THE DISTRIBUTION OF INTERSTELLAR DUST

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ABSTRACT. We propose the interstellar matter structural function as a tool to derive the features of the interstellar dust distribution. We study that function resolving some ideal dust distribution models. Later, we describe the method used to find a reliable computing algorithm for the observational case. Finally, we describe the steps to build a model for the interstellar matter composed by spherically symmetrical clouds. The density distribution for each of this clouds is

$$D(r) = D_o \cdot e^{-(r/r_o)^2}$$

The preliminary results obtained are summarised.

I. INTRODUCTION

A statistical method to analyse the interstellar reddenings to obtain information about the interstellar dust distribution is presented. In the second section the properties of the interstellar matter structural functions and some ideal dust distribution models are described. The third section briefly describes the computing problems and the observational material to calculate the observed interstellar matter structural function and the fourth section the dust distribution model that we made to match it.

II. INTERSTELLAR MATTER STRUCTURAL FUNCTIONS

The interstellar matter structural function (hereafter IMSF), is closely related with the angular correlation coefficients and has been used by some authors to investigate the interstellar dust distribution.

The first in to use it was K. Serkowski (1958) and in fact, he defined the IMSF for the reddening. His work was a statistical study of the reddenings and polarization in the double cluster in Perseus and, by making use of the dust model distribution of Chandrasekhar and Münch (1952), he found a size scale for the dust condensations of 1.75 pc. as lower limit. The use of that model, that supposes randomly separated dust concentrations, lead him to obtain a monotonous IMSF. His comparison with the observations is suggestive in this sense.

Later, Schaffler (1966) extends the IMSF use to the general galactic reddening problem, and in order to match it, he develops a two type cloud model distribution. As well as Chandrasekhar and Münch's his model has a stocastic distribution of the dust condensations and for this reason his work, the same as Serkowski's does not provide a method to take into account the probable periodicity of the dust distribution. We will refer to this fact as the structure of the distribution.

In order to later use as test quantity the IMSF we must study it and investigate its qualities response to the dust distribution changes and at the same time to the com-

puting methods.

a) Definition and properties

The IMSF $\mathcal{E}(\varphi)$ is defined by taking two points in the sky plane, separated by the angular distance φ , taking the difference between the color excesses, squaring and averaging over a definition region.

$$\mathcal{E}(\varphi) = \overline{(E - E_\varphi)^2} \tag{1}$$

developing

$$\overline{(E - E_\varphi)^2} = \langle E^2 - 2 E E_\varphi + E_\varphi^2 \rangle = 2(\overline{E^2} - E E_\varphi) \tag{2}$$

and introducing the autocorrelation coefficient

$$\pi(\varphi) = \sigma_{11}^2 / (\sigma_{20}\,\sigma_{02}) = (\overline{E E_\varphi} - \overline{E^2})/\sigma_E^2 \tag{3}$$

into (2) and solving for $\mathcal{E}(\varphi)$ one obtains

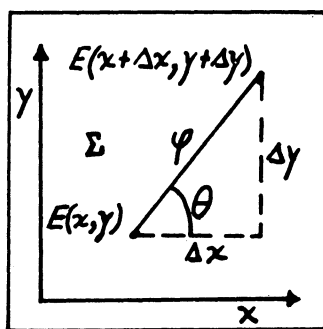$$\mathcal{E}(\varphi) = 2\,\sigma_E\,(1 - \pi(\varphi)) \tag{4}$$

Fig. 1

The autocorrelation coefficient $\pi(\varphi)$ has necessarily its value equal to unity for the zero value of the argument, so that for (4) one sees that

$$\lim_{\varphi \to 0} \mathcal{E}(\varphi) = 0$$

and this is natural as we are looking through the same interstellar material. Furthermore, tends to zero when the argument tends to infinite, and for (4) one sees that

$$\lim_{\varphi \to \infty} \mathcal{E}(\varphi) = 2\,\sigma_E$$

Making explicit refference to the variables involved in the problem, we can rewrite the equation (1) as

$$\mathcal{E}(\varphi) = \left(\frac{1}{2\pi\Sigma}\right) \iint_\Sigma dx\,dy \int_0^{2\pi} d\theta \left[E(x,y) - E(x + \varphi\cos(\theta), y + \varphi\sin(\theta)) \right]^2 \tag{5}$$

where the meaning of the variables is described in the Fig. 1

b) Ideal dust distribution models

It is intuitive that if the interstellar matter distribution has concentrations separated by preferential distances, the IMSF will have in concordance with this fact local max-

ima and minima. Unlike with this, the model used by Serkowski lead to obtain a monotonous increasing IMSF. In order to know more about this behaviour of the IMSFs some simple ideal dust distribution models were solved, making use of the definition (5), the differential color excess

$$d\mathcal{E}(\lambda) = C \, \mathcal{S}(\lambda) \, d\lambda \qquad (6)$$

where C is the differential absortion coefficient, and if it is possible the Chandrasekhar and Munch form for the space density

$$\mathcal{S}(\lambda) = \mathcal{S}_0 \, (1 + f(\lambda)) \qquad (7)$$

In this equation f(s) is an arbitrary function of the space point s(x,y,z) and its mean value must be zero.
The results are now described:

1) "Bar" model.
The "Bar" model is defined by the function

 f(s)=f(x)=sin(2πx/a)
where a is a parameter. In this case, the resulting IMSF is

$$\mathcal{E}(\varphi) = (\bar{E})^2 \left\{ 1 - J_0 \left(\frac{2\pi\varphi}{a} \right) \right\}$$

where Jo is a Bessel function of the first class, zero order.
2) "Chess-board" model.
 The "Chess-board" model is defined by putting in (7)
 f(s)=f(x,y)=sin(2πx/a).sin(2πy/b)
and the resulting IMSF is

$$\mathcal{E}(\varphi) = (\bar{E})^2/2 \left\{ 1 - J_0 \left[\left(\frac{2\pi\sqrt{a^2+b^2}}{a\,b} \right) \varphi \right] \right\}$$

 These two models lead one to the obvious conclusion that, for the same mean color excess the "Bar" model, that is more constrasted than the "Chess-board" model, gives an IMSF that is twice as great as the "Chess-board" one. Taking in account the zeros of the Bessel function of the first class, first order, it is possible to see that the first maxima of the IMSF for the "Bar" model are roughly separated by the distance a. And in the "Chess-board" model roughly by the distance ($\sqrt{2}$ a)/2. The first is the least separation between the maxima (minima) of the density distribution in the "Bar" model and the second is a half of the same one in the "Chess-board" model. This models lead us to confirm our conclusion that the regular distribution of dark matter will give an IMSF with some inflections and that the separation between maxima and minima of this IMSF are closely related with the statistical features of the dust distribution.

 Now, one may ask the following question: Are these conclusions completely general? The doubt is based on the fact that the f(s) functions used in the previous models are intrinsically periodic functions. In order to answer this question the following models were studied:
3) "Single Gaussian cloud" model.
 The single "Gaussian cloud" model is defined by the following density distribution:

$$\mathcal{S}(\lambda) = \mathcal{S}_0 \; e^{-(\lambda/\lambda_0)^2} \qquad (8)$$

here, $\mathcal{S}_0$ is the central and maximum density of the cloud, and λ_0 is a typical radial distance that may be used as a size representative parameter. This distribution has not the Chandrasekhar and Munch form, and will be integrated on the infinite space.

 The resulting IMSF is:

$$\mathcal{E}(\varphi) = 9 \, (\bar{E})^2 \left\{ 1 - e^{-\frac{1}{2}(\varphi/\varphi_0)^2} \right\}$$

 This model lead us to the conclusion that a single cloud does not give structure to the IMSF.
4) "Gaussian cloud array" model.

 The "Gaussian cloud array" model is defined by the following density distribution

$$\mathcal{G}(x,\gamma) = \mathcal{S}_0 \sum_{n=1}^{N^2} e^{-\frac{(x-x_n)^2 + (y-Y_n)^2}{\hbar_0^2}}$$

here, Xn and Yn are the rectangular coordinates of the nth. point in wich one single cloud has its center. If the array has a constant of h, the previous equation may be written as:

$$\mathcal{G}(x,\gamma) = \mathcal{S}_0 \sum_{q=1}^{N} \sum_{m=1}^{N} e^{-\frac{(x-mh)^2 + (y-nh)^2}{\hbar_0^2}}$$

and the resulting IMSF will be

$$\mathcal{E}(\varphi) = \frac{(\bar{E})^2 36}{\pi N^4} \left(1 + \frac{Nh}{6\hbar_0}\right)^2 \left\{ N^2 \left(1 - e^{-\frac{\varphi^2}{2\hbar_0^2}}\right) + 4N \sum_{n=1}^{N-1}(N-n) e^{-\frac{n^2 h^2}{2\hbar_0^2}} \left(1 - e^{-\frac{\varphi^2}{2\hbar_0^2}} I_c\left(\frac{nh\varphi}{\hbar_0^2}\right)\right) + \right.$$

$$\left. + 4 \sum_{q=1}^{N-1} \sum_{m=1}^{N-1} (N-m)(N-n) e^{-\frac{(n^2+m^2)h^2}{2\hbar_0^2}} \left[1 - e^{-\frac{\varphi^2}{2\hbar_0^2}} I_c\left(\sqrt{n^2+m^2} \frac{h\varphi}{\hbar_0^2}\right)\right] \right]$$
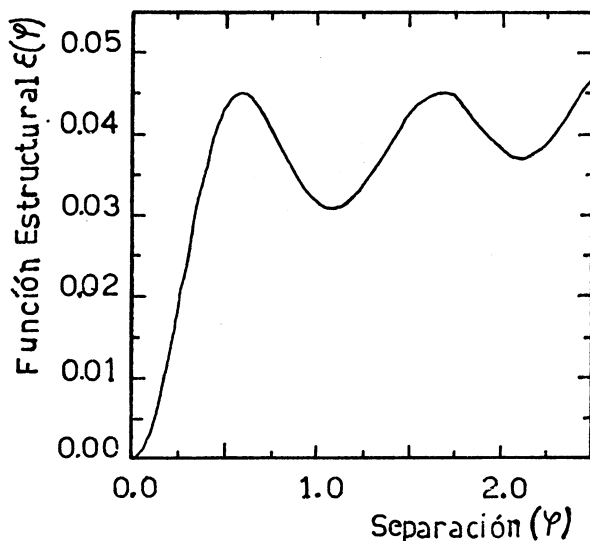

Fig. 2

where Io is the modified Bessel function of zero order. It is clear that this IMSF result of the sum of N^2 IMSF terms of a single cloud (one for each cloud of the array) plus "interferential" terms. In spite of the regular array, none of the functions involved in the IMSF are intrinsically periodical. However, the plot of the previous equation shows the features of a non-monotonous IMSF (See Fig. 2).

The ideal models presented let us to conclude that the IMSF is a pretty sensitive parameter for testing the regular features of the interstellar dust distribution.

III.- COMPUTING METHOD AND OBSERVATIONAL MATERIAL

The conclusion of the previous section will be useful if one has a method to calculate the IMSF starting from photometrical reddening observations. It is clear that in (1), the average cannot be extend on the complete sky plane definition surface. How to avoid this intrinsic difficulty is still an open question. The method proposed by Serkowsky (1958) seems to be very rough and the Feinstein and Marraco (1971) method reveals a extraneously peaked IMSF. The algorithm to compute the IMSF is a critical point that must be solved in order to use this function to find conclusions on the distribution of the interstellar matter.

The ideal models presented in the section II lead us to develop "feedback" cycles and compare the computed and the theoretical IMSF for the same distribution model. This was the method used to obtain a reliable computing algorithm for observational IMSF.

Our IMSF are always based on the reddenings of individual stars. The stars belong-
in to an open cluster are used. In order to satisfy the limits of the IMSF the absence of any
kind of circumstellar matter must be demanded. With this condition it is possible to correct
the reddening dispersion, taking of the intrisic color dispersion, and to work with the real
IMSF (Feinstein, Marraco and Mirabel, 1973). In order to compute the IMSF with good precision
reddenings, the presence of a great number of younger than A0 observed stars is required.

IV. MODEL FOR THE INTERSTELLAR DUST DISTRIBUTION

a) The distribution

The third model presented in section II, may be used to build a more realistic inter
stellar matter distribution model. It is conceivable to think that an arbitrary superposition
of "Gaussian" clouds of different characteristic parameters may reproduce the IMSF and other
properties of the interstellar reddenings.

In order to develop this model it is necessary to restrict the distribution of the
$\mathcal{S}_0$ and π_0 parameters, because not all their possible values are acceptable. Six conditions, involv
ing observational and physical conditions were found. In addition we used the following general
form for the distribution function of the parameters

$$N(p)dp = 0 \qquad\qquad p_1 > p > p_2$$
$$N(p)dp = k\, p^{-\alpha}\, dp \qquad\qquad p_1 < p < p_2$$

where p is one of the two quasi-independent parameters to be generated, k is a normalization
constant, p_1 and p_2 are the minimum and maximum respectively permited values for the parameter,
and α is a free parameter that must be fixed by comparison with the observations. We are now in
condition to compute our model.

b) The dust

To reproduce the optical effect of the dust, it is neccesary to adopt a dust model,
and make use of its photometrical characteristics. We take for valid the dust model proposed by
Mathis et al. (1981).Using the extinctions presented in this work and the cosmical abundances
of Cameron (1973) the differential absortion constant per unit of mass $C_{(B-V)} = (\kappa_B - \kappa_V)$, is com-
puted.

c) Preliminary results

Our first observational result (NGC 2516) indicates that a regular , low constrasted
dust distribution could be responsible of the observed reddenings. The scale of distance of the
distribution might be similar to the Serkowski (1958) one for h and χ Persei.

Regardless of the first conclusion, we use a random process for the space distribu-
tion of the clouds. Our objetive is to investigate the probability of building an inflective IMSF
with stocastic spaced dust cloud fields.

Our preliminary models indicate that irrespective of the value chosen (within rea-
sonable values) for the α parameter of the distribution of one variable there is always an α
parameter of the distribution of the other variable (also within reasonable values) that satis-
fies the mean value and dispersion of the observed excesses. That is, in the "α_1, α_2" plane, the
aceptable models will define a continuous curve. However, the IMSF features seem to be sensitive
to the α_1 (α_2) changes.

REFERENCES

Cameron A.G.W., 1973, *Space Sci. Rev.*, 15, 122.
Chandrasekhar S., and Munch G., 1952, Ap. J., 115, 103.
Feinstein A., and Marraco H.G., 1971, *Bol. Asoc. Arg. Astron.*, 16, 16.
Feinstein A., Marraco H.G., and Mirabel I., 1973, *Astr. Ap. Suppl.*, 9, 233.
Mathis J.S., and Wallenhorst S.G., 1981, Ap. J., 244, 483.
Schaffler H., 1967, *Z. fur Astrophys.*, 65, 60.
Serkowski K., 1958, *Acta Astronomica*, 8, 135.

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