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RADIATIVE TRANSFER IN TYPE I SUPERNOVAE ATMOSPHERES

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RESUMEN. Las explosiones de supernova de tipo I son atribuidas a la explosión termonuclear de una enana blanca de carbono oxígeno. Puesto que la única información directa del mecanismo que desencadena la explosión procede de la espectrofotometría de su radiación, es útil invertir un esfuerzo considerable en el estudio del transporte de la radiación en las atmósferas de tales objetos con el fin de introducir restricciones a los modelos teóricos de supernova. En esta comunicacion se analiza el papel jugado por la curvatura de las capas sobre el transporte radiativo.

ABSTRACT. Type I Supernovae are thought to be the result of the thermonuclear explosion of a carbon oxygen white dwarf in a close binary system. As the only direct information concerning the physics and the triggering mechanism of supernova explosions comes from the spectrophotometry of the emitted radiation, it is worthwhile to put considerable effort on the understanding of the radiation transfer in the supernovae envelopes in order to set constraints on the theoretical models of such explosions. In this paper we analyze the role played by the layers curvature on the radiative transfer.

Key words: RADIATIVE TRANSFER -- STARS-ATMOSPHERES

I. INTRODUCTION

In recents years the interest on Type I Supernovae has increased. There are three reasons for that : first, a better understanding of the pheonomenom; second, their influence on the chemical evolution of the Galaxy, basically through iron production, and third ,their possibility of using the outburst as standard candles to determine the distance scale as well as the deceleration parameter (Arnett 1982; Wagoner 1977,1979; Branch 1984).

The currently accepted models for Type I Supernovae are based on a carbon deflagration explosion (Sutherland and Wheeler 1984; Woosley 1986). These models assume a carbon oxygen white dwarf that accretes matter from a companion in a close binary system. The center is compressed and heated up as the white dwarf approaches Chandrasekhar 's mass and finally carbon ignites at the centre. Because of the degeneracy of matter at the centre, thermonuclear runaway occurs and carbon is incinerated to ^{56}Ni , releasing 7.10^{17} erg/g. When this happens, the density is so high ($\rho \gtrsim 2.10^{9}$ g/cm ^{3}) that the overpressures are only of the order of 20% and the burning front cannot propagate as a Chapman-Jouget detonation. Instead, its propagation is driven by conduction and convection (more precisely by Rayleigh-Taylor instability). All the models agree in that the central part of the star is processed to nuclear statistical equilibrium, dominated by the ^{56}Ni, and that the outer layers, that have had time enough to expand, are permitted to incomplete burning. However, the velocity of the flame and other properties as well are uncertain and this introduces serious difficulties as to the detailed nucleosynthesis of the iron peak elements isotopes (for instance, ^{54}Fe and ^{58}Ni are overproduced compared, to the standard values). One way to reduce the uncertainties on the physics of the nuclear flame is to perform detailed observations of the nucleosynthesis products and this involves the construction of detailed models of supernova envelopes.

One of the difficulties in the construction of a model of a supernova envelope is that the size of the region that contributes to the formation of the spectrum is of the order of the radius of the configuration (Fig 1). Furthermore, the mean free path of photons is of the order of the characteristic length of the configuration and, consequently, the curvature of the layers must be taken into account.

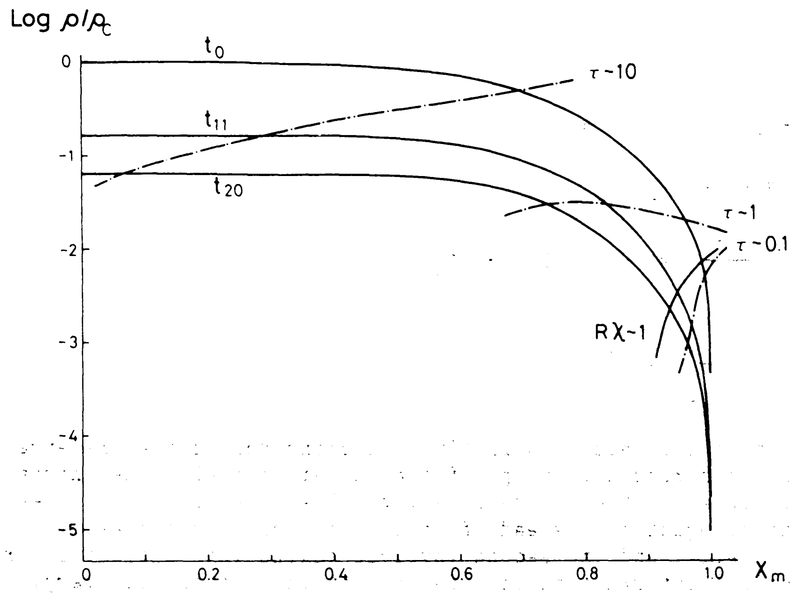

FIGURE 1

Density profile versus mass fraction. These profiles correspond to the optical maximum (t_0), and eleven (t_{11}) and twenty (t_{20}) days after it. The optical depth was calculated assuming a specific opacity of 0.2 cm^2 /g. ($\rho_c = 5.10^{-13}$g/cm^3)

II. THE RADIATIVE TRANSFER PROBLEM

The equation of radiative transfer in spherical coordinates can be written in the laboratory frame as:

$$\frac{1}{c}\frac{\partial I^\nu}{\partial t} + \mu\frac{\partial I^\nu}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I^\nu}{\partial \mu} = -\chi^\nu I^\nu + \sigma^\nu J^\nu + \eta^\nu \qquad (1)$$

where, as usual, χ^{ν} is the opacity coefficient

$$\chi^{\nu} = k^{\nu} + \sigma^{\nu}$$

and σ^{ν} and k^{ν} are the scattering and absorption coefficients, η^{ν} is the emission coefficient and μ has its usual meaning.

Because of the velocity dependence of the coefficients, we can develop them into a series up to order 0 (v/c), to obtain (Mihalas and Weibel-Mihalas 1984):

$$\chi^{\nu} = \chi_0^{\nu}(1 - \mu \, v/c \times e_{\chi}^{\nu})$$

$$\sigma^{\nu} = \sigma_0^{\nu}(1 - \mu \, v/c \cdot e_{\sigma}^{\nu}) \qquad (2)$$

$$\eta^{\nu} = \eta_0^{\nu}(1 + \mu \, v/c \;\; e_{\eta}^{\nu})$$

where we have defined

$$e_{\chi}^{\nu} = 1 + \frac{\partial \ln \chi_0^{\nu}}{\partial \ln \nu}$$

$$e_{\sigma}^{\nu} = 1 + \frac{\partial \ln \sigma_0^{\nu}}{\partial \ln \nu}$$

$$e_{\eta}^{\nu} = 2 - \frac{\partial \ln \eta_0^{\nu}}{\partial \ln \nu}$$

(the quantities subscripted 0 correspond to v = 0).

Substituing in (1) we obtain:

$$\frac{1}{c}\,\frac{\partial I^{\nu}}{\partial t} + \mu\,\frac{\partial I^{\nu}}{\partial r} + \frac{1-\mu^2}{r}\,\frac{\partial I^{\nu}}{\partial \mu} = -\chi_0^{\nu} I^{\nu} + \eta_0^{\nu} + \sigma_0^{\nu} J^{\nu}$$

$$+ \mu\, v/c\, (\chi_0^{\nu} \cdot e_{\chi}^{\nu} I^{\nu} + \eta_0^{\nu} \;\; e_{\eta}^{\nu} - \sigma_0^{\nu} \;\; e_{\sigma}^{\nu} J^{\nu}) \; . \qquad (3)$$

The numerical solution of this equation is time-consuming due to the presence of J in the right hand side and to the strong anisotropy of the radiation field (the intensity peaks arounds the solid angle subtended by the opaque interior). The first point involves the resolution of a partial differential equation instead of an ordinary one (we are assuming that the time dependent terms are zero), and the second involves taking into account a large number of directions in the numerical integrations in order to obtain suitable accuracy. Usually it is better to calculate the momenta of the radiation field. The success of this method depends on the possibility of replacing the infinite set of differential equations by the equations of the low order momenta plus a closure relationship that keeps all the properties of the high order momenta.

The equations for the first and second momenta are:

$$\frac{1}{c}\,\frac{\partial J^{\nu}}{\partial t} + \frac{\partial H^{\nu}}{\partial r} + \frac{2\,H^{\nu}}{r} = -\chi_0^{\nu} J^{\nu} + \eta_0^{\nu} + \sigma_0^{\nu} J^{\nu} + \frac{v}{c}\,\chi_0^{\nu}\, e_{\chi}^{\nu}\, H^{\nu}$$

$$\frac{1}{c}\frac{\partial H^{\nu}}{\partial t} + \frac{\partial K^{\nu}}{\partial r} + \frac{3K^{\nu} - J^{\nu}}{r} = -\chi_c^{\nu} H^{\nu} + \frac{v}{c}\left(\chi_c^{\nu} e_{\chi}^{\nu} K^{\nu} + \frac{1}{3}\eta_c^{\nu} e_{\eta}^{\nu} - \frac{1}{3}\sigma_c^{\nu} e_{\sigma}^{\nu} J^{\nu}\right) \tag{4}$$

where the sphericity effects appear through the terms $2H^{\nu}/r$ and $(3K^{\nu} - J^{\nu})/r$ in (4). They represent the geometrical dilution of the energy and momentum fluxes respectively. The remaining effects of the spherical geometry must be described by the closure relationship.

In order to solve the equations (4) we must to know a linear relationship (the closure relationship) between the momenta. It is usually assumed that the relationship $3K^{\nu} - J^{\nu} = 0$ (the Eddington relationship), valid in the planar geometry, is also valid in the spherical case (Castor 1972; Falk and Arnett 1977). However, in spherical geometry the intensity strongly peaks around the angle μ_c subtended by the opaque interior and the Eddington relationship relies on the hypothesis that the intensity is almost isotropic.

The Eddington relationship can be easily generalized in the spherical case if we know μ_c (Simonneau 1980). We can define a two-zone model ($\mu \leq \mu_c$; $\mu > \mu_c$), similar to the model defined by the outgoing and incoming intensities that characterizes the planar case. In this model, μ_c plays the same role as $\mu = 0$ in the planar geometry. A two region model for the intensity leads to a linear relationship between J^{ν}, H^{ν} and K^{ν} given by:

$$3K^{\nu} - J^{\nu} = 2\mu_c H^{\nu}. \tag{5}$$

A law that reproduces satisfactorily the behaviour of μ_c in the different optical regions is (Simonneau 1980):

$$\frac{d\mu_c}{dr} = \frac{1-\mu_c}{r\mu_c} e^{-\tau^{\nu}/\mu_c} \tag{6}$$

with the boundary condition $\mu_c = 0$ for $r = 0$. The first term in the right hand side takes into account the effect of the geometrical dilution while the exponential term, which represents the probability for a photon travelling in the direction μ_c to scape from the optical depth τ^{ν}, gives the correct opacity dependence on μ_c

The boundary conditions for the equations (4) can be expressed as:

$$r = 0 ;\ H^{\nu} = 0$$

$$r = R ;\ H^{\nu} = 1/2\, J^{\nu}(1 + \mu_c) \quad \text{or} \quad K^{\nu} = 1/3\, J^{\nu}[1 + \mu_c + (\mu_c)^2] \tag{7}$$

where R is the radius of the configuration. We can see from (6) that for

$\tau^{\nu} \longrightarrow \infty$, then $\mu_c \longrightarrow 0$,

and we recover from (5) the Eddington factor $K^{\nu}/J^{\nu} = 1/3$.

For :

$\tau^{\nu} \longrightarrow 0$, then $\mu_c \longrightarrow \sqrt{1 - c/r^2}$,

which represent the geometrical dilution. If

$r \longrightarrow \infty$, then $\mu_c \longrightarrow 1$

and from (7) we obtain the well-known streaming limit $K^{\nu}/J^{\nu} = 1$.

Equations (4) can be simplified considerably by the introduction of the following definitions:

$$\bar{X} = r^2 X$$

$$d\tau^\nu = - \chi_0^\nu \, dr$$

$$\varepsilon^\nu = k_0^\nu / \chi_0^\nu$$

We obtain (assuming LTE):

$$\frac{1}{c\chi_0} \frac{\partial \bar{J}^\nu}{\partial t} - \frac{\partial \bar{H}^\nu}{\partial \tau^\nu} = \varepsilon^\nu (\bar{B}^\nu - \bar{J}^\nu) + \frac{v}{c} e_\chi^\nu \bar{H}^\nu$$

(8)

$$\frac{1}{c\chi_0} \frac{\partial}{\partial t} (r \bar{H}^\nu) - \frac{\partial}{\partial \tau^\nu} (r \bar{H}^\nu) - \frac{1}{r\chi_0^\nu} \bar{J}^\nu = - r \bar{H}^\nu + \frac{v}{c} (r \bar{K}^\nu) + \frac{1}{3} \frac{v}{c} e_\eta^\nu (r \bar{B}^\nu) - \frac{1}{3} \frac{v}{c} (1 - \varepsilon^\nu) e_\sigma^\nu (r \bar{J}^\nu)$$

If we assume a grey opacity, we obtain:

$$e_\kappa^\nu = e_\sigma^\nu = 1$$

$$e_\eta^\nu = x \, (1 - \exp (x))^{-1} - 1 \quad ; \quad x = h\nu/kT$$

and, assuming $\varepsilon^\nu = 1$ (pure absorption), the equations for the momenta become:

$$\frac{1}{c\chi_0} \frac{\partial \bar{J}^\nu}{\partial t} - \frac{\partial \bar{H}^\nu}{\partial \tau^\nu} = \bar{B}^\nu - \bar{J}^\nu + \frac{v}{c} \bar{H}^\nu$$

(9)

$$\frac{1}{c\chi_0} \frac{\partial}{\partial t} (r \bar{H}^\nu) - \frac{\partial}{\partial \tau^\nu} (r \bar{K}^\nu) - \frac{r}{r\chi_0} \bar{J}^\nu = - r \bar{H}^\nu + \frac{v}{c} (r \bar{K}^\nu) + \frac{1}{3} \frac{v}{c} e_\eta^\nu \, r \bar{B}^\nu$$

III. THE IMPORTANCE OF THE CLOSURE RELATIONSHIP

In order to show the importance of the closure relationship we have integrated the equations of the momenta with the usual Eddington factor and its generalization to the extended case for a density and temperature profiles corresponding to the structure of a supernova at zero, eleven and twenty days after the maximum of the light curve (Table 1) as it was calculated by López et al. (1986). For comparison we have also integrated the equation of the intensity for the same conditions.

The anisotropy of the radiation field is very strong as a consequence of the curvature. Figure 2 displays the value of the intensity calculated from the equation (1) as a function of the direction for the optical depths 0.15 and 1. It is evident that the anisotropy diminishes as the optical depth increases but it is important in the region where the spectrum forms and, if we adopt the angular distribution of the radiation field implicit in the Eddington approximation we will introduce severe errors in the predicted properties. For instance, Figure 3 shows the accuracy of the values of J obtained from the two closure

TABLE 1

Density and temperature profiles of the external layers for which the transfer was calculated.

t(days)	r(cm)	v(cm/s)	(g/cm^3)	T(K)	τ
0	3.20E15	2.78E9	9.29E-17	6257	0.0
	3.15E15	2.74E9	2.12E-16	6355	1.34E-3
	3.01E15	2.60E9	5.40E-16	6455	2.22E-2
	2.34E15	2.03E9	2.01E-15	6736	1.84E-1
	1.82E15	1.58E9	1.04E-14	8166	8.31E-1
	1.53E15	1.33E9	1.51E-14	8963	1.57
	1.32E15	1.15E9	4.69E-14	12160	2.89
11	5.76E15	2.78E9	1.26E-17	4480	0.0
	5.53E15	2.67E9	3.52E-17	4500	1.93E-3
	4.76E15	2.30E9	1.15E-16	4505	1.25E-2
	3.28E15	1.59E9	1.13E-15	4640	1.50E-1
	2.38E15	1.15E9	7.20E-15	6040	6.31E-1
	2.04E15	9.92E8	2.30E-14	8470	1.60
	1.93E15	9.50E8	3.39E-14	9500	2.21
20	7.97E15	2.78E9	5.67E-18	3479	0.0
	7.51E15	2.62E9	3.06E-17	3506	1.84E-3
	6.60E15	2.30E9	8.64E-17	3528	1.24E-2
	4.54E15	1.59E9	6.75E-16	3779	1.32E-1
	3.02E15	1.06E9	5.65E-15	5302	6.95E-1
	2.56E15	9.00E8	1.74E-14	7047	1.6
	2.37E15	8.35E8	2.57E-14	7946	2.47

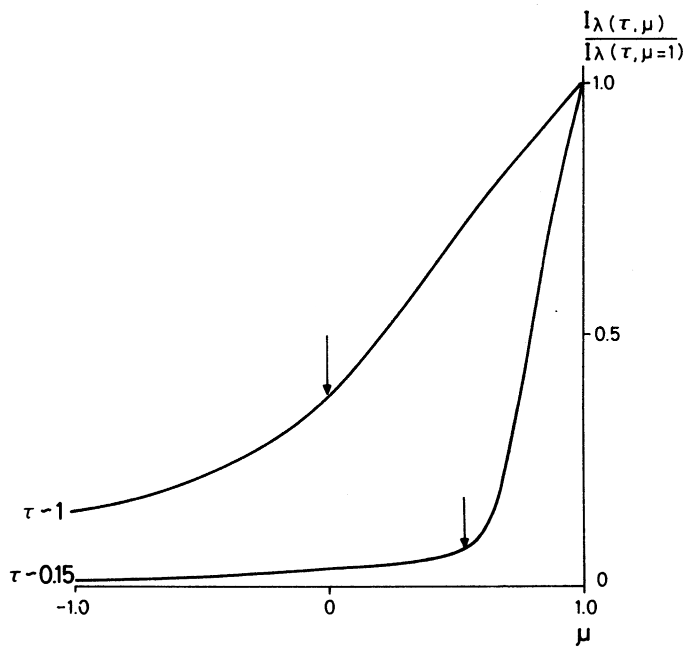

FIGURE 2

The intensity at 5000 A normalized to the value of the intensity in the direction μ= 1 versus the direction for the optical depths 0.15 and 1. The arrows show the value of μ_c

relationships , from the diffusion approximation and that obtained directly from (1). The relationship K/J = 1/3 and the diffusion approximation give inaccurate results in the external layers and can introduce severe errors if, as it ís suspected, the opacity in the atmosphere of a supernova is dominated by the electron scattering. The generalized Eddington factor proposed here gives an accuracy of the order of 10% that is sufficient for preliminary studies. The luminosity of the models are also modified. The models that take into account the curvature effects are systematically dimmer. As we can see from Table 2 the discrepancies are important and they increase with time.

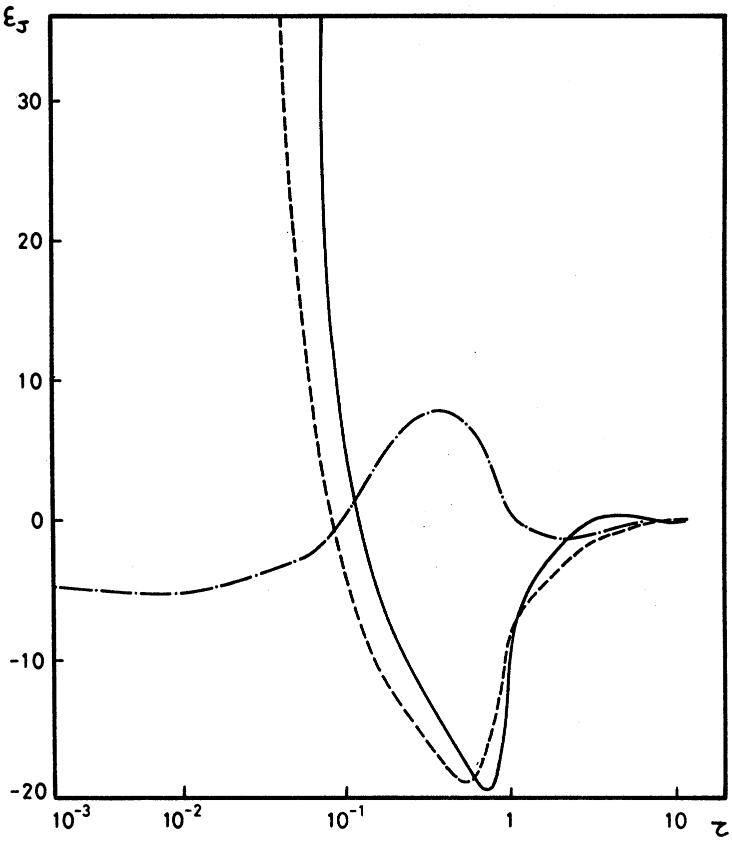

FIGURE 3

Accuracy of J versus the optical depth. The accuracy is defined as $(J_{cal} - J_{true})/J_{true}$ where J_{cal} is the value obtained from the diffusion approximation (continuous line), the generalized Eddington relationship (dash-dotted line) and the Eddington relationship (dashed line).

TABLE 2

Dependence of the luminosity on the selected closure relationship

t(days)	L_{real}	$L_{\mu_c \neq 0}$	$L_{\mu_c = 0}$
0	1.63E43	1.55E43 (5%)	1.87E43 (14%)
11	1.07E43	1.04E43 (3%)	1.50E43 (40%)
20	8.27E42	8.12E42 (2%)	1.19E43 (44%)

IV. CONCLUSIONS

From this work we can conclude that the effects introduced by the extension of supernova envelopes are not negligible. The existence of a suitable relationship that gives a priori the angular dependence of the radiation field simplifies the calculation of the radiative transfer. The solutions obtained in this way have an accuracy of the order of a 10% and they can be obtained in a way that is as simple as in the planar case.

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