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FIRST ORDER PERTURBATION IN THE ENCELADUS-DIONE SYSTEM

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SUMÁRIO: Uma nova órbita para o par de satélites de Saturno, Encélado e Dione, é construída com base na solução intermediária cbtida por Salgado e Sessin (1985). Quando todas as perturbações de primeira ordem são levadas em consideração, termos seculares são agregados à Hamiltoniana. Consequentemente, a nova órbita gerada por essa Hamiltoniana possui uma melhor determinação de seus períodos.

ABSTRACT. A new orbit for the pair of Saturn's satellites Enceladus and Dione, is constructed based on the intermediate solution obtained by Salgado and Sessin (1985). Secular terms are aggregated to the Hamiltonian when all first order perturbations are taken into account. Consequently, the periods of the new orbit generated by this Hamiltonian are better determined.

Key words: PLANETS AND SATELLITES — DYNAMICS

I- Introduction

Salgado and Sessin (1985) obtained an intermediate solution for a pair of satellites, whose periods are commensurable in the ratio 2:1, including the efects of resonance and oblateness of the central body.

In this work, a new orbit for the pair Enceladus-Dione will be constructed based on this intermediate solution, considering all first order perturbation. The central body is oblate and the eccentricities and inclinations are taken as small quantities which are not null.

II - The Auxiliary Equations

Consider the three body system of two mass point, Enceladus and Dione, and a central body of finite dimension Saturn. Let m', m and M be their masses, m' and m of the same order of magnitude, both much smaller than M. Only gravitational forces acts on this system.

The equations of motion of Enceladus and Dione are referred to the Jacobi's coordinate system whose origin is A, the center of mass of Saturn and Enceladus, so the potencial U is written as function of these coordinates, making use of classical theory (Brouwer and Clemence, 1961).

In terms of Delaunay's variables the canonical equations of motion are:

$$\frac{d}{dt}(L_i, G_i, H_i) = \frac{\partial F}{\partial(\ell_i, g_i, h_i)}, \quad \frac{d}{dt}(L_i, G_i, H_i) = -\frac{\partial F}{\partial(\ell_i, g_i, h_i)}, \qquad (1)$$

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where the index i = 1 refers to the satellite Enceladus and i = 2 to Dione. The Hamiltonian F is given by:

$$F = F_o + R,$$

where F_o generates Keplerian movements of Enceladus and Dione with focus A and the disturbing function R is developed in terms of the Delaunay's elements. For these expansions we assume:

$$\frac{m'}{M} = \varepsilon \quad , \quad J_2 \left(\frac{a_s}{a_i}\right)^2 \sim \sigma(\sqrt{\varepsilon}) \quad , \quad J_4 \left(\frac{a_s}{a_i}\right)^4 \sim \sigma(\varepsilon) \tag{2}$$

as the order of magnitude of the small quantities and

$$e_i^2 \sim \sigma(\sqrt{\varepsilon}), \quad s_i^2 \sim \sigma(\sqrt{\varepsilon}) \tag{3}$$

where $s_i = \sin \frac{I_i}{2}$, and a_i, e_i, I_i, are the semi major axis, eccentricity and inclination respectively of the satellite i, and a_s is the equatorial radius of Saturn. The expression for R contains all the terms of these expansions up to the 3/2 power of the small quantities; in the eccentricities and inclinations up to the fourth order for the terms factored by $J_2\left(\frac{a_s}{a_i}\right)^2$ and up to the second order for the other terms.

The short period terms are eliminated considering the existence of the 2:1 commensurability, and a new set of canonical variables is introduced:

$$\begin{array}{llll} x_1 = L_1 + \frac{1}{2} L_2 & x_2 = -\frac{1}{2} L_2 & y_i = G_i - L_i & z_i = H_i - G_i \\ \lambda_1 = \ell_1 + \tilde{\omega}_1 & \Theta = \lambda_1 - 2\lambda_2 & \tilde{\omega}_i & \Omega_i \end{array} \tag{4}$$

where $\lambda_2 = \ell_2 + \tilde{\omega}_2$ and $\tilde{\omega}_i = \omega_i + \Omega_i$.

The Hamiltonian F* is given by:

$$F^* = F_o^*(x_1, x_2) + R^*(x_i, y_i, z_i, \Theta, \tilde{\omega}_i, \Omega_i) \tag{5}$$

so, λ_1 is cyclic and x_1 is a first integral.

The study of the resonant terms is performed in a neighbourhood of the exact resonance, i. e.,

$$x_2 = x_{20} + x \tag{6}$$

where x_{20} is a constant determided by:

$$\left.\left(\frac{\partial F_o}{\partial x_2}\right)\right|_{x=0} = \frac{-\mu_1^3 m_1^3}{(x_1 + x_{20})^3} - \frac{\mu_2^3 m_2^3}{4x_{20}^3} = -n_1^* + 2n_2^* = 0 \tag{7}$$

The so expanded F* is again treated by Hori's method in order to study resonant terms .

As noticed by Salgado and Sessin (1985) if all the first order terms of F* are taken into account in the construction of the Hori auxiliary system, integration does not seem to be possible. Likewise these authors it will be assumed that the oblateness of the central body does not change drastically the topology of the phase-space. So the term:

$$\Delta F = \frac{3}{2} A_{10} \quad (-2c_2 \frac{y_1}{x_{20}}) + \frac{3}{2} A_{20} (\frac{y_2}{x_{20}}) , \tag{8}$$

which is quadratic in the eccentricities and the A_{io} are factored by J_2, will be considered together with the terms of $F^*_{3/2}$ of the Hamiltonian as high order perturbations

The auxiliary system is defined by

$$F^*_1 = F_{o2}\left(\frac{x}{x_{20}}\right)^2 + F_{(1/2,\ 1)}\left(\frac{x}{x_{20}}\right) + \frac{m'}{M}\left\{- P_{30}\sqrt{-2c_2 \frac{y_1}{x_{20}}}\ \cos(\Theta + \tilde{\omega}_1) + \right.$$

$$\left. + P_{40}\sqrt{\frac{y_2}{x_{20}}}\ \cos(\Theta + \tilde{\omega}_2)\right\} - 3A_{10}\ c_2\left(\frac{z_1}{x_{20}}\right) + \frac{3}{2} A_{20}\left(\frac{z_2}{x_{20}}\right) , \tag{9}$$

where F_{o2}, $F_{(1/2\ ,1)}$, P_{30}, P_{40}, c_2 are constants.

Except by the equations of inclinations and nodes which are easily integrable, this auxiliary system is identical to the one given by Salgado and Sessin (1985), so its solution is obtained from their paper, in terms of ten constants of integration E, G, ρ, Θ_o, Θ_1, z_{10}, z_{20}, Ω_{10}, Ω_{20}, τ_o and Λ the parameter of the auxiliary system .

III - High Order Terms (Bevilacqua, 1985)

The expressions of ΔF and $F^*_{3/2}$ written in terms of elliptical integrals, introduced by the solution of the auxiliary system prevents a straight forward application of Hori's method. However, for the Enceladus-Dione pair the solution corresponds to a stable periodic orbit close to a stable equilibrium point in the (H,K) plane (Sessin and Ferraz-Mello, 1984). So, the solution will be expanded in power series of the modulus k of the elliptical integrals. Substituting this development in ΔF and $F^*_{3/2}$ we have:

$$\Delta F + F^*_{3/2} = R^*_s + R^*_p \tag{10}$$

where s and p stands for secular and periodic terms in Λ.

The equations of motion are:

$$\frac{d}{dt}(x^*,\ y^*_i,\ z^*_i) = \frac{\partial F^{**}}{\partial(\Theta^*,\ \omega^*_i,\ \Omega^*_i)} \quad , \quad \frac{d}{dt}(\Theta^*,\tilde{\omega}^*_i,\ \Omega^*_i) = - \frac{\partial F^{**}}{\partial(x^*,y^*_i,\ z^*_i)} ,$$

where the Hamiltonian F^{**} is

$$F^{**} = F^{**}_1 + R^{**} .$$

F^{**}_1 defines the Hori auxiliary system and generates an intermediate orbit which includes resonant effects. R^{**} contains all secular terms of ΔF and $F^*_{3/2}$. It's important to emphasize that internal resonances do not occur with the frequencies associated to the auxiliary system.

The use of the Lagrange variational equations gives the influence of R^{**} over the Hori auxiliary solution (Sessin, 1983.) Except for the energy E, the other integration constants and the parameter Λ are not necessarilly constants in t.

In this case the metric variables G, ρ, z_{10} , z_{20} are also constant functions of the time t and the angular variables Θ_o, Θ_1, Ω_{10}, Ω_{20} and the parameter Λ are linear functions of t:

$\Theta_o = \nu_o t + c_o$, $\Theta_1 = \nu_1 t + c_1$, $\Lambda = \nu t + \Lambda_o$,

$\Omega_{10} = \nu_2 t + c_2$ $\Omega_{20} = \nu_3 t + c_3$,

where

$$\sigma(\nu) = \text{zero}$$

$$\sigma(\nu_o) = \sigma(\nu_1) = \sigma(\varepsilon^{1/2})$$

$$\sigma(\nu_2) = \sigma(\nu_3) = \sigma(\varepsilon)$$

IV - Conclusion

Considering all first order perturbation, secular terms are aggregated to the Hamiltonian which generated the solution of the auxiliary system. The influence of this secular part over the Hori's intermediated solution affects only the angular type variables, giving a better determination of the periods of the new orbit generated by this Hamiltonian.

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