EQUATOR AND EQUINOX DETERMINATIONS FROM SIMULATED PLANETARY OBSERVATIONS

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RESUMEN. Se calcularon observaciones simuladas de los planetas, el Sol y la Luna, considerados como objetos geométricos, para su supuesto período de observación de 10 años, con el 40% de los días y las noches despejados. El análisis de estas observaciones indica que el Sol y los planetas interiores son contribuyentes importantes para el equinoccio y ecuador celestes. En teoría, la Luna debe ser un contribuyente muy importante pero dificultades prácticas con las observaciones lunares, en especial los errores sistemáticos muy serios que están asociados con este tipo de observaciones, hacen poco probable que su potencialidad pueda ser alguna vez verificada completamente. Sin embargo, es posible orientar el sistema celeste de referencia con sólo las observaciones de los asteroides, observaciones que no están afectadas seriamente por errores sistemáticos grandes.

ABSTRACT. Simulated observations of the planets, the Sun, and the Moon, considered as geometrical objects, were calculated for an assumed observing period of ten years, with 40% of the days and nights clear. Analysis of these observations indicates that the Sun and the inner planets are strong contributors to the celestial equinox and equator. In theory the Moon should be a very strong contributor, but practical difficulties with the lunar observations, especially the very serious systematic errors associated with this class of observation, make it unlikely that its potential will ever be fully realized. However, it is possible to orient the celestial reference system from minor planet observations alone, a class of observation not seriously affected by large systematic errors.

I. INTRODUCTION

The determination of the celestial equinox and equator is one of the basic concerns of astrometry. Observations of solar system objects provide a means for this determination. Transit circles, when undertaking a fundamental program, use the Sun, the planets, and the brightest minor planets. Photographic astrometry relies almost exclusively on minor planets, but is not restricted to the brightest ones.

Given the plethora of objects that can be observed, which ones give the best determinations of the equinox and equator? Boiko (1975) and Branham (1980) address this question, but confine themselves to the minor planets. Both of these studies use simulated observations. I know of no similar study that investigates which of the other solar system objects, the planets, the Sun, and the Moon, best determines the equinox and equator. If one examines the Washington series of fundamental catalogs, such as the W450 (Adams and Scott 1968), one will notice that the Sun, Mercury, and Venus are heavy contributors to the equinox and equator solution. But, given the disparity in the number of observations of the various objects and their mean errors of unit weight, it is difficult to assess which object is the strongest contributor.

What about the Moon? The Moon's orbit possesses special features that are not characteristic of the other objects, including the minor planets. For one thing, the orbit is geocentric rather than heliocentric, and the Moon's synodic period of 29.4 days is shorter than that of any of the other objects. Actual studies that have used lunar observations to determine the equinox and equator have given mixed results. Klock and Scott (1970), using U.S. Naval Observatory 6-inch transit circle lunar observations made between 1925 and 1968, conclude: "The validity of the use of the moon to orient the fundamental reference system is questioned". Likewise, Fomin (1980), using the same observational material as Klock and Scott and additionally observations, including observations of the crater Mosting A, made at Greenwich from 1923 to 1940 and at the Cape from 1939 to 1958, states that "... irregularities in the apparent figure of the Moon lead to some uncertainty in deriving zero points of a star catalog from prolonged series of meridian observation of the lunar limb..." On the other hand, H.R. Morgan (1952) found when compiling the N30 fundamental catalog that equinox and equator determinations from observations of the Moon made at Washington from 1925 to 1941 and at Greenwich from 1923 to 1928 were concordant with those from the Sun, Mercury, and Venus, and he used these lunar observations to help establish the equinox of the N30 system.

A way to address the question of which of the solar system objects gives the best equinox and equator solutions and whether the Moon is a useful object for such a purpose is to employ simulated observations. This allows one to circumvent the deleterious effects of possible systematic errors and to concentrate on the orbital properties that permit strong equinox and equator determinations.

II. OBSERVATIONS AND SOLUTIONS

To calculate the simulated observations I decided to parallel closely the work done in my previous study on simulated minor planet observations (Branham 1980). In this way it would be possible to compare results given by the planets, Sun, and Moon with those given by the minor planets. The assumed observing conditions were: the planets, Sun, and Moon would be observed from 2 Jan. 1980 to 1 Jan. 1990; they would be observable from quadrature through opposition to quadrature or, for the Moon, from first quarter through full moon to last quarter; an observation could not be made if the visual magnitude, m_V, of an object exceeded 10.0; the number of clear days and nights during the observing period would be 40%, randomly distributed; and Mercury and Venus could not be observed if they were closer than 5° to the Sun. A random number generator was applied to the days and nights separately. Thus, if a day was clear the night was not necessarily clear, and vice versa. The magnitude criterion excluded Pluto completely, whereas all of the other planets were always brighter than m_V = 10.0. JPL Development Ephemeris 102 served to calculate the simulated observations. Of the 3,652 nights during the assumed observing period 1,444, or 39.54% were clear and of the days 1,472, or 40.31%, were clear. Table 1 shows the number of observations, n, for each object.

TABLE 1

Number of Observations

Planet	σ(1)	n
Mercury	1.″231	1,275
Venus	1.280	1,383
Sun	1.118	1,472
Mars	0.778	419
Jupiter	0.668	660
Saturn	0.736	711
Uranus	0.608	726
Neptune	0.624	724
Moon	1.056	752
Total	0.970	8,122

To calculate equations of condition for the simulated observations I followed the procedures given in detail in a previous investigation (Branham 1979a). Briefly, osculating rectangular coordinates and velocities at epoch JD 2446000.5 (27 Oct. 1984) for each individual object were corrected. Numerical integrations were the basic technique for the calculation of the required partial derivatives. Partial derivatives for corrections to the Earth's orbit, Brouwer and Clemence's (1961) set VI, were included in the equations of condition. The equinox correction, ΔE, is contained in these partial derivatives. The last unknown in the equations of condition is the equator correction, ΔD. No other unknowns were included. The right hand side of the equations of condition, the observed minus the computed position or (O-C), was set to zero.

Normal equations can be formed from these equations of condition in the usual manner. The solution vector itself will be zero because of our setting the (O-C)'s to zero. But we are interested in the mean errors of unknowns. These depend upon the assumed mean error of unit weight, $\sigma(1)$, for the observations, which for our present purpose represents noise in the data, and the elements on the main diagonal of the variance-covariance matrix. If $\underset{\sim}{A}$ denotes the matrix of the normal equations, then the mean error of the j-th unknown, denoted by $\sigma(j)$, is

$$\sigma(j) = \sigma(1)\,(A_{jj}^{-1})^{1/2} \qquad (1)$$

The $\sigma(1)$ to associate with a typical observation of a planet, or the Sun, or the Moon was a vexing question. For my minor planet investigation I had assumed $\sigma(1) = 0\farcs350$, but this value is unrealistic for observations of the other solar system objects, which are generally more noisy. The values that I finally selected, exhibited in Table 1, were taken from the study of Oesterwinter and Cohen (1972), which was based on Washington 6-inch and 9-inch transit circle observations of the planets, Sun, and Moon from 1913 to 1967.

A parameter of interest when one solves a set of linear equations is the condition number, which shows at a glance how ill-conditioned the system is. There are various definitions of this quantity; the one adopted here is the ratio of the largest to the smallest eigenvalue of the matrix $\underset{\sim}{A}$, or

$$\mathrm{COND}(\underset{\sim}{A}) = \lambda_{max}/\lambda_{min} \quad , \qquad (2)$$

where λ stands for eigenvalue.

Normal equations were formed and solved for the simultaneous reduction of all of the observations and for two subsets: the individual objects, and the day objects of the Sun, Mercury, and Venus (called by objects because they are observed during the day with a transit circle). The solutions for the mean error of the equinox, $\sigma(E)$, and the equator, $\sigma(D)$, along with the condition numbers and the number of observations are shown in Table 2.

TABLE 2

Mean Errors from Simulated Observations

Planet	$\sigma(E)$	$\sigma(D)$	COND($\underset{\sim}{A}$)	n
Mercury	0$\farcs$116	0$\farcs$039	2.10^5	1,275
Venus	0.121	0.035	1.10^6	1,383
Sun	0.099	0.037	3.10^5	1,472
Mars	0.268	0.051	8.10^5	419
Jupiter	0.654	0.029	3.10^6	660
Saturn	1.502	0.165	2.10^7	711
Uranus	4.402	1.208	6.10^8	726
Neptune	12.462	5.195	2.10^{10}	724
Moon	0.0004	0.040	2.10^9	752
Combined planet	0.0005	0.017	4.10^{12}	8,122
Combined day object	0.068	0.020	6.10^7	4,130

Having obtained these solutions from simulated observations, a question naturally arises: "How realistic are your results? Do the simulated observations accurately model what one might expect in practice?" Fortunately, one has access to an external check. The Naval Observatory's 6-inch transit circle observed the Sun, Mercury, Venus, Mars, and Jupiter, along with selected minor planets, during its 1963-1971 program. The catalog for this program will be published soon, but in the meantime results for σ(E) and σ(D) from these objects are already available (Scott 1982) and are presented in Table 3. No information is available concerning COND(A). The entry for the Moon comes from Fomin (1980).

TABLE 3

Mean Errors from Actual Observations

Planet	σ(1)	σ(E)	σ(D)	n
Mercury	1".075	0".212	0".065	234
Venus	1.233	0.112	0.034	668
Sun	0.863	0.075	0.022	990
Mars	-	0.239	0.048	253
Jupiter	-	1.102	0.035	269
Combined day object	-	0.061	0.018	1,892
Moon	0.943	0.045	0.16	19,666

A comparison of Tables 2 and 3 shows that the simulated study exaggerates the number of observations that could be obtained, even though Washington has approximately 40% of its days and nights clear. Of course, the real observing program lasted eight, rather than ten, years. Furthermore, during the 1963-1971 6-inch program weekend observations of the Sun, Mercury, and Venus were not regularly scheduled (Smith 1982). But even allowing for this, the number of simulated observations is too high. Two factors, both of which are difficult to model in simulations, militate against the practical attainment of the theoretically possible number of observations: no allowance was made for instrumental down-time; and the loss of an observation because of poor contrast between the planetary disk and the background. This latter factor is especially operative for the day observations of Venus and, particularly, Mercury, where even a slight amount of haze results in a lost observation. Compensating for the fewer number of actual observations is their somewhat better precision, undoubtedly occasioned by the increase in the precision with which an observation can be made in a modern program compared with the precision for the overall period 1913-1967. In general, however, the mean errors from the simulated observations seem realistic for objects other than the Moon.

III. DISCUSSION

Table 2 indicates that objects of high mean daily motion, the Sun, Mercury, Venus, and the Moon, are all strong contributors to the equinox, the Moon strikingly so. This confirms what both Boiko (1975) and Branham (1980), using simulated minor planet observations, encountered: a high correlation between a strong equinox correction and a high mean daily motion.

The extremely good σ(E) for the Moon cannot be considered realistic. It undoubtedly arises from the 1/ρ weighting that occurs when one forms the equations of condition, where ρ is the object's geocentric distance in units of the astronomical units. In practice the Moon is a very difficult object to observe, and the mean errors for equinox and equator determinations from actual lunar observations are far higher than those in Table 2. Fomin (1980) finds σ(E) = 0".045 and σ(D) = 0".16. Klock and Scott (1970), who solve for sinε ΔE, where ε is the obliquity, rather than ΔE quote values of σ(E) = 0".019 and σ(D) = 0".031. Morgan (1952), unfortunately, gives no mean errors in his work.

Part of the reason for this discrepancy between what the Moon should give in theory and what is gives in practice is undoubtedly the assumption made that the objects considered are geometrical points. Table 2 shows that this assumption is not too bad for the planets and the Sun, but it obviously is far short of the mark where the Moon is concerned because of serious systematic errors that enter into the lunar observations, errors that are greater than those for the other objects. Some of these errors are discussed below. Nevertheless, it is still true that at least one investigator, Morgan (1952), obtained reasonable results when he included lunar observations in equinox and equator determination, which indicates that the Moon should not be dismissed out of hand as a potential contributor to the orientation of the celestial reference system.

Several possible sources of systematic error that may account for the discrepancy between what the Moon should contribute to the equinox in theory and what it contributes in practice immediately suggest themselves. The Moon is a difficult object to observe, with its irregular limb profile and high contrast between the brilliant disk and dark sky background. The reduction programs that transit circle installations use for lunar observations account for the irregular limb by use of the Watts' (1963) limb correction charts and determine irradiation corrections on an observer basis. But the Watts' charts suffer from certain deficiencies, and the irradiation corrections are difficult to determine. Mulholland (1981) has indicated some of the problem with the Watts' charts. A way to minimize these two possible sources of systematic error is to use some feature not on the limb, such as the crater Mosting A. The Airy transit circle at the Greenwich Observatory observed this crater from 1905 until 1954. Fomin (1980) analyzed the observations obtained during the period 1923 to 1940 and found that the ΔE from Mosting A was different from that given by the limb whereas ΔD from the two classes of observation was similar; the mean errors of the equinox and equator from the two classes of observation, Mosting A and the lunar limb corrected by the Watts' charts, were similar. An unmodelled systematic error may be operative for the right ascensions of the lunar limb or the observations of the crater may be influenced by shadow effects that depend on the lunar phase. In any event the observations of the crater Mosting A in no way yield results as good as those predicted from Table 2.

When the image dissecting micrometer for the Naval Observatory's 7-inch transit circle becomes operational in 1983, it should be possible to more carefully investigate the role of the irradiation correction and the irregular lunar limb in the observations of the Moon. The image dissector should be insensitive to, or at most only moderately affected by, the irradiation, which is basically a psychological -or just possibly a physiological- effect. The edge detection algorithm used to process the lunar observations will pass a smooth curve through the lunar limb, thus minimizing the effect of the irregular profile.

Another possible source of systematic error, mentioned by Klock and Scott (1970), is the displacement of the vertical wire of the micrometer from the true center of the Moon at the times the measurements in declination are made. This results in a slight error in the time of a declination measurement. This possible error will be eliminated when the image dissecting micrometer becomes operational.

A further source of difficulty arises not in the reduction of the lunar observations, but in their processing. Klock and Scott (1970) solve for 21 unknowns and mention that "... the correction to the orbital latitude, ΔB, is strongly correlated with $\Delta\delta_0$ [ΔD in my notation] in the declination equations, and what separation there is comes from the right-ascension equations". Fomin (1980) solves for up to eleven unknowns, including ΔD and ΔB, and refers to "... the resulting poorly conditioned systems of normal equations". Both of these studies, as mentioned previously, found problems with the use of the lunar observations to orient the fundamental reference system. Morgan (1932), on the other hand, includes only three unknowns and states: "This is the first extensive use of transit circle observations of the Moon for equinox determination and the very close agreement of the results from the Moon and from the Sun and planets, as shown below, is very satisfactory".

It appears as if the linear system formed from the lunar observations is very sensitive to the number of unknowns solved for and that many unknowns can over-burden the system and result in a poorly conditioned problem. It is important to realize that at times we cannot obtain reliable estimates for all of the parameters that we would like. Whether we can or not depends on the condition of the linear system. A good measure of this condition is the condition number given by Eq. (2). Whether Morgan's normal equations are better conditioned than those of Klock and Scott and Fomin cannot be known without access to the original data because none of these studies publishes condition numbers. In the future anyone who uses lunar observations should carefully investigate which unknowns can be adequately determined from his data and publish the condition number of the linear system. Then someone else can see immediately how well -or poorly- conditioned the linear system is.

Some other conclusions are suggested from a perusal of Table 2. The day objects all give strong determinations of ΔE and ΔD. Mercury's contribution is weakened, as Table 3 shows, because this planet is more difficult to observe, in practice, than the others. Nevertheless, observers should make every effort to obtain as many observations as possible of Mercury. Jupiter contributes little to the equinox, but determines the equator well. The planets beyond Jupiter contribute nothing to either the equinox or the equator. Saturn, Uranus, and Neptune should, to provide material for the improvement of their orbits, be observed, but these observations should not be used in equinox and equator determinations.

Having obtained the results summarized in Table 2, it is instructive to compare them with those given by the minor planets (Branham 1980). My previous study assumed the same conditions as those of Sec. II. A simultaneous reduction of 7,400 simulated observations of the nine minor planets 1-4, 6-9, and 15 yielded mean errors of $\sigma(E) = 0''.069$ and $\sigma(D) = 0''.0068$. This mean error for the equinox is identical to the $\sigma(E)$ for the combined day objects in Table 2, whereas the $\sigma(D)$ from the minor planet solution is better. This demonstrates that it is possible to carry out a fundamental observing program by use of minor planets alone and, in fact, has already been done (Branham 1979b). At transit circle installations where day observing may produce an intolerable observing burden it is still possible to undertake fundamental programs by reliance on solely the minor planets to determine the equinox and equator.

IV. CONCLUSIONS

The Sun and the inner planets of Mercury and Venus provide strong determinations of the celestial equinox and equator. The Moon is potentially a very strong contributor, but serious problems with the lunar observations impede the full utilization of its potential although if the observations are carefully handled the Moon can make a meaningful contribution to equinox and equator solutions. In any event, it should not be arbitrarily discounted. It is possible to undertake fundamental observing programs from minor planet observations alone.

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DISCUSSION

Paneth: ¿Cómo se determinó la diferencia entre el centro del disco luminoso y el centro de masa de la luna?

Branham: Puesto que la presente investigación no incluyó deliberadamente los efectos de posibles errores sistemáticos, no se determinó la diferencia entre el disco luminoso y el centro de masa.

Sanguin: ¿Qué condiciones han debido reunir los Asteroides que Ud. utilizó en el cálculo o modelo presentado?

Branham: Las mismas condiciones que las de la presente investigación, o sea: 1) programa de 10 años de duración, 2) 40% de las noches despejadas; 3) observar el asteroide desde cuadratura pasando oposición hasta cuadratura; 4) no poder observar un asteroide cuya magnitud visual es menor que 10.0.

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