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A MODEL OF THE STELLAR VELOCITY FIELD

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RESUMEN. Se ha deducido mediante las ecuaciones hidrodinámicas un modelo cinemático basado en los momentos centrados obtenidos a partir de una amplia muestra de estrellas. Se ha determinado la forma, muy general, de la densidad estelar, el potencial galáctico, las presiones de segundo orden y la velocidad del centroide, siendo esta última estudiada en un interesante caso particular.

ABSTRACT. A kinematic galactic model has been derived through the hydrodynamical equations, based on the central momenta obtained for a wide sample of stars. The very general form of the stellar density, the galactic potential, the second order pressures and the velocity of the local standard of the rest has been determined, being the latter studied in an interesting special case.

Key words: GALAXY-KINEMATICS AND DYNAMICS

I. INTRODUCTION

Hydrodynamical equations obtained by taking momenta of the fundamental equation of stellar dynamics have been for long time used, usually under suitable hypotheses, to derive relations holding for the kinematic parameters that describe the stellar velocities, the potential under which the stars are moving, the forces acting upon them or the density that gives account of their space distribution.

After Erickson (1975) first computed central momenta up to the fourth order of a sample of stars taken from the Nearby Stars Catalogue (Gliese 1969), the hydrodynamical equations have also been used to obtain estimations of parameters other than momenta, mainly their gradients, related to them through the equations. Some recent momenta determinations present values not compatible with general hypotheses for long time accepted on the stellar velocity distribution function. The behaviour of these samples of stars will be therefore better described by kinematic models derived through hydrodynamical equations based on a suitable choice of hypotheses, e.g. a) not contradicting experimental data, b) not too determined that they are not solvable , or c) not too loose that it is not possible to determine their solutions.

II. THE SAMPLE OF STARS

Recently, Figueras (1986) compiled a catalogue of kinematic and astrophysical data based on the S.A.O. Catalogue with astrophysical parameters (Ochsenbein 1980). These parameters have been used to determine spectro-photometric parallaxes for a total of 12824 stars. Luminosity class - spectral type and distance - spectral type distributions of this catalogue can be found in page 21 of the work by Figueras.

Among other important studies, central momenta up to the fourth order have been computed for all the stars in the catalogue with residual velocities with respect to Delhaye (1965) local standard of the rest lower than 65 kms^{-1}, and with residuals lower than 2.5 σ, σ being the error of the determination of the main kinematic parameters of the stars in the catalogue. In computing these momenta, the galactic rotation, obtained previously, has been subtracted to their velocities. The results, reproduced here by kind permission of the author, are shown in table 1. It must be pointed out that the sub-sample constituted by A - M stars has momenta with similar characteristics to the global one, while OB stars have, as many authors have described before, μ_{200}, μ_{020} and μ_{002} nearly equal; this fact suggests that their distribution is nearly spherical and that the hypotheses under which it could be described are different from the ones useful for the rest of the stars.

TABLE 1. HELIOCENTRIC VELOCITY OF THE LOCAL STANDARD OF THE REST AND CENTRAL MOMENTS OF THE RESIDUAL VELOCITY DISTRIBUTION. Units: $km^n\ s^{-n}$ (taken from Figueras 1986).

	9629 *	A - M 7817 *	O B 1812 *
$-U_\odot$	-9.85 ± 0.22	-9.87 ± 0.25	-9.78 ± 0.37
$-V_\odot$	-13.05 ± 0.17	-13.04 ± 0.19	-13.13 ± 0.36
$-W_\odot$	-6.90 ± 0.15	-6.08 ± 0.16	-7.01 ± 0.36
μ_{200}	457 ± 6	506 ± 7	245 ± 9
μ_{110}	52 ± 4	63 ± 4	5 ± 6
μ_{020}	279 ± 4	289 ± 5	240 ± 8
μ_{101}	1 ± 3	-1 ± 4	10 ± 6
μ_{011}	4 ± 3	6 ± 3	-7 ± 6
μ_{002}	205 ± 4	197 ± 4	240 ± 10
μ_{300}	380 ± 205	404 ± 243	374 ± 275
μ_{210}	-558 ± 103	-811 ± 121	542 ± 150
μ_{120}	188 ± 90	120 ± 106	490 ± 151
μ_{030}	-1055 ± 140	-1511 ± 162	926 ± 247
μ_{201}	-82 ± 61	-223 ± 67	500 ± 109
μ_{111}	-19 ± 57	4 ± 66	-126 ± 105
μ_{021}	-61 ± 39	-88 ± 21	49 ± 164
μ_{102}	42 ± 76	-2 ± 86	235 ± 151
μ_{012}	-198 ± 65	-293 ± 71	217 ± 157
μ_{003}	-211 ± 114	-344 ± 119	374 ± 314
μ_{400}	604431 ± 16387	694358 ± 19617	217329 ± 17886
μ_{310}	44145 ± 5460	52119 ± 6507	9540 ± 6843
μ_{220}	127206 ± 3404	140618 ± 4012	69566 ± 4835
μ_{130}	29118 ± 4297	35605 ± 5048	1009 ± 6761
μ_{040}	269095 ± 9769	285813 ± 11392	197623 ± 15715
μ_{301}	194 ± 4951	-811 ± 5911	4371 ± 6383
μ_{211}	4476 ± 1955	6029 ± 2296	-2004 ± 3096
μ_{121}	2831 ± 1799	2506 ± 2092	4254 ± 3150
μ_{031}	4607 ± 3415	6801 ± 3915	-4572 ± 6585
μ_{202}	98714 ± 2841	106902 ± 3330	63608 ± 4509
μ_{112}	4559 ± 1576	6361 ± 1791	-3258 ± 3211
μ_{022}	68536 ± 2314	69038 ± 2571	66566 ± 5291
μ_{103}	-4285 ± 3278	-7339 ± 3693	8911 ± 7059
μ_{013}	1343 ± 2885	3504 ± 3116	-7787 ± 7340
μ_{004}	173818 ± 7156	159805 ± 7356	234632 ± 20857

III. DERIVATION OF THE MODEL

Based on the values of the central momenta given in the two first columns of table 1 we have considered a time depending axi-symmetric model where the perpendicular velocity $Z_\odot$ of the local standard of the rest and their gradients are taken to be zero and, in addition to μ_{200}, μ_{020}, μ_{002} second order momenta and μ_{400}, μ_{220}, μ_{202}, μ_{040}, μ_{022}, μ_{004} fourth order momenta, that would be the only not vanishing momenta if the distribution function is quadratic, second order μ_{110}, third order μ_{300}, μ_{210}, μ_{030}, and fourth order μ_{310} and μ_{130} momenta have also been taken to be different to zero.

We have taken that all the fifth order momenta and also their gradients, which appear in the fourth order equations, vanish. The adopted hypotheses of which are the vanishing momenta can be too restrictive: may be that momenta taken to vanish identically are only locally zero due to the position of the stars near the galactic plane. In that case their gradients would not vanish and the obtained solutions would be only particular solutions of a more general new model.

The model has been derived by using the hydrodynamical equations of stellar dynamics up to the fourth order written in cylindrical coordinates as it is suggested by the geometry of the system. The complete form of these equations, in terms of momenta, has been given by Sala et al. (1985) and they will be referred to along this work; notwithstanding, we have used them written in terms of pressures, to make easier, if possible, this derivation. A similar work, performed under different hypotheses, was previously done by Orús (1980).

The independence of z of the functions Π_0, θ_0, U, P_{002}, P_{202}, P_{022} and P_{004} is obtained from equations 101, 011, 111, and from this latter one combined with 001, 201, 021 and 003, respectively.

Equations 202, 112 and 022 lead to

$$\frac{\partial \Pi_0}{\partial \varpi} = \frac{\Pi_0}{\varpi}$$

and from it

$$\Pi_0 = p_1(t)\varpi \tag{3.1}$$

Equation 012 states

$$\frac{\partial \theta_0}{\partial t} + \Pi_0 \left(\frac{\partial \theta_0}{\partial \varpi} + \frac{\theta_0}{\varpi}\right) = 0 \tag{3.2}$$

and by solving it

$$\theta_0 = \frac{1}{\varpi} G_1(c) \tag{3.3}$$

where

$$c = \frac{b_1(t)}{\varpi} = \frac{1}{\varpi} \exp \int p_1(t)dt$$

and G_1 is an arbitrary function of c.

Galactocentric radial velocity (3.1) of the local standard of the rest has the form given in Chandrasekhar (1960) and also found by Sala (1986) in some particular cases, especially when being in the galactic plane. Rotational velocity (3.3) has a very general form which includes as particular cases those given by Chandrasekhar (1960) and Català (1972), once again, when being in the galactic plane.

We have also determined the stellar density that, unless its z dependence, it is

$$N = \frac{1}{\varpi^2} G_2(c) \tag{3.4}$$

by using equation 000, the second order pressures

$$P_{110} = \frac{1}{\varpi^2} F_1(t,z) \tag{3.5}$$

$$P_{002} = \frac{1}{\varpi^2} G_3(c) \tag{3.6}$$

through equations 010, taking into account (3.2), and 002, respectively, and the fourth order pressures

$$P_{202} = \frac{1}{\varpi^4} G_4(c)$$

$$P_{022} = \frac{1}{\varpi^4} G_5(c)$$

$$P_{004} = \frac{1}{\varpi^2} G_6(c)$$

through equations 202, 022 and 004, respectively. F_1 is an arbitrary function of t and z, while G_2, ..., G_6 are arbitrary functions of c. Equation 112 states the relation

$$\frac{P_{202}}{P_{022}} = \frac{2\dfrac{\theta_0}{\varpi}}{\dfrac{\partial\theta_0}{\partial\varpi} + \dfrac{\theta_0}{\varpi}} = \frac{B - A}{B} \quad , \tag{3.7}$$

where A and B are the Oort constants, and from where it is found

$$G_5 = - c \frac{G_1}{G_1} G_4 \quad ,$$

holding between the functions that determine the rotational velocity of the local standard of the rest and the pressures P_{202} and P_{022}.

If we write

$$M = \frac{\partial\Pi_0}{\partial t} + \Pi_0 \frac{\partial\Pi_0}{\partial\varpi} - \frac{\theta_0^2}{\varpi} + \frac{\partial U}{\partial\varpi} \tag{3.8}$$

equation 102 leads to

$$M = \frac{1}{\varpi^3} G_7(c)$$

with

$$G_7 = \frac{1}{G_3}\left[\left(- c\,\frac{G_1'}{G_1} + 3\right) G_4 + c\, G_4'\right]$$

Then, the potential U can be determined from (3.8), and it is found to be

$$U = \frac{1}{b_1^2}\left(G_8(c) - cG_8'(c)\right) - \frac{\dot{p}_1 + p_1^2}{2}\,\varpi^2 \tag{3.9}$$

where G_8 is an arbitrary function of c.

The stellar density (3.4) and the galactic potential (3.9) are very general expressions. Schmidt (1965) density in the galactic plane of his mass model of the Galaxy is a particular case of (3.4). For the potential (3.9), the second term is the potential at a point inside an homogeneous sphere (Ogorodnikov 1965), while the first one includes, among other, the newtonian central point mass potential and also the potential (Jeans 1923) necessary so that spiral arms of galaxies keep their spiral shape with time, as particular cases. In this latter case, the potential (3.9) will have the form of the potential found by Sala (1986) under Chandrasekhar hypotheses.

Finally, through equation 020, it is found

$$P_{020} = \frac{1}{b_1^4}\left(2c^5 G_1 \int F_1 dt + G_9(c)\right) \tag{3.10}$$

where G_9 is an arbitrary function of c and, from equation 100, a tedious development will lead to the form of P_{200}.

It is important to point out that this model has two features that are not fulfilled by models derived under Chandrasekhar hypothesis: a) the not vanishing third order momenta, and b) that if it holds any of the relations

$$\frac{\mu_{002}}{\mu_{200}\ \mu_{002}} = \frac{\mu_{022}}{\mu_{020}\ \mu_{002}} = \frac{\mu_{004}}{3\ \mu_{002}^2} \tag{3.11}$$

as it is true for Chandrasekhar models (Orús, 1977), then it would be

$$\int F_1(t,z)dt = \phi(c) = \text{constant}$$

and, therefore, $\mu_{110} = 0$, contradicting the hypothesis under which the model has been derived.

From table 1 it can be seen that neither the quotients (3.11) are equal, nor the moment $\mu_{110} = 0$, and, consequently, nor the vertex deviation, defined by

$$\psi = \frac{1}{2} \arctan \frac{2\ \mu_{110}}{\mu_{220} - \mu_{020}}$$

is.

The form of the third order pressures and the fourth order pressures, so far undetermined, can be found through the other hydrodynamical equations, although the general outlook of their forecast does not encourage to calculate them.

IV. ON THE MOTION OF THE LOCAL STANDARD OF THE REST

In the work by Figueras (1986) radial and rotational velocities of the local standard of the rest have been determined to be

$$\Pi_0 = 23.2 \pm 7.1 \text{ kms}^{-1}$$

$$\Theta_0 = 200.5 \pm 13.5$$

The not very large but, anyway, definite expansion at the solar position suggests to adopt models, like this one, where this velocity is not zero.

Rotational velocity is lower than (Kerr and Lynden-Bell 1985)

$$\Theta_0 = 222 \pm 20 \text{ kms}^{-1}$$

but even lower values are usually found in the literature. Special attention can be played to Rohlfs et al. (1986) study on the rotation curve of the Galaxy. In it, it is suggested that, although Oort constants are $A \neq -B$, the galactic rotation curve is basically flat, with two maxima at ϖ = 6 kpc and ϖ = 16 kpc (Θ_0 = 200 kms^{-1}) and one minimum between them at ϖ = 10.5 kpc (Θ_0 = 170 kms^{-1}). In that work, the solar position is taken to be $\varpi_\odot$ = 7.9 kpc and the velocity of its local standard of the rest Θ_0 = 184 kms^{-1}. So, the rotational velocity would, at the position of the Sun, decrease with the distance to the galactic center as most of the suggested forms of the rotation curve do, e. g. Chandrasekhar (1960). The above mentioned shape, with two maxima and a minimum in between, requires a little more complicate form; we point out that it can be given by (Català, 1972)

$$\Theta_0 = \Omega_0 \frac{(1 + \mu\bar{\omega}^2)\bar{\omega}}{1 + \rho\bar{\omega}^2 + \tau\bar{\omega}^4} \tag{4.1}$$

obtained from (3.3) as a particular case by putting

$$G_1(c) = \frac{\Omega_0 b_1^2 c^2 (1 + \mu b_1^2 c^2)}{1 + \rho b_1^2 c^2 + \tau b_1^4 c^4}$$

The function (4.1) should be fitted to the experimental data after an easy computation.

V. SUMMARY

A kinematic galactic model has been derived by using the hydrodynamical equations of the stellar dynamics under suitable hypothesis in order to explain the central momenta up to the fourth order of a wide sample of stars. Some of the not vanishing momenta have made impossible the adoption of the Chandrasekhar hypothesis in this study. The stellar density (3.4), the galactic potential (3.9), the galactocentric radial (3.1) and rotational (3.3) velocities of the local standard of the rest and three of the four second order pressures (3.5, 3.6 and 3.10) have, among other parameters, been obtained, and their variation has been given with the distance to the galactic center.

The form of the functions that describe these parameters is very general, and usually contains, as special cases, results previously obtained by other authors. A special attention has been given to the radial and rotational velocities of the local standard of the rest, and it is suggested for the latter the form (4.1) that agrees with some experimental data and which it can be fitted to.

More work must be done in this field, mainly when more samples of stars are available and they allow to adopt more accurate hypotheses about which is the distribution function of the stellar velocities and, consequently, which are the momenta relevant to its description.

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