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# UNCERTAINTIES IN THE DETERMINATION OF THE UPPER MASS LIMIT FOR ZERO-AGE MAIN SEQUENCE STARS

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RESUMEN. En una investigación reciente Klapp et al. 1987 obtuvieron una masa crítica de 440 $M_{\odot}$ para la inestabilidad de estrellas muy masivas de población I extrema en la secuencia principal. En este trabajo investigamos la dependencia de los resultados de Klapp et al. 1987 en la física del programa. Hemos encontrado que estrellas en el rango 100 - 500 $M_{\odot}$ son marginalmente estables (ó inestables) y que este rango de masas debe ser considerado como una región de transición de estabilidad a inestabilidad de estrellas muy masivas.

ABSTRACT. In a recent investigation Klapp et al. 1987 obtained a critical mass of 440 $M_{\odot}$ for the overstability of very massive extreme population I stars at the main sequence. In this work we investigate the dependence of Klapp et al. 1987 results upon the program input physics. We find that stars in the 100 - 500 $M_{\odot}$ range are marginally stable (or unstable) and that this mass range should be considered as a transition region from stability to overstability of very massive stars.

*Key words:* STARS-MASS -- STARS-STRUCTURE

## I. INTRODUCTION

In a recent investigation Klapp et al. 1987 (hereafter referred to as KLF) obtained a critical mass $M_{crit}$ of 440 $M_{\odot}$ for the overstability of massive extreme population I stars at the main sequence. This value is somewhat higher than previous estimates that have obtained critical masses in the mass range 60 to 260 $M_{\odot}$ (see Table 1). With the exception of Ziebarth 1970, all authors have always used Ledoux's 1941 quasiadiabatic approximation for determining the value of $M_{crit}$ and studying the stability of these objects along evolutionary tracks. In KLF we claimed that if we treat properly the outer nonadiabatic layers then $M_{crit}$ can be increased to a value higher than the usually accepted value of ~ 100 $M_{\odot}$.

In this paper we investigate the dependence of KLF results upon the program input physics and discuss other factors that might affect the value of $M_{crit}$. The effect of the various methods now in use (Ledoux's 1941 quasiadiabatic approximation, Castor's 1971 quasiadiabatic approximation and fully nonadiabatic calculations) on the value of $M_{crit}$ will be discussed in a future communication.

## II. LINEAR NONADIABATIC STABILITY ANALYSIS

For the linear stability analysis KLF coupled the Los Alamos linear nonadiabatic code (LNA) with the Göttingen stellar evolution code. For a detailed description of the numerical technique see KLF. A sequence of models from 130 to 5000 $M_{\odot}$ was constructed with homogeneous composition (X,Z) = (0.687, 0.043). For all models KLF used the new Stellingwerf 1975 fit to the King opacity tables. For the energy generation due to nuclear burning KLF

Table 1.

| Author(s) | Method | $\frac{M_{crit}}{M_\odot}$ | Remarks |
|---|---|---|---|
| Ledoux 1941 | QAD | $\gtrsim$ 100 | Kramers $\varkappa$ |
| Schwarzschild and Härm 1959 | QAD | ~ 60 | Thomson $\varkappa$ |
| Boury 1963 | QAD | ~ 260 | Pure Hydrogen |
| Stothers and Simon 1970 | QAD | ~ 100 | |
| Ziebarth 1970 | NAD | ~ 90 | |
| Papaloizou 1973 | QAD | ~ 85 | |
| Maeder 1985 | QAD | ~ 100 | |
| Klapp et al. 1987 | NAD | ~ 440 | |

Table 1. Results from the linear theory for the critical mass $M_{crit}$ obtained using the quasiadiabatic approximation (QAD) and nonadiabatic theory (NAD).

used the formula

$$\varepsilon_{CNO} = 4.44.10^{27} \rho T_6^{-2/3} X_H X_{CNO} \exp - \frac{152.28}{T_6^{1/3}} \text{ erg g}^{-1}\text{sec}^{-1}, \qquad (1)$$

where $T_6$ is the temperature in units of $10^6$ K, $X_H$ the hydrogen abundance and $X_{CNO}$ the CNO-abundance which is taken as 1/3 the heavy element abundance Z.

Figure 1 shows KLF results (crosses) for the imaginary part of the eigenfrequency $\omega$ of the fundamental mode as a function of the mass of the star. Unstable modes corresponds to negative values of Im($\omega$). From the figure we observe that we have reproduced Ledoux's 1941 classical result in the sense that an instability of the fundamental mode occurs for high enough masses. The major difference to the general picture is that we have encountered the instability at ~ 440 $M_\odot$ rather than already at ~ 100 $M_\odot$. In sections III and IV we will study the sensitivity of $M_{crit}$ upon the program input physics and computational details.

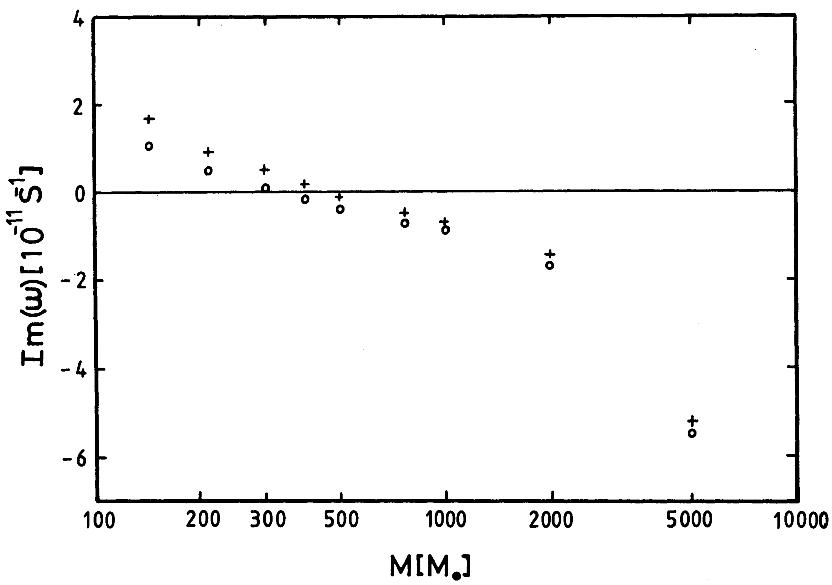


Figure 1. The imaginary part of the eigenfrequency $\omega$ for the fundamental mode vs. the total stellar mass as obtained by KLF (crosses) and by this investigation (circles) where the opacity derivatives $\varkappa_\rho$ and $\varkappa_T$ have been neglected.

## III. SENSITIVITY OF $M_{crit}$ UPON THE PROGRAM INPUT PHYSICS

In order to study the sensitivity of our results to the program input physics we repeated the stability analysis of KLF but neglecting the opacity derivatives $\kappa_\rho$ = d log $\kappa$ /d log $\rho$ and $\kappa_T$ = d log $\kappa$ / d log T. Figures 2 and 3 shows the opacity $\kappa$ and its derivatives $\kappa_\rho$ and $\kappa_T$ as a function of the fractional radius $R/R_*$ for the 208 $M_\odot$ model. The opacity changes from the center to the surface by up to a factor of two and the opacity derivatives attain fairly large values near the surface, specially the temperature derivative $\kappa_T$. The neglect of the opacity derivatives in a sense simulates what we would expect if we were using a pure electron scattering opacity. In figure 1 we also plot the results obtained by neglecting the opacity derivatives (circles). The damping of the outer nonadiabatic layers is reduced and the critical mass is now only about 300 $M_\odot$, i.e. about 25% lower than the KLF value. This indicates that the actual value of the critical mass is rather sensitive upon the opacity law used for the calculation. This is mainly due to the fact that in the 100 - 500 $M_\odot$ range we obtain an almost cancelation between the nuclear driving of the stellar core and the damping of the outer nonadiabatic layers and thus the stability (or instability) is marginal. For this mass range the quantity $\eta$ = Im($\omega$)/Re($\omega$), which is a measure of the nonadiabaticity of the star is only ~$10^{-8}$. For example, for 8 $M_\odot$ star with similar composition $\eta \sim 10^{-6}$ which is ~ 100 times greater than for the KLF models. In the end the actual value of $M_{crit}$ might not be too important and we could consider this mass range as a transition region from stability to overstability of very massive stars.

The effect on Im($\omega$) of (i) different methods of construction of the equilibrium model, (ii) different methods for the stability analysis and (iii) inconsistencies between the stellar structure code and the pulsation code (see section IV) is now under investigation.

## IV. OTHER FACTORS AFFECTING $M_{crit}$

We have seen that the growth rate of the fundamental mode is very sensitive upon the program input physics. This suggest that other factors like computational details and boundary conditions could also affect the results. We have done a great effort in equalizing the input physics and certain computational details in the stellar evolution code and the pulsation code. However, there are still minor differences that because of the great sensitivity of the results could be important. The most important differences are:

(i) Different radiative luminosity formula. The stellar evolution code uses the usual radiative luminosity formula while the pulsation code uses Stellingwerf 1975 luminosity formula.

(ii) Slightly different boundary conditions and

(iii) Different centering of thermodynamic and mechanical variables. This problem was somewhat solved by an interpolation described in KLF but still the accuracy to within the momentum and energy equations are satisfied is lower than in the original LNA code. The LNA code satisfies the equations to almost machine precision while we get one part in $10^4$ for the momentum equation and a slightly lower precision for the energy equation.

Whether these minor differences between the codes produces significant errors in the computations is not clear at the present time. Work is now in progress for constructing a stellar structure program compatible in all respects with Los Alamos linear nonadiabatic code.

Another approach to the problem is that followed by Ziebarth 1970 and Odell 1986 that used a modified version of the LNA code to study the linear stability of very massive stars. The original LNA code was constructed for studying the linear stability of envelope models. The center of the star is treated as a central ball of mass $M_{star} - M_{envelope}$. Ziebarth 1970 modified the code to include nuclear burning and found a critical mass near 100 $M_\odot$. In Ziebarth 1970 code (and also Odell 1986 that used essentially the same code) the user supplies the effective temperature, luminosity, mass, envelope mass and composition and the code constructs an equilibrium model by an inward only integration. For Cepheid type stars this procedure is justified but for massive stars where the integration is carried out all the way to the center, small errors in the central quantities and/or inconsistencies between central and surface quantities could affect the stability results. Another possible source of error in the LNA code is that the usual stellar evolution equations for the stellar interior are used (both in constructing the equilibrium model and solving the linear stability problem) from an optical depth $\tau$ near zero (typically $10^{-3}$) up to the center of the star. The grey atmosphere approxi-

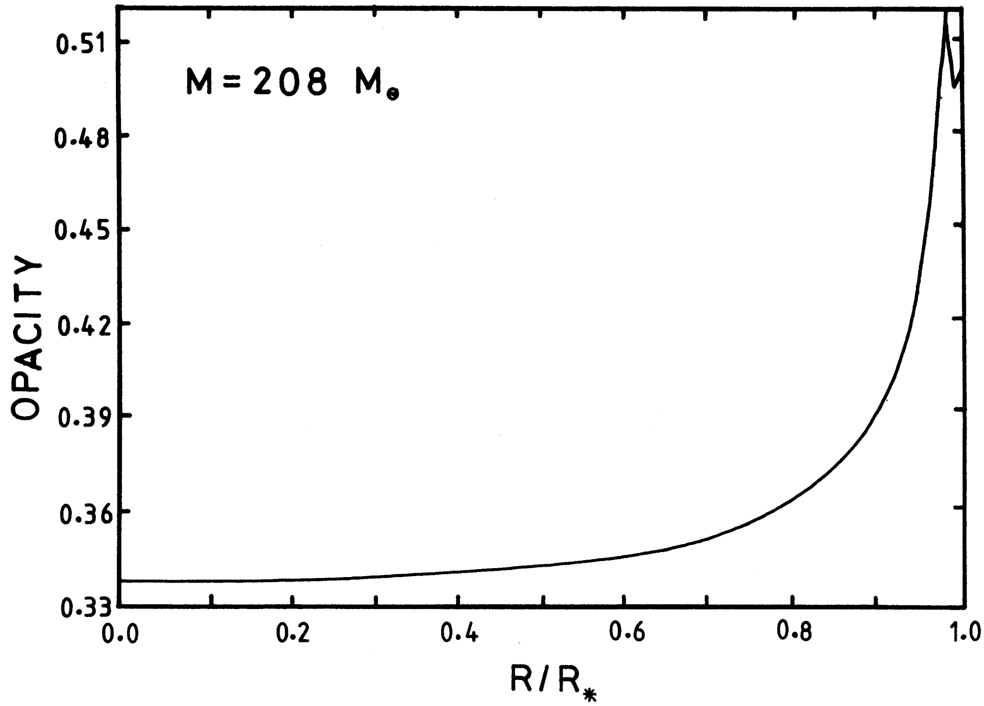


Figure 2. Opacity in cgs units as a function of the fractional radius $R/R_*$ for the 208 $M_\odot$ model of KLF. $R_*$ denotes the star's surface radius.

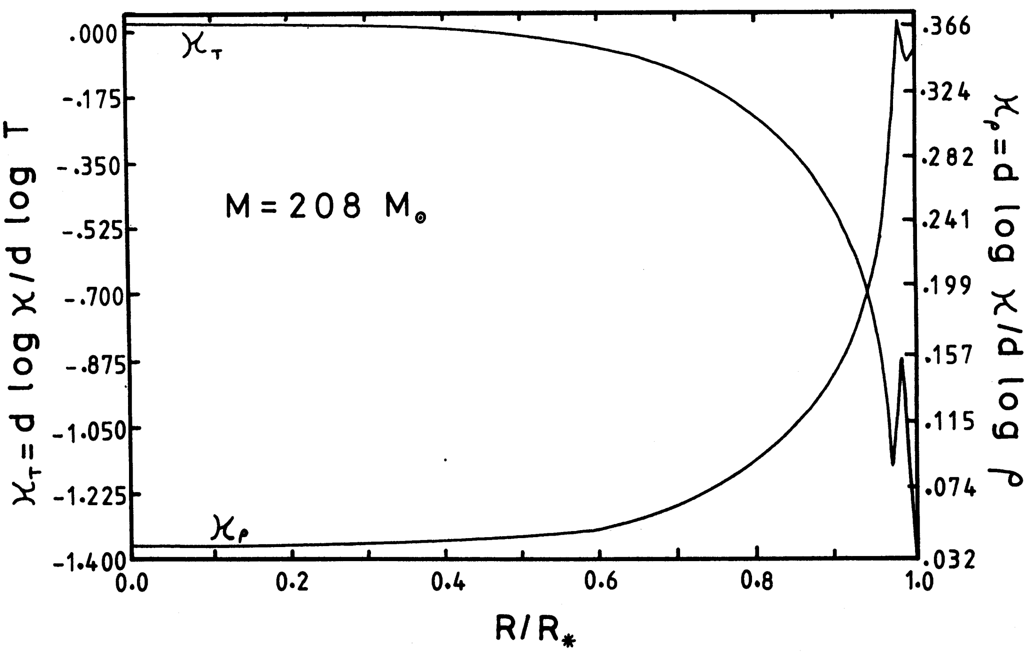


Figure 3. Opacity derivatives $\chi_\rho$ and $\chi_T$ as a function of the fractional radius $R/R_*$ for the 208 $M_\odot$ model of KLF.

mation is only used for the boundary conditions at the outermost mesh point. A more consistent treatment of the problem would be using two sets of equations, the first set for the optically thin regions from $\tau = 0$ up to the photosphere at $\tau = 2/3$ and a second set of equations from $\tau = 2/3$ up to the center of the star. The discontinuity at $\tau = 2/3$ could then be treated as a mathematical discontinuity with Rankine-Hugoniot matching relations. This technique is now under investigation by Shlosman et al. 1986 for treating the hydrogen ionization zone in Cepheid type stars as a front or discontinuity. Other alternatives for treating the optically thin regions are discussed by Castor 1971. For most regions of the upper HR-diagram such refinements might not be important but could be significant for calculations of the stability of very massive stars near the critical mass $M_{crit}$.

## V. CONCLUSIONS

The theoretical critical mass $M_{crit}$ for the overstability of hydrogen burning ZAMS stars has been found to depend significantly upon the program input physics. For the 100-500 $M_{\odot}$ range there is an almost cancelation between the nuclear driving of the core and the damping of the outer nonadiabatic layers that makes the actual value of $M_{crit}$ depend strongly upon the program input physics and certain computational details. Further refinements to our program are now under investigation that might reduce the uncertainties in the determination of $M_{crit}$. We suggest that the 100-500 $M_{\odot}$ range be considered as a transition region from stability to overstability of very massive stars.

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## DISCUSSION

FIRMANI: Creo que este trabajo es muy importante porque pone cierto orden en un material muy controvertido. Quiero señalar que el límite superior de la IMF puede ser debido a la acción del viento estelar durante la fase protoestelar. Los modelos están calculados con una metalicidad de Pop I, ¿qué pasaría con una metalicidad por ejemplo, diez veces menor?

KLAPP: No hemos probado otras metalicidades pero debido a que la estabilidad (o inestabilidad) es marginal, $M_{crit}$ debe depender fuertemente de la metalicidad utilizada. Con respecto a su comentario quisiera mencionar que estos estudios suponen que la estrella se encuentra en equilibrio mecánico y térmico. Pérdida de energía a través de vientos inducidos por radiación u ondas transmitidas a través de la superficie de la estrella (normalmente se supone reflexión total en la superficie) tendrían probablemente el efecto de aumentar el valor de $M_{crit}$. Esta posibilidad la estamos estudiando actualmente.

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