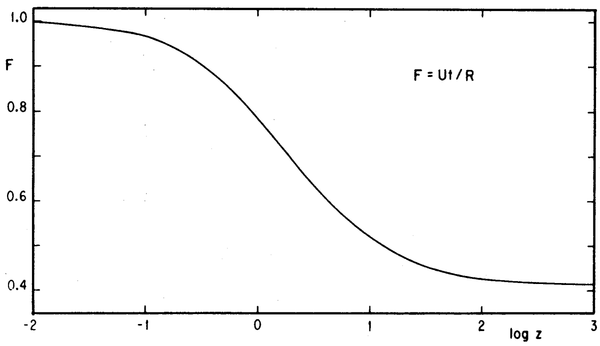

Fig. 1. Plot of the hypergeometric function ${}_1F_2$ (– 1/2,1;4/3;Z/(Z + 1)). This function is equal to Ut/R, where U is the shock velocity, t is the age of the remnant and R its radius.

where U_T is the proper motion in arcsec per year, t_3 is the age in 10^3 year and β is the angular diameter in arcseconds. On the other hand, if we use linear quantities we find

$$ {}_1F_2[-1/2, 1; 4/3; Z/(Z+1)] = 1.02\, U_8\, t_3/R_{pc} \quad (5)$$

where U_8 is the shock velocity in 1 000 km s^{-1} and R_{pc} is the radius of the SNR in parsec. In the following section we will discuss how we obtain some of these quantities and what are the results for various objects.

III. DISCUSSION

The objects that will be considered here are only those for which the age is known from historical records. There are 7 SNRs that are either probably or certainly associated to historical records of SN explosions: RCW86, SN1006, Crab, 3C58, Tycho, Kepler and Cas A (Clark and Stephenson 1977; Ashworth 1980). According to Strom, Angerhofer and Velusamy (1980) CTB80 is the remnant associated with the 1408 AD supernova. This conclusion has been disputed by Becker, Helfand and Szymkowiak (1982), who found that the ratio of X-ray to radio luminosity is much too small, the size is too large and the velocity is too small to identify CTB80 as the remnant of the 1408 AD supernova. Consequently we omit any discussion of this object. The list of objects considered here, together with some relevant parameters associated with them, appears in Table 1.

Of the parameters required to estimate ${}_1F_2$ and the ejected mass, the distance is probably the most uncertain, and we will only indicate how this quantity was selected in each case. The shock velocity is liable to an indirect estimate from the determination of the maximum temperature of the X-ray emitting region, which is related to the gas immediately behind the shock front. Assuming temperature equilibrium between electrons and ions, the Rankine-Hugoniot conditions lead to

$$U_8 = 0.612\, T_7^{1/2} \quad (6)$$

where T_7 is the maximum X-ray temperature in 10^7K. We assumed that the mass per particle is 2×10^{-24} g and will maintain this assumption throughout. Unfortunately this equation leads to velocities that are smaller than those observed in most young SNRs and, in particular, in Cas A (Pravdo *et al.* 1976; Pravdo and Smith 1979). Such a difference can be explained if ions and electrons have not yet reached temperature equilibrium or, more fundamentally, if the 'classical' form of the Rankine-Hugoniot conditions can no longer be applied in shock fronts dominated by the collisionless processes that probably occur in young remnants. In any case equation (6) is an unreliable instrument to determine the shock velocity, leading only to lower limits, and should be used only if there is no other alternative.

To determine the swept-up mass we must establish the value of the mean particle number density of the surrounding interstellar medium. The preshock density can be determined from the density dependent sulphur line ratio I(6717)/I(6731). According to Bohigas *et al.* (1983) it is given by

$$n_0 \simeq 0.01\, (n_e/V_8)^{2/3}\ \text{cm}^{-3} \quad (7)$$

if the shock velocity is larger than 70 km s^{-1}. In this equation n_e is the electron density in cm^{-3} and V_8 is

TABLE 1

PARAMETERS CHARACTERIZING SOME YOUNG SNRs

Object	T_0^a	U_8 (10^8 cm s^{-1})	β (arcsec)	D (kpc)	n_0 (cm^{-3})	$M_e/M_\odot$	E_{50} (10^{50} erg)
RCW86	185	$\gtrsim$ 1.5	2340	1.0	0.10	$\gtrsim$ 0.2	$\gtrsim$ 0.7
SN1006	1006	1.85[b]	1800	1.0	0.19	0.01	0.8
		2.7[c]	...	...	...	0.37	2.0
Crab	1054	1.5	350	2.0	0.8	0.45	0.2
3C58	1181	$\gtrsim$ 1.3	420	2.6	0.5	0.01[d]	$\gtrsim$ 0.1
Tycho	1572	3.0	460	3.0	0.70	0.10	3.2
Kepler	1604	$\gtrsim$ 1.9	190	4.0	2.4	0.05[d]	$\gtrsim$ 0.7
Cas A	1680	7.0	340	3.3	1.0	3.1	29

a. T_0 is the time at which the explosion occurred.
b. Is based on a proper motion.
c. Is based on Lasker's (1981) observations.
d. Rough estimates since the age, radius and velocity lead to conclude that these remnants are in or very close to the self-similar expansion phase.

the velocity of the shock preceding the optically active regions in the SNR. This density is above the mean value, otherwise there would be no optical emission. But since the ram pressure (NV^2) along the shock front is approximately constant, it follows that the mean interstellar density is

$$\langle n_0 \rangle \approx 0.01\,(V_8/U_8)^{4/3}\,(n_e/U_8)^{2/3} \quad . \qquad (8)$$

The mean interstellar density can also be estimated from the X-ray flux of the hottest component. If the volume of the emitting region is $4\pi R^2\ \delta R\ f$ (where $\delta R = 0.1R$ is the width of the remnant's shell and f is the filling factor), it follows that

$$\langle n_0 \rangle \approx 10^4\,[F_{34}(\delta E, T)/\{\beta^3\,D_{kpc}\,f\,P_{-23}\,(\delta E, T)\}]^{1/2} \quad (9)$$

where F_{34} (δE,T) is the flux in the energy range δE of a plasma at a temperature T in units of 10^{34} erg s^{-1} kpc^{-2}, D_{kpc} is the distance to the remnant in kiloparsec and P_{-23} (δE,T) is the volume emissivity in units of 10^{-23} erg s^{-1} cm^{-3}. The volume emissivity is taken from Raymond and Smith (1977), who calculated it for a plasma in equilibrium, which is probably not the case in young SNRs such as Tycho (Pravdo *et al.* 1980) and Cas A (Pravdo and Smith 1979). If so, the real volume emissivity might be up to 10 times larger (Shull 1982), so that the value of $\langle n_0 \rangle$ tends to be overestimated when this assumption is followed. On the other hand, the value of the filling factor is difficult to determine. We will assume throughout that f = 1, which probably implies that $\langle n_0 \rangle$ is underestimated. Overall we expect that the effects of these two assumptions (P_{-23} in equilibrium and f = 1) tend to cancel each oter.

a) *RCW86*

Clark and Stephenson (1977) identify this object as the probable remnant of the 185 AD supernova event which, on the basis of the long period of visibility is favored as being type I. Based on the apparent magnitude at maximum the authors place the object at a distance of 1 kpc from us, slightly more than Minkowski's (1968) estimate but identical to the value favored by Ruíz (1981). The optical size of the object is 480 arcsec X 1900 arcsec (van den Bergh, Marscher, and Terzian 1973), whereas the radio diameter is 2 340 arcsec (Clark and Caswell 1976). Assuming the latter to be the correct value we find a radius of 5.6 parsec if the object is 1 kpc away from us. There are no direct determinations of the shock velocity, but X-ray observations by Winkler (1978) and equation (6) indicate that $U_8 \gtrsim 1.5$. All these leads to $M_{SW}/M_e \lesssim 10.71$. Optical observations lead to an electron density of 1 300 cm^{-3} and a shock velocity of roughly 100 km s^{-1} (Ruíz 1981). With these values and $U_8 = 1.5$ we find that $\langle n_0 \rangle \simeq 0.03$ from equation (8). On the other hand it can be deduced from Winkler (1978) that $F_{34} \simeq 2.24$ in the energy band of 2 to 10 keV which leads to $\langle n_0 \rangle \simeq 0.11$ for a plasma at 6×10^7 °K. This is reasonably close to the value deduced from the optical observations. Consequently, taking $n_0 \simeq 0.1$, we obtain $M_{SW} \simeq 2.2\ M_\odot$, $M_e \gtrsim 0.2\ M_\odot$ and $E \gtrsim 7 \times 10^{49}$ erg. The ejected mass, though a lower limit, seems to support the idea of a type I progenitor for RCW86. The energy is approximately one seventh of what is expected in an SNI (Chevalier 1977), but still larger than the energy associated with the Crab.

b) *SN 1006*

According to Clark and Stephenson (1977) the remnant of the 1006 AD supernova event is G327.6 + 14.5 which is at a distance of 1 ± 0.3 kpc. Based on the $\Sigma - d$ relation Clark and Caswell (1976) place the object at a distance of 1.3 kpc and give a radio diameter of 2040 arcsec. On the other hand, Pye *et al.* (1981) determined an X-ray diameter of 1800 arcsec and a mass between 5 and 15 solar masses for the object. This mass estimate is based on the assumptions that the plasma is in equilibrium and has a cosmic composition. Both are probably wrong and lead to an unrealistically small volume emissivity and, consequently, to an excessively large X-ray mass (Long, Dopita, and Tuohy 1982). There are no direct measurements of the shock velocity, though Hesser and van den Bergh (1981) found a proper motion of 0.39 ± 0.06 arcsec yr^{-1} which corresponds to a linear velocity of 1 850 km s^{-1} if the object is at a distance of 1 kpc. With an angular diameter of 1800 arcsec this motion leads to $M_{SW}/M_e \simeq 170$, which implies that the ejected mass is almost completely diluted in the interstellar medium. Such a conclusion is in disagreement with the high Fe II abundance discovered by Wu *et al.* (1983). On the other hand, based on the model of Chevalier and Raymond (1978), Lasker (1981) deduced a shock velocity of at least 2 700 km s^{-1}, which is closer to the velocity that Wu *et al.* (1983) deduced from the width of Fe^+ absorption lines. A shock velocity of 2 700 km s^{-1} leads to $M_{SW}/M_e \simeq 5.4$, and in this case one must conclude that the remnant is still far from the self-similar expansion phase. The mean interstellar density can be estimated from the X-ray observations of Winkler *et al.* (1979), who found a plasma temperature of 1.9×10^7 K (leading to a velocity of 850 km s^{-1}) and F_{34} = 4.86 in the energy band between 0.2 and 1 keV. This leads to $\langle n_0 \rangle \simeq 0.19$, which is very similar to the value of 0.25 that Hesser and van den Bergh (1981) give from purely qualitative arguments. This density leads to a swept-up mass equal to 2 $M_\odot$, somewhat smaller than the disputable range of values given by Pye *et al.* (1981), but similar to the determination of Hesser and van den Bergh (1981). We find that the proper motion measured by Hesser and van den Bergh leads to $M_e \simeq 0.01\ M_\odot$ and $E \simeq 8 \times 10^{49}$ erg, whereas Lasker's results lead to $M_e \simeq 0.37\ M_\odot$ and $E \simeq 2 \times 10^{50}$ erg. It is evident that both possibilities indicate a type I progenitor for the remnant, as is also suggested by the supernova light curve (Clark and Stephenson 1977), by the high iron abundance (Wu *et al.* 1983) and by the large distance of the remnant above the galactic plane.

Of the two alternatives we find Lasker's more attractive, not only because it leads to a smaller dilution for the ejected mass, but also because a value of 0.01 $M_\odot$ for it is too small. Even if we did underestimate the swept-up mass by a factor of 5, we still find that an ejected mass of 0.5 $M_\odot$ is hard to believe. On the other hand an ejected mass of 0.37 $M_\odot$ is quite similar to the value that Utrobin (1978) obtained with a very different approach.

c) *Crab*

There is convincing evidence indicating that this remnant has been accelerated since the 1054 AD supernova explosion (Trimble 1968). This situation is obviously beyond the set of hypotheses used here and the model is of no use unless the energy deposited in the ejecta at the time of the explosion is larger than the energy that the remnant has received during its lifetime. We will assume that this is so.

The optical size of the object is 290 arcsec × 420 arcsec (van den Bergh *et al.* 1973) which leads to a mean angular diameter of 350 arcsec. Situated at a distance of 2 kpc this implies a radius of 1.7 pc. On the other hand, Trimble's radial velocity measurements indicate that $U_8 \simeq 1.3$-1.5, leading to $M_{SW}/M_e \simeq 1.35$-0.54. With this quantity and the observed helium relative abundance, A(He), it is possible to calculate the fractional helium mass, f(He), in the mass ejected by the supernova explosion from the following equation

$$f(He) \approx (1 + M_{SW}/M_e)/[1 + 0.25/A(He)] - 0.28\ M_{SW}/M_e \quad (10)$$

The relative abundance of helium is somewhere between 0.4 and 1 (Davidson 1979; Henry and MacAlpine 1982). These abundances are incompatible with $M_{SW}/M_e \simeq 1.35$. On the other hand the lower limit (0.54) leads to $f(H_e) \simeq 0.9$-1.1, suggesting that the relative abundance of helium in the Crab nebula cannot be much larger than 0.5 if the shock velocity is equal to 1 500 km s.$^{-1}$ In any case it follows that the supernova ejected an envelope which was essentially made up of helium.

It is difficult to estimate the mean interstellar density since our approach is based on the hypothesis that emission from the SNR is dominated by the excitation of the shock wave, which is not the case in the Crab. Yet, it must be observed that equation (7) is based on two premises, namely that compression is limited by the magnetic field and, more importantly, that the total pressure is uniform within the remnant. If this is the case in the Crab, and it seems reasonable to assume so, we can then use this equation to estimate the mean interstellar density. Emission line spectra of different regions in the Crab (Davidson 1979; Fesen and Kirshner 1982) indicate that the mean value of I(6717)I(6731) is approximately equal to 0.9. With $U_8 = 1.5$ we find that the preshock density is approximately equal to 0.8 cm^{-3}. Consequently, the mass of interstellar material that has been swept-up by the Crab is roughly equal to 0.24 $M_\odot$. This leads to an ejected mass of 0.45 $M_\odot$ and an energy of 2×10^{49} erg. According to Utrobin (1978) the mass ejected in the 1054 AD supernova explosion was roughly equal to 0.7 $M_\odot$, which is reasonably close to the value predicted here.

d) *3C58*

This object has been identified as the remnant of the 1181 AD supernova explosion (Clark and Stephenson 1977). The identification was disputed by Becker, Helfand and Szymkowiak (1982), one of their main arguments being the low radial velocities found by Kirshner and Fesen (1978). New spectroscopic observations of faint filaments located near the center of 3C58 (Fesen 1983) showed that these velocities were probably due to projection effects and that the expansion velocity is larger than 1 000 km s^{-1}, indicating that 3C58 is indeed the remnant of the 1181 AD SN event. The distance to the object is now in dispute. Goss, Schwarz and Wesselius (1973), gave a distance of 8 kpc to the SNR which leads to a mean expansion velocity of 10 000 km s^{-1} and implies a recent and sudden deceleration since the present expansion velocity is roughly 10 times smaller. This seems very unlikely. However, recent observations lead to a distance of 2.6 kpc (Green and Gull 1983), and a better agreement between the mean and the present expansion velocities is obtained. Adopting this value, and taking into account that the remnant is an ellipsoid whose size is 5 arcmin × 9 arcmin (Fesen 1983), we obtain a radius of 2.6 parsec. This leads to a shock velocity of at least 1 300 km s^{-1} if the remnant is not beyond the self-similar expansion phase.

Adopting this velocity, and with a mean value of 1.06 for I(6717)/I(6731) (Kirshner and Fesen 1978), we obtain $\langle n_0 \rangle \cong 0.5$ and $M_{SW} \cong 0.54\ M_\odot$. This means that the ejected mass cannot be much larger than 0.01 $M_\odot$. This is very unlikely, and it probably indicates that the object is not yet in the self-similar expansion phase and has a larger velocity. Assuming that $U_8 = 2$ we obtain $M_{SW}/M_e \cong 4.3$, $\langle n_0 \rangle \cong 0.4$, $M_{SW} \cong 0.43\ M_\odot$ and, consequently, $M_e \cong 0.1\ M_\odot$ and $E \cong 2 \times 10^{49}$ erg. These numbers are similar to those characterizing the Crab which is, as is well known, an SNR with similar features in the radio domain.

e) *Tycho*

The 1572 event has become the typical example of a Type I supernova. The optical, X-ray and radio angular diameters are all very similar, with the mean value being approximately equal to 460 arcsec (van den Bergh *et al.* 1973; Clark and Caswell 1976; Fabbiano *et al.* 1980; Strom, Goss and Shaver 1982). Kamper and van den Bergh (1978) found that the proper motion of the optical filaments is approximately equal to 0.25 arcsec yr^{-1}, which is remarkably similar to the value found by Strom *et al.* (1982) from radio observations. When applied to equation (4) all these quantities lead to $M_{SW}/M_e \cong 34$, which means that the remnant is strongly decelerated but not quite yet in the self-similar expansion phase. There has been some controversy as to the distance to the object. Based on the width of the hydrogen optical lines Chevalier, Kirshner and Raymond (1980) concluded that the shock velocity must be approximately equal to $2\,300 \pm 500$ km s^{-1}, which, combined with the measured proper motion, leads to a distance of 2.3 ± 0.5 kpc. This is 2-3 times less than the estimates based on radio absorption (Williams 1973; Schwarz, Arnal and Goss 1980). Most authors have discarded these radio estimates, opting for a distance of 3 kpc, which leads to a radius of 3.3 pc and a shock velocity of 3 000 km s^{-1}. We will follow suit. Because of the peculiar optical spectrum of Tycho (Kirshner and Chevalier 1978) it is impossible to determine the preshock density from the sulphur line ratio. On the other hand, X-ray observations by Davison, Culhane and Mitchell (1976) show that the spectrum can be fitted with the model of Raymond and Smith (1977) only if there are two components at different temperatures. The spectrum is in the 1.5-6 keV energy band and the hottest component is at a temperature of 4.1×10^7 K. With $F_{34} \simeq 1$ we obtain $n_0 \simeq 0.7$ cm^{-3}, very similar to the result obtained by Seward, Gorenstein and Tucker (1983), so that $M_{SW} \simeq 3.2\ M_\odot$, $M_e \simeq 0.10\ M_\odot$ and $E \simeq 3.5 \times 10^{50}$ erg. Both the ejected mass and the energy are similar, though slightly less than what is expected in type I supernovae (Chevalier 1977).

f) *Kepler*

The explosion that produced this SNR occurred in 1604 AD and the event has been classified as a type I supernova (Clark and Stephenson 1977). The radio angular diameter of the object is approximately equal to 190 arcsec (Clark and Caswell 1976). Based on estimates of the absolute blue magnitude of type I supernovae, Dennefeld (1982) finds that the distance to the remnant is approximately equal to 4.1 kpc. This number compares favorably with distance estimates based on the Balmer decrement and on the ratio of the S^+ auroral to transauroral lines (Danziger and Goss 1980; Dennefeld 1982), both leading to a distance of roughly 3 kpc. Thus, assuming a distance of 4 kpc to the object we find a radius of 1.8 pc for it. The mean proper motion of the optical filaments has been determined by van den Bergh *et al.* (1973), who found it to be equal to 0.03 arcsec yr^{-1}, and van den Bergh and Kamper (1977), who give a much smaller value of 0.005 arcsec yr^{-1}. At a distance of 4 kpc these two estimates lead to radial velocities of 570 and 100 km s^{-1}. The latter might be identified as the velocity of the shock wave preceding the optical filaments, but not as being representative of the whole object. This is evident from the X-ray observations of Becker *et al.* (1980), who observed this remnant in the energy range of 1-3 keV. Fitting their spectrum using the model of Raymond and Smith (1977) and assuming that the plasma is composed of two components at different temperatures, they find that the highest temperature is equal to 4.6×10^7 K; and it can be inferred from their paper that $F_{34} \simeq 1.3$. This temperature leads to a minimum shock velocity of 1 300 km s^{-1}, and a minimum value of 0.28 for the hypergeometric function, indicating that the swept-up mass is much larger than the ejected mass and implying that the remnant is already in the self-similar expansion phase, which means that the shock velocity is 1 900 km s^{-1}. The optical spectrum of Dennefeld (1982) leads to a shock velocity approximately equal to 120 km s^{-1} for the optical region and an electron density of 15 000 cm^{-3}. From equation (8) we find a mean interstellar density of 0.1 cm^{-3}, a reasonable mean interstellar value, and a swept-up mass of only 0.07 $M_\odot$. Since the remnant apparently is in the self-similar expansion phase we must conclude that the ejected mass cannot be much larger than 0.001 $M_\odot$. This is clearly absurd. On the other hand X-ray observations lead to $\langle n_0 \rangle \simeq 2.4$, which is an unexpectedly large value for such a high latitude (an even larger value of 3.8 cm^{-3} has been given by Long, Dopita and Tuohy 1982). This density implies $M_{SW} \simeq 1.7\ M_\odot$, which means that the ejected mass is roughly equal to 0.05 $M_\odot$ and $E \simeq 7.5 \times 10^{49}$ erg. The ejected mass still seems abnormally small. Of course the velocity used here is only a lower estimate, and it could well be that the remnant is not yet in the self-similar expansion phase. If this were the case (that is, if $U \gtrsim 1900$ km s^{-1}) both the ejected mass and the energy would be closer to the normally accepted values and to Utrobin's (1978) results. It is also possible that we have underestimated the size of the remnant. But it still seems puzzling that such a high density can be found at such a height above the plane, and it does not seem reasonable to suggest that this is caused by the presence of circumstellar material, since the type of progenitor expected for a type I supernova makes this possibility unlikely (Chevalier 1982*b*).

g) *Cas A*

According to Ashworth (1980) the supernova event that produced the Cas A SNR occurred in 1680 AD and was observed by Flamsteed. Assuming a constant expansion velocity the kinematics of the fast moving knots lead to an age of 325 years. (Kamper and van den Bergh 1976), indicating that the remnant is already beyond the free expansion phase. Minkowski (1959) determined that the velocity of the fast-filaments is in the range of 6 000 to 7 400 km s^{-1}. The optical and radio angular diameters of the object are very similar and close to 245 arcsec (Minkowski 1958; Clark and Caswell 1976). On the other hand an X-ray map indicates an angular diameter of 340 arcsec though there is also an annulus coincident with the position of the fast filaments and the maximum of the radio emission (Murray *et al.* 1979). The authors observed that the annulus is characterized by a lower temperature and suggested that this structure is a reverse shock wave moving into the ejected mass in the manner described by Gull (1973, 1975) and by McKee (1974). The outer regions are characterized by a higher temperature. Thus, if the angular diameter of the object is 340 arcsec we find a radius of 2.7 pc, for a distance of 3.3 kpc (Clark and Caswell 1976), which leads to $M_{SW}/M_e \simeq 0.78$ if $U_8 = 7$. As expected, this ratio indicates that the remnant is already decelerated, as Bell (1977) and Murray *et al.* (1979) also suggest. This is not in contradiction with the results of Kamper and van den Bergh (1976), who did not observe significant deceleration in 36 filaments, since the optical structures are much denser than the mean and consequently it is more difficult to detect any deceleration in them. The mean interstellar density can be determined from the observations of Peimbert and van den Bergh (1971), who found that $I(6717)/I(6731) \simeq 0.51$, leading to an electron density in the S^+ region of 7 500 cm^{-3}. With $U_8 = 7$ we find from equation (7) $\langle n_0 \rangle \simeq 1.05$ cm^{-3}. The mean interstellar density can also be estimated with equation (10), since Davison *et al.* (1976) studied Cas A in the 1.5-6 keV energy range. They also discovered two components, and gave $F_{34} = 1$ and a temperature of 6×10^7 K for the hottest component. According to Pravdo and Smith (1979) this temperature may be as high as 2×10^8K. In any case $P_{-23} \simeq 1$, so that equation (9) leads to $\langle n_0 \rangle \simeq 0.88$, which is almost identical to the result we have just determined from the optical observations. Consequently we assume that $\langle n_0 \rangle = 1$, so that $M_{SW} \simeq 2.4$ $M_\odot$, $M_e \simeq 3.1$ $M_\odot$ and $E \simeq 2.9 \times 10^{51}$ erg. Based on the momentum conservation equation Brecher and Wasserman (1980) estimated an ejected mass approximately 4 times larger than the value given here. This discrepancy stems from the different value used for the interstellar density (they use 2 cm^{-3}) and from the fact that there is a tendency to underestimate M_{SW}/M_e with their approach. In any case the large value of the ejected mass indicates that Cas A was probably caused by a type II supernova event.

IV. CONCLUSIONS

1. The mean ejected mass and energy of the four remnants that are suspected to be related to type I SNe (RCW86, SN1006, Tycho, and Kepler) are approximately equal to a 0.2 $M_\odot$ and 2×10^{50} erg. There is a marked difference between this group and the other remnants, indicating that they do form a separate class with approximately equal properties. This is consistent with the high degree of homogeneity found in the light curves and spectra of type I SN explosions (Barbon 1978). The values obtained here for the ejected mass and energy are very similar to those obtained by Colgate, Petschek, and Kriese (1980) from a model for the post-maximum light curve based on the radioactive decay of ^{56}Ni in which a neutron star is left as a stellar remnant after the SN explosion. Our results are inconsistent with the light curve model developed by Chevalier (1981). In this case an identical energy source is assumed but the presupernova star, a white dwarf, is completely disrupted and no stellar remnant is left after the SN explosion.

2. It has long been recognized that the Crab nebula was produced by a SN event that cannot be classified with any certainty, though the nebula itself has been taken as the prototype of a class of radio remnants known as 'plerions' or 'filled center' (Weiler and Panagia 1978). Its position above the galactic plane and peculiar chemical composition led Wheeler (1978) to propose a moderate helium star as a likely candidate for the 1054 AD SN event. These stars are thought to have approximately 2 $M_\odot$. If we assume a mass of roughly 1 $M_\odot$ for the neutron star at the center of the Crab nebula we find, from our estimated ejected mass, that the mass of the presupernova star should have been approximately equal to 1.5 $M_\odot$. The coincidence is very suggestive. Furthermore, our estimate for the mass of helium contained in the ejected mass is consistent with the large helium abundances observed in moderate helium stars (Wheeler 1978), supporting the idea that one of these produced the 1054 AD supernova. An important property of the Crab is that its total energy, and consequently its initial expansion velocity, is substantially smaller than in the other remnants. If this is correct we can then expect the Crab to be a rather short-lived remnant. A similar conclusion was obtained by Weiler and Panagia (1978) from the slowing down timescale of the Crab's pulsar and from the paucity of old plerions.

3. There is no other remnant as similar to the Crab as 3C58. Being equally young it also shows a similar radio morphology and spectrum (Wilson and Weiler 1976) and it might also contain a central collapsed object (Becker *et al.* 1982). Furthermore, the optical filaments of 3C58 are not unlike the optical filaments in the Crab as far as their morphology and spectrum are concerned (Fesen 1983). Our model indicates that the ejected mass and energy are likewise similar to those of the Crab. But there are other elements indicating that 3C58, though similar, is a different kind of object. First of all it is underluminous relative to the Crab throughout the spectrum. More fundamentally, if 3C58 is at a distance of 2.6 kpc, then the SN event that produced it was much fainter than normal. Such a behavior is not unique, as is exemplified by Cas A, but it does suggest that the presupernova star that was associated to the 1181 AD event is not of the same kind as the one that produced the Crab. It is evident that better optical observations would be priceless for they would help to establish the chemical

composition of the filaments and, consequently, to determine the type of star that produced 3C58.

4. The energy and ejected mass deduced from the Cas A SNR indicate that a type II supernova was associated with this object. Lamb (1978) reached the same conclusion and suggested that the nitrogen rich OBN stars are the precursors of Cas A type SNRs (such as N132D and NGC 4449). These stars have an initial mass of at least 9 $M_{\odot}$ and lose up to 40% of it during their evolution. Thus, if Lamb's hypothesis is correct, our estimate of 3 $M_{\odot}$ for the ejected mass leads to a precursor mass of 8 $M_{\odot}$, which is not unlike the initial mass an OBN star is expected to have.

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