KINETIC EQUATIONS AND DEPARTURES FROM LTE

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RESUMEN. Se expone aquí un método que permite resolver en forma numérica las ecuaciones cinéticas que presentamos y que describen las funciones de distribución de los componentes de un gas formado por fotones, electrones, protones, átomos e iones. Estas ecuaciones integrodiferenciales, que se aplican en los casos de validez de las hipótesis estadísticas, pueden describir situaciones apartadas del equilibrio termodinámico como en el caso de las atmósferas estelares, dando distribuciones que difieren de la de Boltzmann y aún de la de Maxwell.

Se plantean las ecuaciones para el caso unidimensional y se propone utilizar el método de Newton-Raphson para resolver las ecuaciones, suponiendo conocidas las condiciones de contorno, y se muestra cómo calcular los apartamientos de las funciones de distribución respecto de la de Maxwell para los casos en que sean pequeños.

Luego, a través de esos apartamientos, se muestra cómo calcular los coeficientes de transporte y los límites de validez de la teoría de esos coeficientes.

ABSTRACT. We present a method for the nummerical solution of the kinetic equations for a gas composed by photons, electrons, atoms and ions. The gas is assumed to satisfy the statistical hypothesis. We show the integro-diferential equations that determine the distribution functions, for situations departing from thermodynamical equilibrium as in stellar atmospheres. These functions differ from Boltzmann's and even from Maxwell's function. We give the equations for a one-dimensional problem and propose the use of the Newton-Raphson method to solve the equations for given boundary conditions. We also show how to compute first order deviations from Maxwell's distribution, and, from these departures, how to compute the transport coefficients and their range of aplicability. We further suggest correction procedures for saturated fluxes.

I. INTRODUCTION

In the calculation of stellar model atmospheres is usual to write certain equations to describe the state of the gas composed by photons and several species of particles.

For describing the state of photons, the radiation intensity is usually chosen, it is directly related to the distribution function of photons. The variation of intensity is described by the radiative transfer equation (Athay 1974) which corresponds to the Boltzmann equation, on the assumption that the local derivative respect to time is zero. This is reasonable since, through the characteristic lenght of variation of the atmospheric macroscopic parameters (hereafter called *scale height*), the flight time of photons is small.

For the deepest regions of the atmospheres, local thermodynamical equilibrium (LTE) holds; this means that departures from the Planck-Boltzmann-Maxwell distribution functions are small and can be accounted for by a first order term proportional to macroscopic parameters gradients.

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In this situation, macroscopic parameters (temperature, velocity, and density) are determined by hydrodynamical equations subject to boundary conditions. In these equations there appear fluxes proportional to macroscopic temperature gradients; for instance, from the radiative transfer equation, the radiation flux becomes proportional to the temperature gradient and this flux has to be considered in the impulse and energy equations.

The involved factors are the transport coefficients, not always well known; for instance, in a paper by Verga (1982) there appear many coefficients which reflect the interaction between radiation and matter. Examples of the aplication of LTE theory with a simplified set of transport coefficients are found in Eddington's (1926) early papers.

As density decreases with increasing altitude in the atmosphere, there is some height at which LTE becomes a bad approximation to the actual situation because the free path of photons at some relevant frequencies becomes of the order or larger than the scale height. In such a case, usually it is assumed that particles and photons depart significantly from Boltzmann and Planck distributions, but particles follow the Maxwell law with a first order correction which is assumed to be proportional to the macroscopic parameter gradients. We call this approximation *partial LTE* (PLTE). The usual procedure is to solve simultaneously the radiative transfer equation for photons at the relevant frequencies, the statistical equilibrium equations for the different species of particles and the hydrodynamical equations for the gas of particles, including transport phenomenae, radiative force and radiative energy loss. Work of this kind assuming stationary state were done by many authors as, for instance, Mihalas (1975), Heasley and Milkey (1976) and Fontenla and Rovira (1983), and for a non-stationary case by Kneer and Nakagawa (1976).

Examples of transport coefficients not well known are the thermal conductivity and viscosity in partially ionized plasmas in PLTE, for the range of pressures, temperature and ionization typical of stellar atmospheres; values for other cases were calculated and compared with measurements, for instance, by Devoto (1968). At even smaller densities, in what is usually called the transition region, even the PLTE can be a bad approximation because of the comparable free path of some particles relative to the scale height.

In that case, particles may also depart significantly from Maxwell's distribution, as it was shown by Rousell-Dupree (1980), and, in the extreme cases, it can be a nonsense to write hydrodynamical equations and we are left with the kinetic equations only.

One of such problems is that of the radiatively driven stellar winds, where, assuming n_e = 1E10 cm^{-3} and T = 1E4 K, and using Allen's (1973) formulae, one can calculate the elastic collisions rate between electrons and ions of, say, C IV, which comes out to be 1E5 s^{-1}. If we assume an impulse exchange by collision of $(2\ Me\ k\ T)^{1/2}$, the impulse exchange rate becomes 4E-15 gr cm s^{-2}. On the other hand, the rate of excitations is from 1E5 to 1E9 s^{-1}, for radiation temperatures from 1E4 to 1E5 K, respectively, and the rate of impulse exchanged with the radiation field (highly anysotropic) is 1E-16 to 1E-12 gr cm s^{-2}, respectively. These figures show clearly that in the conditions proposed for the region, which we can call the external atmosphere, the mean velocity of the ions can depart significantly from that of the electrons, giving rise to frictional forces, and may even develop two stream instabilities, all processes leading to macroscopic dissipative mechanisms which can explain some observational facts, as it was suggested by Fontenla *et al.* (1981).

Another point to consider is the elastic impulse exchange between ions themselves. Again, according to Allen's formulae, the rate of such elastic collisions is 1E3 s^{-1} in the previously mentioned conditions, and since $Ni \sim Np \sim Ne$, if we assume an impulse exchange by collision of $(2\ Mi\ k\ T)^{+1/2}$, the impulse exchange rate becomes 2E-15 gr cm s^{-2} which, when compared with the radiative rate, justifies the use of a Maxwellian distribution function for the ions, only for some cases. In other cases, the departures can be large and even lead to a *runaway* process similar to the one well known in the laboratory for plasmas subject to electric fields.

In view of the considerations made, it can be suspected that the real stellar winds are quite far from the classical models by Lucy and Solomon (1970) and by Castor *et al.* (1975), and realistic models need a more elaborated treatement of the interaction between radiation and matter.

The first impulse when facing this kind of problems is to use the Montecarlo technique (James 1980) for simulating the problem by a computer, but one can see that, on one hand, because the non-linear effects, critical *bias* functions can predetermine the solutions, and boundary conditions or other restrictions sometimes become obscure, collisions, at small deflection angles between charged particules, require very long calculations and stability analysis is long and difficult. On the other hand, a system of kinetic equations can describe better the behaviour of the gas subject to boundary conditions, and, for simple symmetries, allow to calculate solutions and their stability as well as the weight of the different terms, in a safer and faster way and to deduce general properties or approximations.

The purpose of this paper is, on one hand, to set up a system of kinetic equations for describing the general problem, and on the other hand, to show how the equations can be solved numerically for a one-dimensional case and can be applied for the calculation of transport coefficients for LTE and partial LTE cases.

II. THE KINETIC EQUATIONS

We have developed the kinetic equations in the more general way following the Ehler and Kohler's (1977) formalism. Assuming that all requirements related to statistics are satisfied, we describe the macroscopic system by the one particle distribution function, for each of the present species.

We consider the phase space composed by the four dimensional coordinates (x, y, z, ict) and the four dimensional momenta (p_x, p_y, p_z, iE/c) where i is the imaginary unit, c the velocity of light, E the total energy; the other variables have the usual meaning. For a particle of rest mass m, always $|\vec{p}|$ = imc, and for a photon, $|\vec{P}|$ = 0, so, given a particle, the impulse have only three independent variables.

Following Ehler and Kohler, we write the volume measure on the space of orbital phases, for particles of rest mass m

$$d\omega' = (\vec{p} \cdot d\vec{\sigma})\ d\P' \tag{1}$$

where $d\vec{\sigma}$ is an space-time surface element, and d¶' the volume element in four momentum space for that particle. Then,

$$d\P' = \frac{dp_x dp_y dp_z}{E/c} \tag{2}$$

We have assumed a gas of particles which are essentially free; interactions between them are, then, short-lived, compared with the time between such interactions. The particles are subject to a general field which can include the autoconsistent field.

According to the definitions and assuming summation over repeated index, we have

$$N_\Sigma = \int_{\Sigma,\infty} f'\ d\omega' = \int_\Sigma d\sigma_\mu \int_\infty p_\mu f'\ d\P' = \int_\Sigma j_\mu\ d\sigma_\mu \tag{3}$$

where the integral over ∞ means integration over the whole momentum space, N being the number of particle trajectories which cross the surface Σ, and j_μ, the four-vector current density of the species. Then, if N is the volume density of particles,

$$j_4 = i \int_\infty \frac{E}{c} f'\ d\P' = i\,N \tag{4}$$

Redefining f' and d¶' in order to work with the state occupation number and the state density, we write

$$f = \frac{h^3}{w} f' \quad ; \quad d\P = \frac{w}{h^3} d\P' \quad ; \quad d\P^* = \frac{E}{c} d\P \tag{5}$$

w being the multiplicity of the state, and h, the Planck constant; then,

for particules $\quad d\P^* = (2S+1) \left(\frac{mc}{h}\right)^3 \gamma^5 \beta^2\ d\beta\ d\mu\ d\phi \quad ,$

for photons $\quad d\P^* = 2 \left(\frac{\nu_0}{c}\right) \left(\frac{\nu}{\nu_0}\right) d\left(\frac{\nu}{\nu_0}\right) d\mu\ d\phi \quad ,$

s being the spin of the particle, β, the velocity in units of the velocity of light, γ, the relativistic correction $\gamma = (1 - \beta^2)^{-1/2}$ and θ and ϕ, the spherical angles which define the direction of vector $\vec{p}$ in the three dimensional space; $\mu = \cos\theta$ being the projection factor along the z-axis. The quantity ν_0 represents an arbitrary frequency taken to adimensionalize the variable.

The force is assumed to include gravitational and Lorentz terms

$$\vec{F} = mc\gamma \left(\frac{m}{c}\vec{a} + \frac{Ze}{c}\vec{E} + \frac{Ze}{mc^2\gamma}(\vec{p} \cdot \vec{B}) \right) \tag{7}$$

where a is the gravitational acceleration, $\vec{E}$ and $\vec{B}$ the electric and magnetics fiels, respectively, e, the electron charge and Z, the ratio of the charge of particle to that of the electron.

The Liouville equations give for particles (see Ehlers and Kohler 1977), in cases of small $\vec{B}$,

$$mc\gamma \left\{ \beta m_i \frac{\partial f}{\partial x_i} + \frac{1}{c}\frac{\partial f}{\partial t} + \left(\frac{m}{c} a_i + \frac{Ze}{c} E_i + \frac{Ze}{c} \beta (\vec{n} \cdot \vec{B})_i \right) \frac{\partial f}{\partial p_i} \right\} = e \quad ,$$

and, for photons,

$$\frac{h\nu}{c} \left(n_i \frac{\partial f}{\partial x_i} + \frac{1}{c}\frac{\partial f}{\partial t} \right) = e \quad ,$$

where $\vec{n}$ indicates the unitary vector in the direction of the impulse and τ, is the collision density for the particles considered.

It is convenient to write these equations in the following form, for particles,

$$\beta n_i \frac{\partial f}{\partial x_i} + \frac{1}{c}\frac{\partial f}{\partial t} + \Phi_i \left(mc \frac{\partial f}{\partial p_i} \right) = \xi \quad ,$$

and for photons

$$m_i \frac{\partial f}{\partial x_i} + \frac{1}{c}\frac{\partial f}{\partial t} = \xi \tag{9}$$

where

$$\vec{\Phi} = \vec{a}* + q* \left(\vec{E}* + \beta(\vec{n} \cdot \vec{B}*) \right)$$

and $\xi = \dfrac{\tau}{mc\gamma}$, for particles, or $\xi = \dfrac{\tau}{\frac{h\nu}{c}}$, for photons;

$\vec{E}*$ and $\vec{B}*$ are the electromagnetic fields divided by the electron charge, $\vec{a}* = \vec{a}/c$ and $q* = \dfrac{Ze^2}{mc^2}$.

The collision term ξ for a particle of some species with a certain value of the impulse, namely A, can be written with the use of a source term which does not depend on f_A, but depends on the other particle or impulse distribution function f_B, f_C, etc., and a sink term which is proportional to f_A, so that

$$\xi_A = \eta_A - \chi_A f_A \tag{10}$$

Following Ehlers and Kohler, we adopt the definitions

$$f^A = 1 + f_A \quad , \qquad \text{for bosoms} \quad ,$$
$$f^A = 1 - f_A \quad , \qquad \text{for fermions} \qquad (11)$$

Then, for instance, and since $d\sigma = \sigma\, d\omega$, we have for the differential cross-section for the binary collision of A and B that produce particles C and D,

$$\eta_A = \Sigma \int f_C f_D f^B\, Q_{AB}\, \sigma\, d\omega\, d\P_B{}^* \quad ,$$
$$\chi_A = \Sigma \int f_B f^C f^D\, Q_{AB}\, \sigma\, d\omega\, d\P_B{}^* \pm \eta_A \quad , \qquad (12)$$

where the summation is over all collisions which can suffer a particle A, and the minus sign holds when A's are bosons, and the plus sign, when fermions. The expression of Q_{AB} is derived from Ehlers and Kohler's definition of P_{AB}

$$Q_{AB} = (\beta^2{}_A + \beta^2{}_B - 2\beta_A\beta_B \cos\theta_{AB} - \beta^2{}_A\beta^2{}_B \sin^2\theta_{AB})^{1/2} \qquad (13)$$

where θ_{AB} is the angle between $\vec{p}_A$ and $\vec{p}_B$. This expression can be used even when one of the particles is a photon, in which case the corresponding β will be unity.

For the conditions prevailing in normal stellar atmospheres, degeneration is negligible and $f^A = 1$, for particles, thus in what follows we will not consider degeneration, except in the case of photons.

Expression (10) is suitable for calculating the elastic collision densities when the potentials involved are of short range, as collisions between neutral particles and photons. It is also useful when dealing inelastic collisions and in such cases expression (12) include products of all intervening species distribution functions.

When considering collisions we have to consider reactions as well, as elastic collisions, an example of this is the hydrogen atom in which case we have elastic collisions, ionizations, recombinations, excitations, deexcitations, attachements and dissociations, each of these processes has to be accounted for by a different term, and special care has to be taken for including the reverse processes.

In cases of long range binary elastic interactions as for Coulomb potentials, one can start from the expression

$$\xi_A = \Sigma \int (f_C f_D - f_A f_B)\, Q_{AB}\, \sigma\, d\omega\, d\P_B{}^* \qquad (14) \quad ,$$

where we assume that particles C and D correspond, respectively, to A and B, before the collision takes place.

In the last formulae one sees that when the deflection angle of particle A goes to zero, the parenthesis also goes to zero and in the case of the Coulomb potential the cross-section goes to infinite, leading to an undefined expression.

Writing $W = Q_{AB}$ and $p_C = p_A + p$ with p small, then, $p_D = p_B - p$. At this point we follow Landau's (1936) formalism and develop the functions W, f_C and f_D up to second order in p replace them in formulae (14) and simplify terms. Taking into account collision symmetries, we can write

$$\xi_A = \Sigma \int \frac{\partial}{\partial p_{A_i}} \left\{ G_{ij} \left(\frac{\partial f_A}{\partial p_{A_j}} f_B - \frac{\partial f_B}{\partial p_{B_j}} f_A \right) \right\} d\P_B{}^*$$

with (15)

$$G_{ij} = \frac{1}{2} \Sigma\, \delta p_i\, \delta p_j\, W\, d\omega$$

Analogous procedure can be followed in cases where degeneracy has to be taken into account.

By defining $G_{ij}^* = \frac{G_{ij}}{m_A^2 C^2}$ for dimensional purposes, we have

$$\chi'_A = \Sigma \int m^2{}_A C^2 \; \frac{\partial G_{ij}^*}{\partial p_{A_i}} \; \frac{\partial f_B}{\partial p_{B_j}} \; d\P_B^*$$

$$\Phi'_{A_i} = \Sigma \int \left(G_{ij}^* \, m_A C \; \frac{\partial f_B}{\partial p_{B_j}} - m_A C \; \frac{\partial G_{ij}^*}{\partial p_{A_j}} \; f_B \right) d\P_B^* \qquad (16) \; ,$$

$$\psi_{ij} = \Sigma \int G_{ij}^* \, f_B \, d\P_B^* \qquad ,$$

where summation is over all particles which can deflect projectile A, including A itself. With this definition the kinetic equations for charged particles become modified as follows

$$\beta\eta_i \frac{\partial f}{\partial x_i} + \frac{1}{c}\frac{\partial f}{\partial t} + (\chi + \chi')f + (\Phi_i + \Phi'_i)(mC \frac{\partial f}{\partial p_i}) = \eta + \psi_{ij}(m^2C^2 \frac{\partial^2 f}{\partial p_i \partial p_j}) \quad , \qquad (17)$$

and, after some geometrical considerations, we have

$$G_{ij}^* = \frac{2\P Q_{AB}}{m_A^2 \; C^2} \left(\frac{Z_A Z_B C^2}{m_A C^2} \frac{g}{\gamma\beta^2} \right)^2 \left(A^* \, \delta_{ij} + (B^* - A^*) \frac{g_i g_j}{g^2} \right) \qquad (18) \; ,$$

where $g = \vec{p}_{A_{CM}}$. A* and B* are coefficients which depend on the differential cross-sections as function of the deflection angle ε, β and γ are given by the projectile velocity of the center of mass and $_{ij}$ is the standard Kronecker's delta.

Analogous treatement of deflections for non-relativistic cases can be followed from many books as, for instance, Spitzer's (1962) it leads to the Focker-Planck equation. We call here *deflections* the kind of binary elastic interactions which result in small angular deviations of the particles; we treat them relativistically and we sum their corresponding collision densities to those we call properly *collisions* and can be described by the standard Boltzmann equation.

One important point is that the preceding derivation of Focker-Planck collision terms correpsonds to binary collisions. As De Witt and Detoeuf (1960) have demonstrated in cases where PLTE holds, these terms can also account for the main of the collective interactions and there remains a contribution associated with the energy exchange between plasma waves and charged particles. The latter can be calculated by including *plasmon pseudoparticles*, but this is beyond the scope of the present paper. It has to be noted that, in principle, we cannot drop out any term in (17) since the phenomenon we are interested in is precisely the effect of the convergence of the terms.

In formulae (17), we can express the derivatives respec to impulse from the corresponding spherical coordinates and write

$$mC \frac{\partial}{\partial p_i} = \frac{m_i}{\gamma^3} \frac{\partial}{\partial \beta} + \frac{(\delta_{i_2} - \mu\eta_i)}{\gamma\beta} \frac{\partial}{\partial \mu} + \frac{(\eta_y \delta_{ix} - \eta_x \delta_{iy})}{\gamma\beta(1 - \mu^2)} \frac{\partial}{\partial \phi} \qquad (19) \; .$$

We can write the electromagnetic equations for a non-polarizable gas as

$$\vec{\nabla}.\vec{E}\,^* = 4\pi\,\Sigma\,Z \int f\, d\pi^* \quad ,$$

$$\vec{\nabla}.\vec{B}\,^* = 0 \quad ,$$

$$\vec{\nabla}\mathrm{x}\vec{E}\,^* = -\frac{1}{c}\frac{\partial \vec{B}\,^*}{\partial t} \quad , \qquad (20)$$

$$\vec{\nabla}\mathrm{x}\vec{B}\,^* = \frac{1}{c}\frac{\partial \vec{E}\,^*}{\partial t} - 4\pi\,\Sigma\,Z \int \vec{n}\,\beta\, f\, d\pi^* \; ,$$

where $\vec{\nabla}$ is the gradient operator and summations are over all species. In regard to the coefficients in formula (15), they result from considering the system of reference of the center of mass; their derivation can be followed from any book by considering the relativistic modifications in each step. Then, for the spherical coordinate system in the space of the impulses, in which the direction of the impulse of particle A was along the z-axis prior to the deflection by an angle ε, we have

$$G_{xx} = G_{yy} = \frac{\pi}{2} Q_{AB} g^2 \int_{\varepsilon_0}^{\varepsilon_1} \sigma \sin^3\varepsilon \, d\varepsilon \quad ,$$

$$G_{zz} = \pi Q_{AB}\, g^2 \int_{\varepsilon_0}^{\varepsilon_1} \sigma (1 - \cos\varepsilon)^2 \sin\varepsilon \, d\varepsilon \qquad (21)$$

the other components of tenser G_{ij} being null.

The values ε_0 and ε_1 are the cutoff settings of the integrals and, in principle, are respectively, equal to zero and to π, but since we are assuming small angles ε, the value of ε_1 must be small for equations (15) to be valid. Anyway, since for the Coulomb potential the integrals diverge when ε_0 goes to zero, the critical parameter is ε_0, one has, in consequence, to consider Debye's shielding by the electrons (Spitzer 1962) and this can be done in two ways. The first one is by considering a modified potential, Debye's shielded potential (Devoto 1968), for the interaction, and the second one is by setting a limit of Debye's lenght l_D (Spitzer 1962) to the impact parameter b in the interaction. The first alternative is, of course, theoretically, the more correct one, but the error by taking the second one is unimportant for our calculations, according to De Witt and Detoeuf (1960).

Thus, we have adopted the criteria of setting in such way that the corresponding impact b_0 is equal to Debye's lenght l_D. By definition,

$$\sigma \sin\varepsilon \, d\varepsilon = b \, db \qquad (22)$$

which, due to the properties of σ, that implies

$$b_0^2 = 2 \;\; {}_{\pi}^{\varepsilon_0} \;\; \sigma \sin\varepsilon \, d\varepsilon \qquad (23)$$

For interactions between electrons we have adopted the cross-sections given by Akhiezer and Berestetskii (1965) in the center of mass system

$$\sigma = \frac{e^2}{mc^2}\;\frac{2\gamma^2-1}{2\gamma^3\beta^2}^{\,2} \left[\frac{4}{\sin^4\varepsilon} - \frac{3}{\sin^2\varepsilon} + \frac{\gamma^2-1}{2\gamma^2-1}^{\,2}\; 1 + \frac{4}{\sin^2\varepsilon} \right] \qquad (24)$$

Since the deflection angle ranges between 0 and $\pi/2$, it is convenient to adopt the adimensional parameter

$$\lambda = \sin^{-1} \varepsilon \quad ,$$

With some algebra we obtain an expression for b as a function of λ; the expression is very complicated but, since λ is much larger than unity because of the smallnes of ε, one can write

$$b \simeq \left(\frac{e^2}{mc^2}\right)\left(\frac{2\gamma^2-1}{\gamma^3\beta^2}\right)\lambda \quad , \tag{25}$$

and

$$\lambda_0 = \frac{l_D\,\gamma^3\beta^2}{\left(\frac{e^2}{mc^2}\right)(2\gamma^2-1)}$$

Integrals of equation (21) can be perfomed by introducing formula (24) and the definition of λ and by simplifying the results because λ>>1; then, we have

$$G_{xx} = G_{yy} = \frac{\pi}{2} Q_{AB} g^2 \left(\frac{e^2}{mc^2}\right)^2 \left(\frac{2\gamma^2-1}{\gamma^3\beta^2}\right)^2 \left[\ln(2\lambda) - \frac{3}{4} + \left(\frac{\gamma^2-1}{2\gamma^2-1}\right)^2 \frac{5}{4}\right]_{\lambda_1}^{\lambda_0} \quad ,$$

$$G_{zz} = \pi Q_{AB} g^2 \left(\frac{e^2}{mc^2}\right)^2 \left(\frac{2\gamma^2-1}{\gamma^3\beta^2}\right)^2 \left[\frac{3}{4} - \frac{3}{2}\ln 2 + \left(\frac{\gamma^2-1}{2\gamma^2-1}\right)^2 \left(2\ln 2 - \frac{2}{3}\right)\right]_{\lambda_1}^{\lambda_0} \tag{26}$$

Comparison between equations (26) and (18) gives the values of A* and B* for the interaction between electrons and, assuming $\ln \lambda_0 >> \ln \lambda_1 >> 1$, we obtain that

$$A^*_{ee} \simeq \left(\frac{2\gamma^2-1}{2\gamma^2}\right)^2 \ln \lambda_0 \quad ,$$

$$B^*_{ee} \simeq 0 \tag{27}$$

For interactions between electrons and ions we adopt the differential cross-section given by Akhiezer and Beresteeskii (1965) for the Coulomb potential, which, in the laboratory system of coordinates, is

$$\sigma = \left(\frac{Z\,e^2}{m_p c^2}\right) \left[\frac{1-\beta_p^2 \sin^2\left(\frac{\varepsilon'}{2}\right)}{4\,\gamma_p^2\beta_p^4 \sin^4\left(\frac{\varepsilon'}{2}\right)}\right] \tag{28}$$

with ε' ranging from 0 to π; m_p, is the projectile mass and, β_p and γ_p are given by the projectile velocity.

This formula has to be integrated over ε', thus it can be used without transforming the angle to the center of mass system.

Chosing

$$\lambda = \frac{\cos\left(\frac{\varepsilon'}{2}\right)}{\sin\left(\frac{\varepsilon'}{2}\right)}$$

we have, for λ>>1,

$$A^{*}_{ei} \simeq \left(\frac{\gamma\ \beta^2}{\gamma_p \beta^2_p} \right)^2 \ln \left(\frac{\lambda_0}{\lambda_1} \right) \quad ;$$

in the case of $\ln \lambda_0 >> \ln \lambda_1 >> 1$,

$$A^{*}_{ei} \simeq \left(\frac{\gamma \beta^2}{\gamma_p \beta^2_p} \right) \ln \lambda_0 \quad ,$$

$$B^{*}_{ei} \simeq 0 \qquad (29).$$

The value of λ_1 can be estimated from the partition of the impulse space and, since Δp the smallest variation in our numerical partition,

$$\lambda_1 = \frac{g}{\Delta p} \qquad (30)$$

this is the limit up to which we can consider an interaction as a deflection, that is, when the deviation is not strong enough to bring a particle from one volume element in the space of the impulses to another one; otherwise, we consider the interaction as a collision and use Boltzmann's collision density term.

For deflections between identical ions we believe that it is not a bad approximation to use also expressions (27) and for different ions, to use formula (29), adequately modified. A more accurate treatement should be to calculate the integrals from (21) and (23) for specific elastic scattering differential cross-sections when available.

The β and γ for expressions (24) through (29) follow from the velocity of the projectile in the center of mass system of coordinates, whose velocity is given by

$$\vec{u} = \frac{\vec{p}_A + \vec{p}_B}{p_{A_4} + p_{B_4}} \quad ,$$

and since

$$p_{A_4} = m_A c \gamma_A \quad ,$$

and

$$\beta = \left| \frac{\beta_A \vec{m}_A - \vec{\mu}}{1 - \beta_A (\vec{n}_A . \vec{u})} \right| \quad ,$$

we have

$$\beta^2 = \gamma_A^2 \gamma_B^2 \frac{\beta_A^2 + \beta_B^2 - 2\beta_A \beta_B \cos\theta_{AB}}{\frac{m_A}{m_B} + \sqrt{1 + \gamma_A^2 \gamma_B^2 Q^2_{AB}}} \qquad (31)$$

where θ_{AB} is, again, the angle between the impulse of particles A and B.

III. THE ONE-DIMENSIONAL CASE

Let us assume that all macroscopic parameters are function only of the z-coordinate and time, we can integrate equations (6) over angle ϕ and average equations (9) and (17); thus,

$$d\tau^* = 2\pi(2S+1)\left(\frac{mc}{h}\right)^3 \gamma^5\beta^2\, d\beta\, d\mu ,$$

$$d\tau_f^* = 4\pi\left(\frac{\nu_0}{c}\right)^3\left(\frac{\nu}{\nu_0}\right)^2 d\left(\frac{\nu}{\nu_0}\right) d\mu ,$$

$$\beta\mu\frac{\partial f}{\partial Z} + \frac{1}{C}\frac{\partial f}{\partial t} + (\chi + \chi')f + (\Phi_2 + \Phi_2')\left(\frac{\mu}{\gamma^3}\frac{\partial f}{\partial \beta} + \frac{(1-\mu^2)}{\gamma\beta}\frac{\partial f}{\partial \mu}\right) = \qquad (32)$$

$$= \eta + \alpha_{\beta\beta}\frac{\partial^2 f}{\partial \beta^2} + \alpha_{\mu\mu}\frac{\partial^2 f}{\partial \mu^2} + \alpha_{\beta\mu}\frac{\partial^2 f}{\partial\beta\partial\mu} + \alpha_\beta\frac{\partial f}{\partial \beta} + \alpha_\mu\frac{\partial f}{\partial \mu} ,$$

$$\mu\frac{\partial f_f}{\partial Z} + \frac{1}{C}\frac{\partial f_f}{\partial t} + \chi^f = \eta ,$$

where the index f denotes the functions corresponding to photons. The coefficients α can be defined from the mean, over the angle ϕ, of equations (16), (17), (18) and (19).

The force $\vec{\Phi}$ is dependent on the electromagnetic fields $\vec{E}^*$ and $\vec{B}^*$, which, in the present case, have only components along z-axis. Then, the electromagnetic equations become

$$B_2^* = \text{cte.} ,$$

$$\frac{\partial E_Z^*}{\partial Z} = 4\pi\, \Sigma\, Z \int f\, d\tau^* \qquad (33)$$

$$\frac{1}{C}\frac{\partial E_Z^*}{\partial t} = -\, 4\pi\, \Sigma\, Z \int \beta\mu f\, d\tau^* ,$$

and the force Φ is given by

$$\Phi^2 = a_2^* + q^* E^* \qquad (34)$$

When calculating the momenta of the kinetic equations numerical cancelation problems can appear, which lead to wrong values for the fluxes. This can be avoided by decomposing all functions of μ in symmetrical and antisymmetrical components with respect to μ; then,

$$f^S = \frac{1}{2}\left[f(+\mu) + f(-\mu) \right] ,$$

$$f^a = \frac{1}{2}\left[f(+\mu) - f(-\mu) \right] ,$$

and, with μ ranging between 0 and 1,

$$d\tau^* = 4\pi(2S+1)\left(\frac{mc}{h}\right)^3 \gamma^5\beta^2\, d\beta\, d\mu ,$$

$$d\tau_f^* = 8\pi\left(\frac{\nu_0}{c}\right)^3\left(\frac{\nu}{\nu_0}\right)^2 d\left(\frac{\nu}{\nu_0}\right) d\mu ,$$

$$\frac{\partial E_Z^*}{\partial Z} = 4\pi\, \Sigma Z \int f^S\, d\tau^* ,$$

$$\frac{1}{c}\frac{\partial E_Z^*}{\partial t} = -4\pi\, \Sigma\, Z \int \beta\mu\, f^a\, d\tau^* \qquad (35)$$

$$O_p^a f^a + O_p^s f^s = \eta^s + D_p^a f^a + D_p^s f^s ,$$

$$O_p^s f^a + O_p^a f^s = \eta^a + D_p^s f^a + D_p^a f^s ,$$

where the operator expressions are

$$O_p^a = \beta\mu \frac{\partial}{\partial Z} + (\chi^a + \chi'^a) + (\Phi_Z^s + \Phi'^s_Z) \left(\frac{\mu}{\gamma^3} \frac{\partial}{\partial \beta} + \frac{(1-\mu^2)}{\gamma\beta} \frac{\partial}{\partial \mu} \right) ,$$

$$O_p^s = \frac{1}{C} \frac{\partial}{\partial t} + (\chi^s + \chi'^s) + (\Phi'^a_Z) \left(\frac{\mu}{\gamma^3} \frac{\partial}{\partial \beta} + \frac{(1-\mu^2)}{\gamma\beta} \frac{\partial}{\partial \mu} \right) ,$$

$$D_p^a = \alpha_{\beta\beta}^a \frac{\partial^2}{\partial \beta^2} + \alpha_{\mu\mu}^a \frac{\partial^2}{\partial \mu^2} + \alpha_{\beta\mu}^s \frac{\partial^2}{\partial\beta\partial\mu} + \alpha_\beta^a \frac{\partial}{\partial \beta} + \alpha_\mu^s \frac{\partial}{\partial \mu} ,$$

$$D_p^s = \alpha_{\beta\beta}^s \frac{\partial^2}{\partial \beta^2} + \alpha_{\mu\mu}^s \frac{\partial^2}{\partial \mu^2} + \alpha_{\beta\mu}^a \frac{\partial^2}{\partial\beta\partial\mu} + \alpha_\beta^s \frac{\partial}{\partial \beta} + \alpha_\mu^a \frac{\partial}{\partial \mu} .$$

Equations (35) can be expressed numerically by setting a partition in the (Z, t, β, μ) space, writing the derivatives, as finite differences and by impossing boundary conditions closely related to the problem one is solving. Because of the definition of f^s and f^a, for μ=0, it is straightforward to write

$$\frac{\partial f^s}{\partial \mu} = 0 \quad , \quad \frac{\partial^2 f^s}{\partial \mu^2} = \frac{2\Delta f^s}{\Delta \mu^2} ,$$

$$\frac{\partial f^a}{\partial \mu} = \frac{\Delta f^a}{\Delta \mu} \quad , \quad \frac{\partial^2 f^a}{\partial \mu^2} = 0 \qquad (36a)$$

being $\Delta f = f(\Delta\mu) - f(0)$. Since, the inversion of the sign of β is equivalent to the inversion of the sign of μ plus a rotation of ϕ in ¶, we have, for β=0,

$$\frac{\partial f^s}{\partial \beta} = 0 \quad ; \quad \frac{\partial f^a}{\partial \beta} = \frac{\Delta f^a}{\Delta \beta} \quad ; \quad \frac{\partial^2 f^s}{\partial \beta^2} = \frac{2\Delta f^s}{\Delta \beta^2} \quad ; \quad \frac{\partial^2 f^a}{\partial \beta^2} = 0 ,$$

being $\Delta f = f(\Delta_\beta) - f(0)$

For large β's one has to chose a cutoff value at which boundary conditions have to be defined by the problem and do not have much influence on the results since the particles with large β have negligible interaction with the bulk of the gas particles.

For the limit, $\mu = 1$,

$$\frac{\partial f^s}{\partial \mu} = \frac{\Delta f^s}{\Delta \mu} \quad ; \quad \frac{\partial f^a}{\partial \mu} = \frac{\Delta f^a}{\Delta \mu}$$

where $\Delta f = f(1) - f(1 - \Delta\mu)$.

We suggest to use a minimum of three values for μ and ten values of β; in this way, one obtains dimensions accesible to medium computers.

Once one has expressed numerically equations (35), one can solve the resultant system of equations by the Newton-Raphson technique, starting from initial values for the distributions and assuming stationary state. In this way, one can calculate the spatial evolution of f^S and f^a; after this, it is possible to use eigenvalue techniques to evaluate stability, normal modes and relaxation times for departures from stationary state.

The simplest case is that of only binary interactions. By using the previous definition of W and superscript indexes to show the directions along the z-axis of p_A, p_B, p_C and p_D, respectively, we write, a point indicating that the terms that correspond to both signs are to be added,

$$\chi^S \pm \eta^S = \Sigma \int (W^{++\cdot\cdot} + W^{+-\cdot\cdot})\ f^S_B\ d\omega\ d\tau^*_B \quad ,$$

$$\chi^a \pm \eta^a = \Sigma \int (W^{++\cdot\cdot} - W^{+-\cdot\cdot})\ f^a_B\ d\omega\ d\tau^*_B \quad ,$$

$$\eta^S = \Sigma \int \left[(W^{+\cdot++} + W^{+\cdot--})\ (f^S_C f^S_D + f^a_C f^a_D) + (W^{+\cdot-+} + W^{+\cdot+-})\ (f^S_C f^S_D - f^a_C f^a_D) \right] d\omega\ d\tau^*_B \quad ,$$

$$\eta^a = \Sigma \int \left[(W^{+\cdot++} - W^{+\cdot--})\ (f^S_C f^a_D + f^a_C f^S_D) + (W^{+\cdot-+} - W^{+\cdot+-})(f^S_C f^a_D - f^a_C f^S_D) \right] d\omega\ d\tau^*_B \quad , \qquad (37)$$

and similar expressions for χ', Φ' and all α values.

The sign on the left-hand side depends on A particle statistics, it is plus for bosons, and minus for fermions.

These properties simplify the problem of coefficient derivatives, which can be stored when calculating the values of the coefficients.

IV. TRANSPORT COEFFICIENTS IN LTE

Starting with the Planck-Boltzmann-Maxwell distribution functions,

$$F\ \alpha (e^{E/kT} + 1)^{-1} \quad ,$$

$$F_f\ \alpha (e^{E/kT} - 1)^{-1} \qquad (38)$$

where E, is the total energy, k, the Boltzmann constant, and T, the temperature.

For photons, $E = h\nu$, and, for particles,

$$E = mc^2(\gamma' - 1 - \Phi_2 Z) + \varepsilon \qquad (39)$$

where γ' depends on the velocity β' in the fluid system, ε is the internal energy of that species of particles and Φ_2 is the equilibrium force. Since we are dealing with a non-degenerate plasma we can write in the present approximation

$$f^S = \frac{N}{K} e^{-E/kT} + \delta f^S \quad ,$$

$$f^S_f = \frac{N_f}{K_f} (e^{E/kT} - 1)^{-1} + \delta f^S \quad ,$$

$$f^a = \delta f^a$$

$$f^a_f = \delta f^a_f \qquad (40)$$

where $\delta f^{S,a}$ are the first order departures of the symmetric and antisymmetric parts of the distribution function, N, the density of the species involved, and K, the corresponding normalization factors.

In the transport approximation one assumes that the Lionville operator applied to the zero order term of the distribution function is a first order quantity, and the operator, when applied to the first order term of the distribution function, gives a second order term.

Since the zero order part of the distribution function depends only on the thermodynamical variables, the results from equations (35), for a stationary case, allow us to write, up to first order terms,

$$\beta\mu \frac{\partial f^a}{\partial Z} + \Phi_Z^S \left(\frac{\mu}{\gamma^3} \frac{\partial f^a}{\partial \beta} + \frac{(1-\mu^2)}{\gamma\beta} \frac{\partial f^a}{\partial \mu} \right) = 0 ,$$

$$\beta\mu \frac{\partial f^S}{\partial Z} + \Phi_Z^S \left(\frac{\mu}{\gamma^3} \frac{\partial f^S}{\partial \beta} + \frac{(1-\mu^2)}{\gamma\beta} \frac{\partial f^S}{\partial \mu} \right) = \Gamma \qquad (41)$$

$$\Gamma = \frac{\partial F}{\partial N} \frac{\partial N}{\partial Z} + \frac{\partial F}{\partial \beta_0} \frac{\partial \beta_0}{\partial Z} + \frac{\partial F}{\partial T} \frac{\partial T}{\partial Z} + mc^2 \delta \Phi_Z^S F ,$$

where β_0 is the macroscopic velocity along z and, $\delta\Phi_Z^S$, the perturbation on the force relative to equilibrium force.

By using the definition of F, one obtains, $K \propto T^3$ for photons, and then

$$\Gamma_f = \mu F \left\{ \frac{d \ln N}{dZ} + \left(\frac{h\nu}{kT} \frac{e^{h\nu/kT}}{e^{h\nu/kT}-1} - 3 \right) \frac{d \ln T}{dZ} \right\} ; \qquad (42)$$

for particles, we have

$$\Gamma = \beta\mu F \left\{ \frac{d \ln N}{dZ} + \left(\frac{E}{kT} - \frac{d \ln K}{d \ln T} \right) \frac{d \ln T}{dZ} - \frac{d\gamma'}{d\beta_0} \left(\frac{mc^2}{kT} \right) \frac{d\beta_0}{dZ} + q* \left(\frac{mc^2}{kT} \right) \right.$$

The last equation can be simplified when treating non-relativistic plasmas, since in such a case $K \propto T^{3/2}$

$$\frac{d \ln K}{d \ln T} = \frac{3}{2} ; \qquad \gamma' \simeq 1 + \frac{\beta'^2}{2} ,$$

and

$$\frac{d\gamma'}{d} = \beta_0 - \beta\mu .$$

When calculating for a system where fluid is locally at rest, one can write

$$\frac{d\gamma'}{d\beta_0} = -\beta\mu ; \qquad \Gamma = \Gamma_N \frac{d \ln N}{dZ} + \Gamma_{\beta_0} \frac{d\beta_0}{dZ} + \Gamma_T \frac{d \ln T}{dZ} + \Gamma_{E*} \delta E_Z^* ,$$

where, for particles,

$$\Gamma_N = \beta\mu F ,$$

$$\Gamma_{\beta_0} = \beta^2\mu^2 F \left(\frac{mc^2}{kT} \right) ,$$

$$\Gamma_{E_Z^*} = \beta\mu F\, q* \left(\frac{mc^2}{kT} \right) ,$$

$$\Gamma_T = \left(\frac{E}{kT} - \frac{3}{2} \right) \beta\mu F \qquad (43)$$

and, for the photons,

$$\Gamma_N = \mu F ,$$

$$\Gamma_{\beta_0} = \Gamma_{E_Z^*} = 0 ,$$

$$\Gamma_T = \left(\frac{h\nu}{kT} \frac{e^{h\nu/kT}}{e^{h\nu/kT}-1} - 3 \right) \mu F$$

In the present approximation, the zero order collision terms cancel and we are left with the first order terms. Since the expressions of χ^a, χ'^a, Φ'_Z and D_p^a contain factors f^a, these coefficients become first order, while χ^S, χ'^S, Φ'^a, and D_p^a, instead, can be expanded in zero order and first order terms.

Because of the mentioned reasons and equations (35), we can write

$$O_p\ \delta f^S = \delta\eta^S + \delta D_p^S F - (\delta\chi^S + \delta\chi'^S) - \delta\Phi'^a_Z \left(\frac{\mu}{\gamma^3} \frac{\partial F}{\partial\beta} \right) ,$$

$$\Gamma + O_p\ \delta f^a = \delta\eta^a + \delta D_p^a F - (\delta\chi^a + \delta\chi'^a) - \delta\Phi'^S_Z \left(\frac{\mu}{\gamma^3} \frac{\partial F}{\partial\beta} \right) \qquad (44a)$$

with

$$O_p = (\chi^S + \chi'^S) + \Phi'^a_Z \left(\frac{\mu}{\gamma^3} \frac{\partial}{\partial\beta} - \frac{(1-\mu^2)}{\gamma\beta} \frac{\partial}{\partial\mu} \right) - D_p^S$$

The values $\delta\eta$, $\delta\chi$, $\delta\chi'$, $\delta\Phi'$ and δD_p are the corresponding first order terms and can be expressed from their derivatives with respect to $\delta f_{B'}$; thus,

$$\delta\chi^S = \left(\frac{\partial\chi^S}{\partial f_B^S} \right) \delta f_B^S + \left(\frac{\partial\chi^S}{\partial f_B^a} \right) \delta f_B^a ,$$

$$\delta\eta^S = \left(\frac{\partial\eta^S}{\partial f_C^S} \right) \delta f_C^S + \left(\frac{\partial\eta^S}{\partial f_C^a} \right) \delta f_C^a + \left(\frac{\partial\eta^S}{\partial f_D^S} \right) \delta f_D^S + \left(\frac{\partial\eta^S}{\partial f_D^a} \right) \delta f_D^a \qquad (44b)$$

and similar expressions for $\delta\chi'$, $\delta\Phi'$ and δD_p. The values of these derivatives can be from equations (37) for binary collisions and deflections, and, in other cases, they can be easily obtained from the collision terms expressions.

In the present case, the derivatives in (44b) give matrices with null diagonal terms, and, if one consider only binary collisions, some of these derivatives become null. This implies that the first equation (46a) states that $\delta f^S = 0$, and the second equation (44b) can be written in the matrix form, as

$$\vec{\Gamma} + \bar{\bar{M}}.\vec{\delta f}^a = 0 \qquad (44b')$$

$$\bar{\bar{M}} = \bar{\bar{O}}_p + \left(\frac{\partial\chi^a}{\partial f_B^a} + \frac{\partial\chi'^a}{\partial f_B^a} + \frac{\partial\Phi'^S_Z}{\partial f_B^a} \frac{\mu}{\gamma^3} \frac{\partial F_B}{\partial\beta} - \frac{\partial\eta^a}{\partial f_B^a} \frac{\partial D_p^a F}{\partial f_B^a} \right)$$

where in the expression of M all terms are matrix operators.

The solution of equation (44b') is, then,

$$\vec{\delta f}^a = - \bar{\bar{M}}^{-1} . \vec{\Gamma} = - \bar{\bar{M}}^{-1} . \left(\vec{\Gamma}_N \frac{d\ \ln N}{dZ} + \vec{\Gamma}_{\beta_0} \frac{d\beta_0}{dZ} + \vec{\Gamma}_T \frac{d\ \ln T}{dZ} + \vec{\Gamma}_{E_Z^*} \delta E_Z^* \right) \qquad (44c)$$

By comparing δf^a with F one can check the hypothesis of small departures from equilibrium for a given value of the thermodynamical variable gradient.

It is also possible to take account of the flux saturation efects in a way similar to the one proposed by Shvarts *et al.* (1981), by replacing the value of δf^a, for the harmonic mean $(\delta f^{a^{-1}} + F^{-1})^{-1}$.

By taking the appropiate momenta of δf^a, one can calculate the fluxes which enter in the hydrodynamical equations; for instance, the fluxes of mass

$$J_m = \Sigma \int m\beta\mu\delta f^a \, d\tau^* \quad ,$$

the internal energy $J_\varepsilon = \Sigma \int \varepsilon\beta\mu\delta f^a \, d\tau^*$,

the electric charge (current density) $J_q = \Sigma \int Ze\beta\mu\delta f^a \, d\tau^*$,

the impulse along the z-axis $J_{p_2} = \Sigma \int mc\beta^2\mu^2\delta f^a \, d\tau^*$ (45a)

and

the thermal energy (non-relativistic cases) $J_T = \Sigma \int mc^2 \frac{\beta^3}{2} \mu\delta f^a \, d\tau^*$

where the summation is over all species of particles. There, the particle fluxes for each species is given by

$$J_N = \Sigma\beta\mu\delta f^a \, d\tau^* \qquad (45b)$$

Expressions (45a) and (45b) can be written as a vector product by expressing numerically the integral; thus,

$$J = \vec{V}.\vec{\delta f}^a \qquad (46a)$$

being

$$V_{p_2} = mc^2\beta^2\mu^2\vec{w} \quad ,$$

$$V_T = mc^2 \frac{\beta^3}{2} \mu \vec{w} \qquad (46b)$$

$$V_{N_A} = \vec{w}_A$$

where the vectors $\vec{w}$ and $\vec{w}_A$ represent the integration weight for the distribution function over the space of the impulses for all species of particles in the first case, and, only for specie A, in the second case.

From (44c) and (46a), the general expression for the fluxes becomes

$$J = - \vec{V}.\bar{\bar{M}}.\vec{\Gamma} = - \vec{V}.\bar{\bar{M}} \left(\vec{\Gamma}_N \frac{d \ln N}{dZ} + \vec{\Gamma}_{\beta_0} \frac{d\beta_0}{dZ} + \vec{\Gamma}_T \frac{d \ln T}{dZ} + \Gamma_{E_Z}^* \delta E_Z^* \right) \qquad (47)$$

From equation (47) one can define transport coefficients for the fluxes defined in (45), but the *usual* coefficients are sometimes combinations of the coefficients that come from (47).

In LTE, all related species (each one of which can become transformed into ther one) densities are functions of some total *element* density and temperature through the Saha-Boltzmann and equivalent formula, and photon density is set by the temperature; thus, there are linear relations between the different

$$\frac{d \ln N}{dZ} \quad ; \quad \frac{d \ln T}{dZ} \quad ; \quad \frac{d \ln N_A}{dZ}$$

where N_A is element A density.

Element densities are, however, independent of each other, and, for them, it is easy to define the self and mutual diffusion coefficients, and the mobility, which are given by

$$C_{N_A N_A} = -(\vec{V}_{N_A} . \bar{\bar{M}} . \vec{\Gamma}_{N_A}) \; N_A^{-1} \quad ,$$

$$C_{N_A N_B} = -(\vec{V}_{N_A} . \bar{\bar{M}} . \vec{\Gamma}_{N_B}) \; N_B^{-1} \quad ,$$

$$C_{N_A E_Z^*} = -(\vec{V}_{N_A} . \bar{\bar{M}} . \vec{\Gamma}_{E_Z^*}) \qquad (48)$$

respectively.

It is customary to define the other transport coefficients in a way that all fluxes but the one in question are zero; thus, when applying a macroscopic variable gradient, it is necessary to set values for the other gradients in order to set all fluxes but one equal to zero.

It results, then, that customary transport coefficients are linear combinations of the coefficients C, which result from (47); it is not our present purpose to show how they can be constructed, but the question of their values will be the subject of further work.

In LTE one can consider the particles and photons as components of one gas which has transport coefficients defined by both components, or one can separate the effects of particles and photons by writing separate flux contributions in the hydrodynamical equations.

For instance, in the second formulation one can define the diffusion coefficient for radiative energy diffusion, which can be expressed in terms of medium opacity, while in the first formulation radiative energy diffusion is accounted for by the thermal energy flux and fixed by the conduction coefficient and the temperature gradient.

In partial LTE the above developed formalism can be followed except by the fact that related species have densities given by the statistical equilibrium equations, and photon distribution cannot be approximated to Planck function, so we are forced to solve the equations of radiative transfer from (32), statistical equilibrium equations derived integrating formula (32) over the space of the impulses (there we can drop the elastic interaction terms since they cancel) and the hydrodynamical equations for the particles.

In the present approximation, equation of radiative transfer and statistical equilibrium can be solved as a complete set and the result depend on the boundary conditions stated for the radiation field, the solution is straightforward for optically thin plasmas. In this scheme we think in the distribution function of photons (or, equivalently, the intensity) as a variable which enters in all equations and whose value is defined by them.

The hydrodynamical equations contain, as in LTE, various fluxes which are proportional to macroscopic variable gradients, but in PLTE, the photons impulse and energy fluxes have to be explicity calculated and cannot be included in the transport coefficients. The transport coefficients in PLTE do not coincide with those in LTE, and can also depend on the medium intensity of the radiation field. In PLTE there also appears some extra (radiative) transport coefficients which relate the radiation flux (H_ν) with all remaining fluxes, some of them can be called, for instance $C_{N_A H_\nu}$, C_{qH_ν}, C_{TH_ν}, for the A particle, the electric charge and the thermal energy fluxes, respectively. The complete set of radiative transport coefficients can be calculated from equations (46a) by setting them for particles only, separating the parts of $\delta\eta^a$ and $\delta\chi^a$ which are proportional to δH_ν, and including them in as term

$$\int \Gamma_{H_\nu} \delta H_\nu \, d\nu$$

V. CONCLUSIONS

We have presented a general form of kinetic equations which contain in itself the Boltzmann, Focker-Planck, radiative transfer and Vlasov equations.

A simplified one-dimensional case is set by separating equations in symmetrical and antisymmetrical components, which for the radiative transfer equations correspond to Feutrier method (Mihalas 1978). Is is shown how they can be solved by the Newton-Rapson method under certain boundary conditions.

Finally we showed a method which can be used in order to calculate transport coefficients in the cases of LTE and partial LTE, giving values for the transport coefficients and in the partial LTE case, also for photoeffects as, for instance, the ones that correspond to photoelectrical or photodiffusion effects. This method permits, also, to estimate the range of validity of the transport coefficient approximation.

We are developing a computer program to run on a VAX 750 machine, in order to compute the mentioned transport coefficients for a photon, electron, proton an hydrogen atom gas, taking account of ionizations and recombinations as reactive procesess, elastic collisions and deflections. The program is in an advanced state of development, but we are not yet ready to publish results.

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