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FORMATION OF STRUCTURES IN THE VERY EARLY UNIVERSE

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ABSTRACT. We sketch an alternative picture of cosmological phase transition and study its implications on the formation of structures in the very early Universe. We show that the condensation of walls at high temperatures might lead to fluctuations which are in accordance to all necessary conditions to the formation of structures in the Universe.

Key words: Cosmological Phase Transition, Domain Walls, Galaxy Formation.

. INTRODUCTION

The standard cosmology based on Friedmann's model assumes that matter and radiation ave been homogeneous and isotropically distributed during all the history of the Universe. onsequently the formation of nowadays observed structures of the Universe such as galaxies, lusters of galaxies and superclusters, demands the occurrence of small fluctuations in the niform energy density.

The magnitude of the primordial density fluctuations was established by Zel'dovich Zel'dovich 1972) through the compatibility with the barion-photon ratio that is presently valuated as $r = (n_B/n_\gamma) \sim 10^{-9\pm1}$ with temperature fluctuations observed in cosmic microwave ackground radiation ($\delta T/T \sim 10^{-4}$) and with the quantity of primordially synthetised elements consistent with r),

$$\left(\frac{\delta\rho}{\rho}\right) \sim 10^{-4} \tag{1}$$

The most accepted scenario (see the review by Shandarin, Doroshkevich, Zel'dovich 983) is based on the hypothesis that the large scale of the Universe observed today emerged rom density fluctuations which resulted from processes operating close to the singularity. In is context cosmological phase transitions might play a relevant role. This is due to the fact at when a phase transition takes place, one expects the appearance of inhomogeneities in the stem (Kibble 1980). Examples of such inhomogeneities are the Bloch walls in ferromagnetism.

In the paper we will explore the possibility that objects analogous to the Bloch alls in Grand Unified theories (deffects) might lead to contrast densities of the desired order f magnitude (1) as well as lead to a consistent picture for the formation of structure in the ery early Universe. Our approach differs from the usual one, based on the behavior of the effective potential (Linde 1979), due to the fact that the symmetry restauration occurs as a result of condensation of deffects. Consequently one expects global symmetry restauration, but ot local.

II. ALTERNATIVE METHOD FOR THE STUDY OF PHASE TRANSITIONS

Unified models of the interactions admit the existence of macroscopic solitons. These solutions interpolates between different vacua of the theory, and consequently divide the space into domains, functioning therefore as Bloch walls.

At first sight one feels like discarding these solutions since the partition function associated to such a configuration is proportional to $\exp - EA/T$ where A is the soliton area and E the energy per unit area, and it becomes zero in the thermodynamical limit ($V \to \infty$, $A \to \infty$). Consequently the soliton seems not to be thermodynamically relevant. Nevertheless the emergence of a soliton alters the entropy of the system and consequently if we want to decide whether or not a soliton is thermodynamically favored the correct analysis is to consider the free energy associated to such a Bloch wall per unit area, that is (Aragão de Carvalho, Bazeia, Éboli, Marques, Silva, Ventura 1985; Ventura 1981)

$$f_{wall}(T) = E - Ts(T) \quad . \tag{2}$$

At low temperatures $f_w(T)$ is positive and a Bloch wall will not appear in the system. As the temperature increases the entropy term in (2) takes over the energy term and, in accordance with Peierls arguments (Kosterlitz and Thouless 1973), walls will sprout in the system. One then expects that there is a critical temperature T_c for which

$$f(T_c) = 0 \quad . \tag{3}$$

Above the critical temperature the cost in energy in order to introduce a domain wall in the system is zero and there will be a condensation of such objects above the critical temperature.

For the SU(5) Grand Unified Theory, for which the Higgs field potential is written as (we follow the notation of Daniel Vayonakis 1981)

$$V(\phi) = -\frac{1}{2}\mu^2 T_r \phi^2 + \frac{a}{4}(T_r\phi^2)^2 + \frac{b}{2} T_r \phi^4 \quad . \tag{4}$$

An explicit soliton is written as

$$\phi_S = v \tanh \frac{\mu x}{2} \begin{pmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & -3/2 & \\ & & & & -3/2 \end{pmatrix} \tag{5}$$

It has been shown (Éboli, Guerra, Marques 1985) that, being g_5 the gauge coupling, the free energy of a domain wall in the high temperature limit is given by

$$f_{wall} = 5\sqrt{2}\, v^2 \left(1 - \frac{T^2}{60\mu^2}\left(\frac{225}{2} g_5^2 + 13(15a+7b) + 50b\right)\right) \tag{6}$$

where v in (5) and (6) is given by

$$v = \frac{\mu}{\sqrt{\frac{15}{2}a + \frac{7}{2}b}} \quad . \tag{7}$$

From (6) and (3) it follows than that the critical temperature is

$$T_c^2 = \frac{60\mu^2}{\frac{225}{2} g_5^2 + 13(15a+7b) + 50b} \quad . \tag{8}$$

Up to now we have shown that walls will emerge in the system but nothing has been said on counting them. At first sight one might think that there will be produced an infinite number of domain walls. However, as pointed by Ventura (Ventura 1981) this is not so. We will repeat here his proposal for counting domain walls.

The theory is defined for the volume $V = AL = L^3$ and the N^{th} configuration contains $3N$ solitons (N solitons parallel to each of the volume faces which involve the system). The system tends to produce many solitons, because they are thermodynamically favoured configurations and that could make it collapse. Collapse does not occur because of soliton interactions which are supposed to be proportional to the intersections between them, that is, $\alpha\mu^2/gL$. If Δ is the distance between neighbouring solitons and $\Delta = L/N$, then the system's free energy of noninteracting walls shall be

$$F_N = 3NA\, f_{wall}(T) = \frac{3L^3}{\Delta} f_{wall}(T) \quad . \tag{9}$$

But, if we take into account interactions occurring in the intersections and remember that there are $3N^2$ intersections to the proposed geometry, we get:

$$F_N^{Int} = 3N^2\, \alpha\, \frac{\mu^2 L}{g} = 3\alpha\, \frac{\mu^2}{g}\, \frac{L^3}{\Delta^2} \tag{10}$$

therefore, the total free energy would be

$$F_N = 3V \left(\frac{f_{wall}}{\Delta} + \frac{\alpha\mu^2}{g\Delta^2}\right) \tag{11}$$

the stability is obtained with $3N_0$ solitons ($3N_0 = L/\Delta_0$), which minimize (11). It is easy to show that in these circunstances the average distance between neighbouring solitons is given by:

$$\frac{1}{\Delta_0(T)} = \frac{1}{d_0}\left[\left(\frac{T}{T_c}\right)^2 - 1\right] \tag{12}$$

with

$$d_0 = \frac{2\alpha}{5\sqrt{2}\,\mu} \quad . \tag{13}$$

We see that the average distance between the solitons must obviously be greater than their typical width which is approximately $1/2\ \mu^{-1}$. This fact establishes a limit temperature to the validity of the proposed approach

$$T_L \cong \left(1 + \frac{2\sqrt{2}}{5}\alpha\right)^{1/2} T_c \quad . \tag{14}$$

The energy density of the solitons may easily be calculated by:

$$\rho_{wall} = \frac{E_{wall}}{V} = A\, E_{class}\, \frac{3N_0}{L^3} = \frac{3E_{class}}{\Delta_0} \tag{15}$$

and so by using (9) one gets

$$\rho_{wall}(T) = \frac{75\mu^2}{\alpha}\, u^2 \left[\left(\frac{T}{T_c}\right)^2 - 1\right] \qquad T > T_c \quad . \tag{16}$$

The symmetry of the system is recovered at $T > T_c$ because the solitons (5) that come up split the space into regions sometimes with the Higgs field with a value ϕ_V, others with the value $-\phi_V$, so that in the average $\langle\phi\rangle = 0$.

The discussion above can be summarized as follows: this theory describes domain walls (solitons) with a natural thickness $\sim 1/\mu$. This means that for $T \gtrsim T_c$ the average distance between two neighbour walls cannot be smaller than $\Delta_0 \sim 1/\mu$ (otherwise the solitons are so superimposed that one can no longer speak of domains or domain walls). Then for $T \sim T_c$ one has an estimate for the number of domains in the system. If the Universe undergoes a supercooling, this number of domains is going to be preserved till the system reaches the lower temperatures, at which it starts decaying and reheating again. Within this picture one then expects that the number of structures in the Universe should be equal to the number of seeds that generate them (which we call aglutination centers). From the counting of domains one can predict the number of aglutination centers. This calculation and other cosmological implications will be discussed next.

III. COSMOLOGICAL IMPLICATIONS

1. EXISTENCE OF DOMAINS

Unification theories that have $\phi \rightarrow -\phi$ symmetry, have regions at a temperature lower than T_c with expectation values as many as $\pm\sigma$ and could therefore be separated by walls. In the effective potential method this fact is a problem because the superficial energy density of a GUT wall (Lagarides, Shafi, Walsh 1982) is

$$E_{wall} = \alpha_{GUT} \cdot M_H^3 \sim (10^{44}-10^{48})\,g\,cm^{-2} \tag{17}$$

and if these walls are expanding as fast as the horizon (Barrow 1983), it follows that a wall would have a size of the order of our present horizon ($d_H(0,t_p)$), that is

$$R_{wall} \sim d_H(0,t_p) \sim 10^{28}\,cm \tag{18}$$

which implies that the energy associated to one wall divided by the energy of Universe is given by:

$$\frac{E_{wall}}{E_{univ}} \sim \frac{E_{class} \cdot d_H^2(0,t_p)}{\rho_c\, d_H(0,t_p)^3} \sim 10^{46} \sim 10^{50}!! \tag{19}$$

where ρ_c is the critical density energy, $\rho_c \cong 10^{-29}\,g\,cm^{-3}$. Therefore, the discrete symmetry cannot be accepted in these approaches (a term $\propto \mathrm{Tr}\,\phi^3$ is usually introduced in the Higgs potential so as to break the symmetry by hand, and in consequence forbid the existence of walls). However, walls can be very interesting to the formation of structures in the Universe, as we will see.

Within the alternative approach there is a natural solution to this problem since the walls are the solitons which can appear only above the critical temperature, as discussed above, and are forbidden below T_c.

2. THE COSMOLOGICAL CONSTANT PROBLEM

In the usual approach it is possible to show that $\rho_{vacuum} \propto T_c^4$ (Linde 1979, Kibble 1980) and considering the GUT and Weinberg-Salam phase transitions, we have:

$$\rho_{vac} \sim T_c^4 \sim \begin{cases} 10^{78}\,g\,cm^{-3} & \text{GUTS} \\ 10^{25}\,g\,cm^{-3} & \text{G.W.S.} \end{cases} \tag{20}$$

In the present the energy density of the vacuum is estimated by supposing that it does not dominate the dynamics of superclusters of galaxies and so:

$$\rho_{vac} < \rho_{sc} \sim 10^{-29}\,g\,cm^{-3} \tag{21}$$

then, assuming that ρ_v saturates the bound in (21) one gets

$$\frac{\rho_{vGUT}}{\rho_{vac}} \sim 10^{107} \qquad \text{or} \qquad \frac{\rho_{vW.S.}}{\rho_{vac}} \sim 10^{50}!!! \tag{22}$$

nd these huge differences have the explanation within this point of view.

In the alternative approach the energy density of the condensate of walls may be in- erpreted as a "cosmological constant" and as we have seen, the contribution of solitons is mall and tends to zero below the critical temperature.

V. A PROPOSITION ON THE FORMATION OF STRUCTURES IN THE UNIVERSE

The use of elementary particles spectrum to generate primordial density fluctuations nd solve galactic dynamical problems is almost a sort of tradition. The list of examples is ery wide and includes massive neutrinos (Schramm, Steigman 1981), gravitinos (Blumental, Pagels, rimack 1982), photinos (Sciama 1983), and topological objects as strings (Vilenkin 1981) and lso domain walls (Holdon 1982) in spite of dramatic estimations as (14). Following these teps we propose that the remnant of the walls that emerged from the alternative conception to he study of phase transition should work as structure seeds. The following conditions must be ullfiled for the proposition to be consistent:

1) The structures that act as seeds should not dissipate until recombination. This s possible if we keep in mind that:

) topological conservation laws assure the non-dissipation of structures such as walls (solitons);

) although the behaviour of walls becomes unknown below T_c within the equilibrium thermodymical approach, it is believed that the walls would close as "bubbles" with a diminishing radius until zero temperature when the system reaches a unique phase.

2) The presence of walls should not alter significantly Hubble expansive flux. This an be demonstrated if we suppose that the "bubbles" are uniformly distributed and integrating riedman's cosmologic dynamic equation in the presence of solitons, that is:

$$\dot{R}^2(t) + k = \frac{8\pi G}{3} (\rho_{particles} + \rho_{walls}) R^2(t) \quad . \tag{23}$$

The relation between $R(t)$ and T is obtained from the covariant conservation of he matter and radiation energy-momentum tensor and from this it resulted that RT is constant Barrow 1983). This relation is a very good approximation when solitons are present because heir contribution is subdominant.

One can integrate in an approximate way equation (23). In $\rho_{particles}$ one uses Barrow 1983)

$$\rho_{particles} = \frac{\pi^2}{30} n(T) T^4 \tag{24}$$

here $n(T)$ is the effective number of degrees of freedom at the temperature T. (In a GUT such s SU(5) $n_{GUT} \sim 160$). For ρ_{walls} one uses (16). Furthermore if one makes the approximation $(T)g \sim 1$ and $\alpha = 1$, then from the explicit integration in powers of T_c/T one gets

$$t = 2.3\ 10^{-2} \frac{M_p}{T^2} + 3.7\ 10^{-6} \frac{M_p}{T^2} \left[1 + \frac{2T_c^2}{T^2} - \frac{4}{3} \frac{T_c^4}{T^4} + \ldots \right] \tag{25}$$

here M_p in (25) is the Planck mass ($M_p = G^{-1/2}$). As can be seen from (25) the presence of lomain walls just represents a subdominant contribution to that predicted by the standard Friedan model (the first term in the right hand side of (25)).

3) It becomes necessary to satisfy Zel'dovich's (1) condition for the proposed sceario. The contrast density in our case is given by

$$\frac{\delta\rho}{\rho} \equiv \frac{\rho_{wall}}{\rho_{total}} \quad . \tag{26}$$

By using (16) and (24) one can predict that the contrast density will be given, in the SU(5) model, by

$$\frac{\delta\rho}{\rho}(T) = \frac{75\,\frac{\mu^2 v^2}{\alpha}\left(\frac{T^2}{T_c^2} - 1\right)}{n(T)\,\frac{\pi^2 T^4}{30} + 75\,\frac{\mu^2 v^2}{\alpha}\left[\frac{T^2}{T_c^2} - 1\right]} \quad . \tag{27}$$

The determination of the contrast density depends on the value of α, about which we have only numerical estimates ($\alpha \cong 3$) and on the temperature in which we are computing the contrast density. In $T_L < T < 1.001\ T_c$ and $5 < \alpha < 75$ one gets

$$1.1\ 10^{-4} < \frac{\delta\rho}{\rho} < 1.3\ 10^{-2} \tag{28}$$

In this way one can see that GUTS might lead to contrast densities compatible with (1).

4) The length of fluctuation must be greater than Jeans length, so as to enable it to trigger the gravitational mode when recombination occurs.

The length of fluctuation that is proposed here is essentially the distance betweer two walls and it is given by (12) when T is close to T_c. This dimension can be evaluated supposing that the remnants of the walls below T_c, expand conformally keeping the ratio betwee the distance between solitons and the horizon distance constant and so during recombination:

$$L^{GUT} = \frac{d_0^{GUT}}{d_H(0, 2.10^{-37}\,\text{seg})} \times d_H(0,t_R) = \frac{10^{-28}\,\text{cm}}{2.2\ 10^{-37}\,\text{seg}} \times 2\ 10^5\ \text{years} \cong 1.4\ 10^{21}\,\text{cm} \tag{29}$$

that is larger than the Jeans lenght $\lambda_J \sim d_H(0,t_R)$. So the fluctuations generated by the objects produced during the GUT phase transition obey all necessary conditions to the formation of structures in the Universe. The corresponding mass to (28) is:

$$M^{GUT} = \frac{4\pi}{3}\,\rho_{rec}\,L^3_{GUT} \sim 10^{10}\ M_\odot\ \Omega_p \tag{30}$$

which fits very well in the galactical mass spectrum and is probably consistent with all of the if the dynamics of the "bubbles" below T_c is considered.

If the same path is followed for the Weinberg-Salam phase transition, it is possibl to show that the generated fluctuations are non-relevant because $L^{W.S.} << \lambda_J$. As the walls dc not change the photonic bath, the proposed fluctuations are isothermal and so consistent with the hierarchical scenario. A legitimate conclusion would be that the number of aglutination centers is roughly the number of great structures observed in the Universe today. In fact, one can estimate the number of aglutination centers. This number is roughly given by

$$n_{aglu\ center} \cong \left(\frac{d_H(0,t^{GUT})}{d^{GUT}}\right)^3 \cong 1.9 \times 10^6 \quad . \tag{31}$$

The greater known structures are the superclusters of galaxies that consist of groups with an average of 10^5 galaxies, that have densities close to critical $\rho_c \sim 10^{-29}\,\text{g cm}^{-3}$ and spread over dimensions from 50 to 100 Mpcs (from 1.5 to 3.0×10^{26} cm). The number of these structures (sub-clusters) may be estimated by the ratio

$$n_{sc} = \left(\frac{d_H(0,t_p)}{d_{sc}}\right)^3 \cong 7.10^5 - 6.10^6 \tag{32}$$

ecause $t_p \sim 10^{10}$ years and $d_H(0,t_p) = 3t_p \cong 2.7 \times 10^{18}$ cm.

The results from (31) and (32) are quite close to each other. If it is taken into ccount that superclusters have peculiar speeds of 100 km/s, we may conclude that during ubble's period these structures must have moved just some Mpcs and that their distribution is herefore cosmological making thus the above coincidence very interesting.

V. CONCLUSIONS

We sketched in this paper an alternative picture of cosmological phase transitions. t differs from the orthodox one in many respects. This new picture is based on the idea that ymmetry restauration will take place as a result of condensation of topologically non trivial ield configuration being thus very close to the Kosterlitz-Thouless picture of phase transiions.

The most impressive results however, from the point of view of cosmology, are conerned with the formation of structures of the Universe. Up to now much effort has been made wards obtaining Zeldovich's contrast density. We have shown that domain walls provid the density ontrast of the required order of magnitude for Grand Unified Theories and the Weinberg-Salam odel. If one imposes further that the length of fluctuations do exceeds Jeans' length then nly the fluctuations generated by GUT phase transition satisfies this requirement. We have hown that fluctuations originated from GUT phase transition obey all necessary conditions to he formation of structures in the Universe. Furthermore a rough estimate of the number of glutination centers is equal to the number of great structures observed in the Universe today. he fact that the distribution of superclusters is cosmological makes this coincidence even ore interesting.

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