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A CORONA PLUS DISC MODEL FOR CYGNUS X-1

C. Meirelles Filho

Instituto Astronómico e Geofísico
Universidade de São Paulo
Brasil

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RESUMEN

Hemos construído un modelo para Cygnus X-1 el cual consiste de un disco exterior ópticamente grueso y geométricamente delgado, así como un disco interior ópticamente delgado y geométricamente grueso, y sobre éste una región coronal que dispersa fotones del disco exterior hacia el disco interior en donde sufren de efectos compton inverso. Este modelo reproduce el espectro observado de Cygnus X-1 bastante bien. Usando un criterio de estabilidad local y tomando en consideración la contribución de las pulsaciones de la presión turbulenta en equilibrio hidrostático, se muestra que la región interior es térmicamente inestable contra pequeñas perturbaciones en la dirección radial. Para el disco exterior se ha visto que la inestabilidad en el mismo ocurre mientras la presión de radiación excede un valor crítico. Este valor, sin embargo, depende del número de turbulencia de Mach.

ABSTRACT

We have constructed a model for Cygnus X-1 consisting of an outer optically thick and geometrically thin disc, an inner optically thin and geometrically thick disc and above it, a coronal region that scatters photons from the outer disc to the inner disc, where they are inverse comptonized. This model reproduces the observed spectrum of Cygnus X-1 quite well. Using a local stability criterion, and taking into account the contribution of the turbulent pressure pulsations in hydrostatic equilibrium, it is shown that the inner region is thermally unstable against small perturbations in the radial direction. For the outer disc it is seen that instability sets in the disc as long as the radiation pressure exceeds a critical value. This value, however, depends on the turbulent Mach number.

Key words: ACCRETION DISCS – RADIATIVE TRANSFER

I. INTRODUCTION

More than 15 years after its first identification with the massive single line spectroscopic binary HDE 226868, Cygnus X-1 keeps both its candidacy as a black hole, as well as the belief that the primary energy source for the X-rays is the accretion of gas from the supergiant primary companion. However, how the infall proceeds and how the gravitational energy gets converted into X-rays are still unsettled questions (Liang and and Nolan 1984).

At present, the inverse comptonization by hot thermal electrons ($T_e \sim 10^9$), in a region optically thin for true absorption, but with electron scattering depth $\tau_{es} \sim 1$–5, appears to be the most natural explanation for the observed spectrum (Shapiro, Lightman, and Eardley 1976; Liang and Nolan 1984). A persistent requirement of this idea would be the copious production of soft X-rays photons.

Thorne and Price (1975) set forward the idea that the inner disc around Cygnus X-1 could be interpreted as the result of an instability in the outer optically thick disc. According to that idea a secular instability present in the cool disc would drive the inner region to a hot, optically thin and geometrically thick disc, which should explain the observed spectrum near 100 keV. Shapiro *et al.* (1976) using this idea constructed a model for Cygnus X-1 assuming unsaturated comptonization of an external soft photon source, so that Y = 1 always, together with the assumption of non thermal equilibrium between electrons and protons. Instabilities that could provide further coupling are absent. The ionic temperature T_i, much greater than T_e, besides permitting the ionic pressure to dominate over the radiation pressure, makes the disc structure in the inner region highly dependent on T_i. This condition $T_i >> T_e$ is simply assumed for a given range of the parameters $(M, \dot{M}, \alpha)$ suited for Cygnus X-1.

However, it has been shown by Meirelles (1986) that the thin disc approximation is incompatible with the assumption of gas pressure dominated disc and $T_i >> T_e$. A two temperature solution does exist for $(M, \dot{M}, \alpha)$ quite different from the assumed parameters of Cygnus X-1. There is also a supersonic solution in the case of radiation pressure dominated disc. This solution, however, besides blowing the disc up is meaningless in the sense that viscous dissipation puts most of the energy into the ions only in the case of sub-sonic turbulence.

Another model compatible with observational data is the model of a hot corona ($T_e \sim 10^9 - 10^{10}$), heated by non thermal processes, surrounding a cooler standard accretion disc (Icke 1976; Liang and Price 1977; Bisnovatyi-Kogan and Blinnikov 1977). This corona would inverse comptonize the soft photons from the disc. A very serious drawback to this model consists on how to obtain a disc with an energy output smaller than the corona, a necessary requisite for invoking the existence of the latter (Lyubarskii 1984). However, there is not, to the moment, a detailed self consistent non thermal model solution for Cygnus X-1. These facts provide strong motivation for the model we propose.

In this paper we try to incorporate to the inverse compton model, in which some observational features are already taken into account, a self consistent dynamics. For that, we propose a model which is a blend of the thin disc and of the coronal thick disc.

The outline of this paper is as follows. In § II the model is presented, in § III the dynamics of the accretion disc is considered and, in § IV some comments about the turbulence in the disc are made. In § V we restrict the equations to the steady state, which solutions are given in full detail. In § VI the emission spectrum is discussed in terms of the inverse comptonization. In § VII, given some boundary conditions, all the parameters are expressed as a function of the disc luminosity. In § VIII the conditions for stability are analyzed in linear approximation. In § IX conclusions are drawn.

II. THE MODEL

This model Cygnus-1 considers an accretion disc surrounding a very compact object, the accreted matter being supplied by a massive primary companion. This accretion disc is quite different from the usual ones, since it may be thought as a composite of a hot inner region, hereafter denoted by 1, optically thin and geometrically thick; a cooler peripheral region, hereafter denoted by 2, optically thick and geometrically thin, both surrounding the region 3, hotter than region 2, optically thin and geometrically thick. The region 3 may also be thought of as a corona or a very dynamical disc. We are, obviously, assuming that part of the energy generated in region 2 is not thermalized there and may result in non thermal modes of energy transport that are pumped to and dumped in region 3.

Unfortunately, owing to existing difficulties associated with the formulation of a model, i.e., lack of adequate energy generation theory for the disc as well as the knowledge of the role of magnetic fields in the propagation of energy and the energy pumping from region 2, make the physical description of region 3 quite problematic. For the moment, we shall admit the existence of the region 3, with the role of emitting its own soft photons and reflecting the soft photons produced in region 2 to region 1, where they will be inverse comptonized. Though in a very qualitative way, a description and the equations governing the corona may be found in the paper of Liang and Price (1977). Figure 1 is a schematic representation of the model for Cygnus X-1.

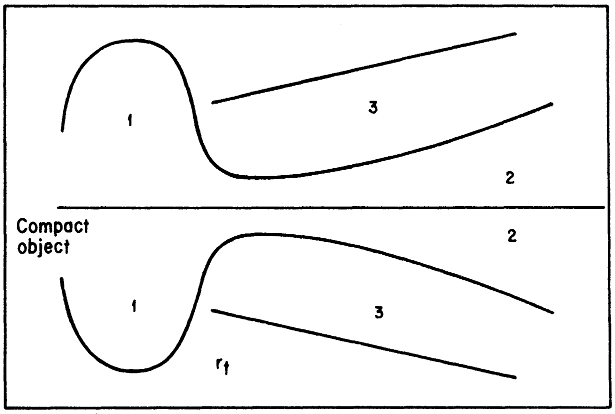

Fig. 1. Schematic representation of the model for Cygnus X-1.

Region 1 may be the result from a thermal instability that develops in region 2, at R_t, the transition point between regions 1 and 2, making the disc geometrically thick and optically thin (Thorne and Price 1975).

III. DYNAMICS OF THE ACCRETION DISC

In this section we shall present a description of the disc which differs from others which have been presented up to now in the literature. Examples of alternative descriptions can be found in the works of Pringle and Rees (1972), Shakura and Sunyaev (1973, 1976), Lynden-Bell and Pringle (1974), Stewart (1975), Shapiro *et al.* (1976) and Pringle (1981).

As usual, we describe the disc by using cylindrical polar coordinates (R, ϕ, z). z=0 is taken as the symmetry plane of the disc. The assumption of Keplerian velocity, thin disc, radiative energy transport only in z direction are also made. Hydrostatic equilibrium holds in regions 1 and 2 till terms of order ℓ^2 (ℓ is the semi-scale height of the disc), so $V_z = 0$ to the same order.

One of the differences of our work with regard to previous treatments is that we consider a dependence of V_r and ρ (and other variables) with z. This dependence we take, however, as a very mild one. More specifically, we assume that in a power expansion about z = 0, the dependence is such that one can keep only terms of second order in z. That is, for example:

$$\rho(r,z) \cong \rho_o + \frac{z^2}{2}\left[\frac{\partial^2}{\partial z^2}\rho\right]_o , \quad (1)$$

$$V_r(r,z) \cong V_{ro} + \frac{z^2}{2}\left[\frac{\partial^2}{\partial z^2}V_r\right]_o , \quad (2)$$

where o stands for variables calculated at z = 0. To expand the above variables we have used a symmetry condition. We now write the equation of continuity as (Landau and Lifshitz 1971)

$$\frac{\partial}{\partial t}\int_V \rho dV + \int_V \mathrm{div}(\rho\bar{v})\, dV = 0 \ , \qquad (3)$$

defining $U = 2\int_0^{\ell} \rho dz$, (3) becomes

$$\frac{\partial}{\partial t} U + \frac{2}{r} \int_0^{\ell} \frac{\partial}{\partial r}(r\rho V_r)\, dz = 0 \ . \qquad (4)$$

Equation (4) may be cast in a much more useful form in terms of the radial flux of matter at R, inside regions 1 and 2, i.e.,

$$\frac{\partial}{\partial t} U - \frac{1}{2\pi r}\frac{2\partial}{\partial r}\dot{M} + 2\,(\rho V_t)_{z=\ell}\frac{\partial \ell}{\partial r} = 0 \ ; \qquad (5)$$

the absence of the last term of equation (5) in the analysis of Shakura and Sunyaev (1976) is due to their boundary condition $\rho = 0$ at $z = \ell$.

For $0 < z < \ell$, the assumption is made that the decrease in density is compensated by an increase in radial velocity, in such a way as to keep ρV_r constant. This is equivalent to the boundary condition

$$V_{ro}\left[\frac{\partial^2}{\partial z^2}\rho\right]_o = -\rho_o\left[\frac{\partial^2}{\partial z^2} V_r\right]_o \qquad (6)$$

which implies

$$\dot{M}(r) \cong -4\pi\rho_o V_{ro}\, r\ell + O(\ell^5) \ , \qquad (7)$$

and

$$(\rho V_r)_{z=\ell} \cong \rho_o V_{ro} + O(\ell^4) \ , \qquad (8)$$

using (7) and (8) we write the continuity equation as

$$\frac{\partial U}{\partial t} - \frac{\ell}{2\pi r}\frac{\partial}{\partial r}\frac{\dot{M}}{\ell} = 0 \ . \qquad (9)$$

We now define $\omega_{r\phi}$, the viscous stress tensor, in such a way that $2\pi\omega_{r\phi}r^2$ is the torque due to viscous forces acting between one layer of matter and another. The time dependence of the ϕ velocity component can be justifiably neglected, so that the equation for momentum transfer can be written

$$V_r\frac{\partial}{\partial r}V_K + \frac{V_r V_K}{r} = -\frac{1}{\rho r}\frac{\partial}{\partial r}(W_{r\psi}\, r^2) \ , \qquad (10)$$

where V_k is the Keplerian velocity. Integrating (10) over z it becomes

$$\dot{M} = \frac{4\pi\ell}{\Omega r}\frac{\partial}{\partial r}\frac{W_{r\psi}\, r^2}{\ell} \ , \qquad (11)$$

where $W_{r\phi} = 2\int\omega_{r\phi}dz$ and Ω is the Keplerian angular velocity. From the hydrostatic equilibrium equation

$$\frac{\partial}{\partial z}P = -\rho\frac{GM}{r^3}\, z \ , \qquad (12)$$

setting $P(z = \ell) = 0$ we have, keeping terms till second order in z,

$$P = \frac{\rho_o GM}{2r^3}(\ell^2 - z^2) \ . \qquad (13)$$

Taking the average and setting $\langle P \rangle = P$, yields to second order,

$$P = \frac{U\,\Omega^2\ell}{6} \qquad (14)$$

as equation of state one usually has for the pressure the contribution from the gas, the radiation and the turbulence. If we define β as the relative radiation pressure and α 1/2 as the turbulent Mach number, we have

$$P = \frac{UkT}{m_H\ell} + \beta P + \alpha P \qquad (15)$$

T is the temperature, k is the Boltzmann constant and m_H is the hydrogen mass. We have assumed a completely ionized gas.

Finally we may write the energy equation as follows:

$$\frac{d}{dt}E = \frac{P}{\rho^2}\frac{d}{dt}\rho + q^+ - q^- \ , \qquad (16)$$

in which E is the internal energy per unit mass, q^+ and q^- are respectively the amount of heating and radiative cooling. E should include contributions from the gas as well as from radiation. Clearly, $E = \epsilon/\rho$, where ϵ is the total internal energy density. From (15) we have

$$\epsilon = \frac{3}{2} P(1+\beta-\alpha) \qquad (17)$$

integrating (16) over z and using the equation of continuity, we have up to second order terms in ℓ,

$$P\frac{\partial \ell}{\partial t} + \ell \frac{\partial}{\partial t}\epsilon = -\frac{1}{r}\frac{\partial}{\partial r} \times$$

$$\times \{rV_r(p+\epsilon)\} + Q^+ - Q^- , \qquad (18)$$

where Q^+, Q^- are the integrals over z of q^+ and q^-. Inserting (11), (13) and (17) into (18), yields

$$\frac{U\Omega^2 \ell}{6}\frac{\partial \ell}{\partial r} + \ell\frac{\partial}{\partial t}\frac{U\Omega^2\ell}{4}[1+\beta-\alpha] = \frac{1}{r} \times$$

$$\times \left[\frac{\partial}{\partial r}\frac{\ell^3\Omega}{12}(5+3\beta-3\alpha)\frac{\partial}{\partial r}\frac{W_{r\psi} r^2}{\ell}\right] +$$

$$+ Q^+ - Q^- . \qquad (19)$$

IV. ON THE DISC TURBULENCE

It is generally assumed that turbulence in the disc may be due to convection and to large shear stresses caused by differential rotation.

Despite these two quite distinct processes, the main results we obtain are not dependent upon the way turbulence is generated in the disc. So, up to second order in ℓ, the energy dissipation between two adjacent layers in the disc, assumed to rotate with Keplerian velocity, is

$$W_{r\psi} = 3\eta\ell\Omega , \qquad (20)$$

where η is the average turbulent dynamical viscosity. Neglecting magnetic fields and assuming isotropic and homogeneous turbulence

$$\eta = 3\sqrt{3}\,\alpha^{1/2}\rho\ell^2\Omega , \qquad (21)$$

to obtain this expression, we have used for the sound velocity

$$v_s = \Omega\ell/\sqrt{3} ; \qquad (22)$$

substituting this in the former expression, and using the hydrostatic equilibrium equation, we obtain the well known result of Shakura and Sunyaev (1976)

$$W_{r\psi} = \sqrt{3}\,\alpha^{1/2} p\ell , \qquad (23)$$

to allow a comparison of our results with those obtained by Shakura and Sunyaev (1973), we make

$$W_{r\psi} = 2\alpha^{1/2} p\ell$$

V. THE STATIONARY DISC

To solve the equations for the stationary accretion disc we shall assume that region 2 is gas pressure dominated and region 1 is radiative pressure dominated. Besides this, we shall assume black body emission for region 2 and optically thin emission for the inner region.

The separation point for these regions is located at r_t, in units of inner radius. In both regions we have as solution for the continuity equation

$$\dot{M}(r) = C\ell , \qquad (24)$$

with the constant to be determined from given boundary conditions.

a) *The Outer Region 2*

From the hydrostatic equilibrium equation, neglecting all the contributions for the pressure, that of the gas,

$$\ell = \left[\frac{6k}{m_H}\right]^{1/2}\frac{T^{1/2}}{\Omega} , \qquad (25)$$

the energy equation in the stationary regime is written (Lynden-Bell and Pringle 1974)

$$-\frac{4}{4}\frac{\partial}{\partial r} r^2\Omega\dot{M} = -\Omega\dot{M}r + \frac{8\pi}{3}\frac{\partial}{\partial r}\frac{r^2 D}{\Omega} , \qquad (26)$$

in obtaining (26) allowance for the variation of M with r was made. D is the dissipation rate, which in the stationary regime equals the radiative cooling rate, thus

$$D = \sigma_B T^4 , \qquad (27)$$

where σ_B is the Stefan-Boltzmann constant. The energy equation may be rewritten as

$$-r^{3/3}\frac{\partial}{\partial r}\left[r^{13/16}\,y\right]=\nu\frac{\partial}{\partial r}y^{8}\ , \qquad (28)$$

with r in units of the inner radius and

$$\nu=2.32\times10^{-42}\frac{M_{34}}{C_o^8}\ ,$$

and

$$Y=\Omega_r^{1/16}\ \dot{M}. \qquad (29)$$

Usually $8\nu y^7 >> r^{25/16}$, so the approximate solution to equation (28) is

$$Y^7\cong-\frac{r^{25/16}}{2\nu}+C_1\ , \qquad (30)$$

C_1, is another constant to be determined from the boundary condition. The solution for the flux of matter $\dot{M}$ is

$$\dot{M}=3.65\times10^{-4}\,M_{34}\left[-\frac{1}{2\nu}+\frac{C_1}{r^{25/16}}\right]^{1/7} r^{9/7}\ ; \qquad (31)$$

if $\dot{M}_\infty$ and $\dot{M}_t$ are respectively the flux of matter at the outer radius and at the transition point, we shall have

$$\nu\cong\frac{8.62\times10^{-25}\,M_{34}^7\,r_D^{25/16}}{\dfrac{\dot{M}_o^7}{r_t^{119/112}}-\dfrac{\dot{M}_\infty^7}{r_D^{119/112}}} \qquad (32)$$

$$C_1=1.16\times10^{24}\ M_{34}^{-7}\left[\frac{\dot{M}_t}{r_t^{9/7}}\right]^7 . \ r_t^{25/16}. \qquad (33)$$

From the definition of ν, equation (32) becomes

$$2.69\times10^{-18}\,\ell_t^8\ M_{34}^{-6}\left\{\left[\frac{\dot{M}}{r_t^{17/112}}\right]^7-\left[\frac{\dot{M}_\infty}{r_D^{17/112}}\right]^7\right\}=r_D^{25/16}\,\dot{M}_t^8\ , \qquad (34)$$

r_D is the disc length in units of the inner radius. Finally we may write the remaining physical variables for the (outer) region 2,

$$\ell=5.74\times10^{-2}\,M_{34}^{3/4}\ \times$$
$$\times\left\{\left[\frac{\dot{M}_t}{r_t^{17/112}}\right]^7-\left[\frac{\dot{M}_\infty}{r_D^{17/112}}\right]^7\right\}^{-1/8} r_D^{25/128}\,r^{9/7}\ \times$$
$$\times\left[-\frac{1}{2\nu}+\frac{C_1}{r^{25/16}}\right]^{1/7}\ ; \qquad (35)$$

$$T\mp5.06\times10^{-5}\,M_{34}^{-1/2}\left\{\left[\frac{\dot{M}_t}{r_t^{17/112}}\right]^7-\left[\frac{\dot{M}_\infty}{r_D^{17/112}}\right]^7\right\}^{-1/4} r_D^{25/64}\ \times$$
$$\times\ r^{-3/7}\left[-\frac{1}{2\nu}+\frac{C_1}{r^{25/16}}\right]^{2/7} . \qquad (36)$$

If the ratio of the angular momentum of the flow to the Kleperian angular momentum is δ at $r = 1$,

$$\rho=2.8\times10^{-5}\,M_{34}^{-5/4}\ \alpha^{-1/2}\ \times$$
$$\times\left\{\left[\frac{\dot{M}_t}{r_t^{17/112}}\right]^7-\left[\frac{\dot{M}_\infty}{r_D^{17/112}}\right]^7\right\}^{3/8} r_D^{-75/128}\ \times$$
$$\times\, r^{-15/14}\left[1-\delta r^{-1/2}\right]\left[-\frac{1}{2\nu}+\frac{C_1}{r^{25/16}}\right]^{-2/7} , \qquad (37)$$

$$V_r=-4\times10^{-6}\,M_{34}^{-1/2}\,r_D^{25/64}\,\alpha^{1/2}\ \times$$
$$\times\left\{\left[\frac{\dot{M}_t}{r_t^{17/112}}\right]^7-\left[\frac{\dot{M}_\infty}{r_D^{17/112}}\right]^7\right\}^{-1/4}\ \times$$
$$\times\ r^{1/14}\left[-\frac{1}{2\nu}+\frac{C_1}{r^{25/16}}\right]^{2/7} . \qquad (38)$$

It is worth to remark that the dissipation rate in this region is

$$D=3.74\times10^{-22}\,M_{34}^{-2}\,r_D^{25/16}\ \times$$
$$\times\left\{\left[\frac{\dot{M}_t}{r_t^{17/112}}\right]^7-\left[\frac{\dot{M}_\infty}{r_D^{17/112}}\right]^7\right\}^{-1}\ \times$$
$$\times\left[-\frac{1}{2\nu}+\frac{C_1}{r^{7/4}}\right]^{8/7} r^{-12/7} . \qquad (39)$$

b) *Region 1*

We shall write the solution for the continuity equation in a slightly modified way, i.e.,

$$\dot{M} = \frac{\dot{M}_o}{\ell_o} \ell , \qquad (40)$$

where the subscript 0 means values evaluated at r = 1. Assuming that the main contribution to opacity comes from electron scattering (and that heat production equals radiative cooling), from the hydrostatic equilibrium equation we have

$$D = \frac{C}{3\pi} \frac{m_H}{\sigma_T} \Omega^2 \ell , \qquad (41)$$

C is the velocity of light, σ_T the Thomson scattering cross section. If we make the substitution

$$\dot{M} = \frac{2}{\Omega r} \frac{\partial}{\partial r} Y , \qquad (42)$$

(Y now differs from the one previously defined by equation (29) for region 2) the energy equation reads

$$\Omega (2 + 2\pi A) \frac{\partial}{\partial r} Y = \frac{3}{2r} Y , \qquad (43)$$

where

$$A = \frac{2 c m_H}{3\pi\sigma_T} \frac{\ell_o}{\dot{M}_o} .$$

Solving equation (43) we obtain

$$\dot{M} = \dot{M}_o r^{\frac{1 - 2\pi A}{4(1 + \pi A)}} , \qquad (44)$$

$$\ell = \ell_o r^{\frac{1 - 2\pi A}{4(1 + \pi A)}} , \qquad (45)$$

$$\rho = \frac{8.7 \times 10^{-5}}{\alpha^{1/2}} \frac{\dot{M}_o}{\ell_o^3} M_{34}^2 r^{\frac{2 + 5\pi A}{2(1 + \pi A)}} \times$$

$$\times (1 - \delta r^{-1/2}) , \qquad (46)$$

$$V_r = - \frac{2 \times 10^{-4} \alpha^{1/2}}{M_{34}^2} \ell_o^2 r^{-\frac{4 + 7\pi A}{2(1 + \pi A)}} \times$$

$$\times (1 - \delta r^{-1/2})^{-1} . \qquad (47)$$

We now assume that region 3 supplies a copious amount of soft photons to region 1, so that unsaturated comptonization of these photons is the main cooling mechanism in region 1. Setting the comptonization parameter y, given by

$$y = \frac{4 k T}{m_e c^2} \frac{\sigma_T \rho \ell}{m_H}$$

equal to 1, we obtain for the temperature

$$T = 4.46 \times 10^{13} \times$$

$$\times \frac{\alpha^{1/2} \ell_o^2}{\dot{M}_o M_{34}^2} (1 - \delta r^{-1/2})^{-1} r^{-\frac{5 + 8\pi A}{4 + 4\pi A}} . \qquad (48)$$

in this expression we shall assume that T is constant and that the dependence with r is accounted by α. The expression for the dissipation rate in this region is, from (24) and (19),

$$D = 6.25 \times 10^{16} M_{34}^{-2} \ell_o r^{-\frac{11 + 14\pi A}{4 + 4\pi A}} . \qquad (49)$$

VI. THE EMISSION SPECTRUM

The X-ray spectrum for constant temperature regions, in which inverse comptonization cooling dominates, has been treated in many papers (Illarionov and Sunyaev 1972; Felten and Rees 1972; Zel'dovich and Shakura 1969; Shapiro *et al.* 1976; Shakura and Titarchuk 1980). The generic case when T is not constant is quite complex, and to the present, has not been treated in full detail (Guilbert, Fabian, and Ross 1980). As we assumed constant temperature in the comptonizing region, we shall write the Kompaneets equation as given by Sunyaev and Titarchuk (1980),

$$\frac{1}{x^2} \frac{\partial}{\partial x} x^4 \left[\frac{\partial N}{\partial x} + N \right] +$$

$$+ \gamma N - \gamma \frac{f(x)}{x^3} = 0 ; \qquad (50)$$

$\gamma = \pi^2 m_e c^2 / 3(\tau + 2/3)^2 kT$,
$\chi = h\nu / kT$,
τ = optical depth for electron scattering,
F(x) = Radiation spectrum of the soft photon source,
N = Photon occupation number.

The Green function for equation (50) is Sunyaev and Titarchuk (1980).

$$G(x, x_o) = \frac{\alpha(\alpha + 3)}{\Gamma(2\alpha + 4)} x_o^\alpha x^3 (\exp - x)$$

$$\int_0^\infty t^{\alpha - 1} (1 + \frac{t}{x})^{\alpha + 3} (\exp - t) dt. \qquad (51)$$

(In this section α is an index and not the turbulent parameter), and the radiative flux will be

$$F = \int \frac{f(x_o)}{x_o} G(x, x_o)\, dx \quad , \qquad (52)$$

Γ = gamma function

$$\alpha = \left[\frac{9}{4} + \gamma\right]^{1/2} - 3/2 \ ,$$

for $\alpha = 1$, $y = 1$, the enhancement factor will be

$$B = \frac{4}{\Gamma(6)} \int_0^\infty f(x_o)\, dx_o \times \qquad (53)$$

$$\times \frac{\int_{xo}^\infty x^3 (\exp - x)\, dx \int_0^\infty (1 + \frac{t}{x})^4 (\exp - t)\, dt}{\int_0^\infty f(x_o)\, dx_o} .$$

It is worth to remark that the dependence of F on r is due to α and to f(x). As we have taken both α and f(x) constant, F is constant, which would imply some mechanism for redistribution the energy generated at r assuring constant temperature.

Another point deserving a comment is that the emission spectrum is independent of M and $\dot{M}$, depending only on T. So, if we take the same temperature at the inner radius taken by Shapiro *et al.* (1976), we shall obtain the same spectrum, which is normalized to have the observed luminosity above 10 KeV, L_{10}. (See Fig. 2).

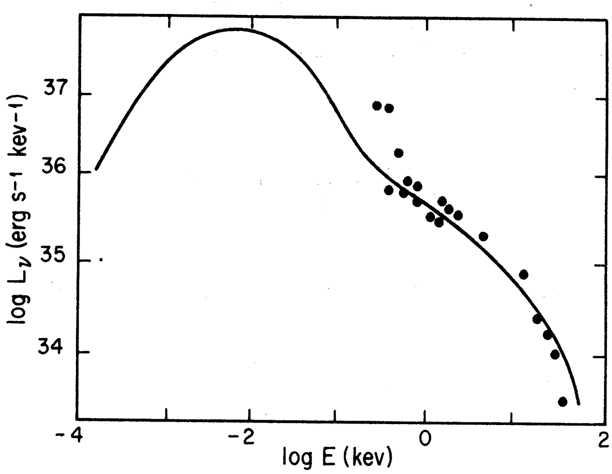

Fig. 2. Solid line gives the predicted spectrum of Cygnus X-1. Points in the graph are observational data, taken from Shapiro *et al.* (1976).

VII. BOUNDARY CONDITIONS

We now express all the integration constants of section 5 in terms of the luminosity of Cygnus X-1, which we shall assume to come mainly from region 1 and essentially in the X-ray portion of the electromagnetic spectrum. With these assumptions we obtain

$$L_x = 1.57 \times 10^{31} \left[\frac{4 + 4\pi A}{3 + 6\pi A}\right] \ell_o \times$$

$$\times \left[1 - r_t^{-\frac{3+6\pi A}{4+4\pi A}}\right] , \qquad (54)$$

according to Shapiro *et al.* (1976), $r_t = 15/4$. Using the definition

$$J = \frac{3 \dot{M}_o + 3.14 \times 10^{11} \ell_o}{4 \dot{M}_o + 2.1 \times 10^{11} \ell_o} . \qquad (55)$$

We have

$$\ell_o = 10^{-11} \dot{M}_o \left[\frac{3 - 4J}{2.1J - 1.14}\right] , \qquad (56)$$

in terms of J equation (54) reads

$$\frac{L_x}{1.57 \times 10^{20} \dot{M}_o} = \frac{1}{J} \left[\frac{3 - 4J}{2.1J - 3.14}\right] . \qquad (57)$$

Clearly

$$0.75 < J < 1.5 \ , \qquad (58)$$

equation (57) may be cast in a much more suitable form, i.e.,

$$2.1\, aJ^2 - J(3.14\, a - 4) - 3 = 0 \ , \qquad (59)$$

with

$$a = L_x / 1.57 \times 10^{20} \quad \dot{M}_0 \ .$$

To solve (59) we look for an a which yields J in the range (58) and holding the thin disc approximation. In Figure 3 we plot ℓ_0 as a function of a

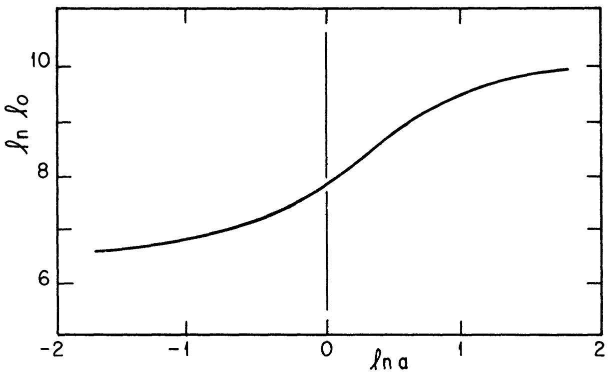

Fig. 3. ℓ_o as a function of a. ℓ_o in units of 10^6.

We shall take a = 0.5. This gives

$$J = 0.89 ,$$

$$M = 1.27 \times 10^{18} L_{38} \text{ g s}^{-1},$$

$$\ell_o = 5.6 \times 10^6 \text{ cm} ,$$

$$a = 7.34 \times 10^{-2} .$$

If the temperature at r = 1 is T_9 and $\delta = .9$ (Apparao and Chitre 1974; Pacheco and Steiner 1975) we shall have for the turbulence parameter α

$$\alpha = 9.74 \times 10^{-5} L_{38}^{-4} M_{34}^{-4} \dot{M}_O T_9 , \qquad (60)$$

with L_{38} the X-ray luminosity in units of 10^{38} erg s^{-1}, T_9, the temperature in units of 10^9, M_O in units of 10^{17}. The luminosity of region 2 is

$$L_2 \cong 1.74 \times 10^{20} r_D^{25/26} r_t^{-0.33} M_{34}^{-8} \dot{M}_t , \qquad (61)$$

taking $r_D \sim 100$

$$L_2 = 1.5 \times 10^{23} M_{34}^{-8} \dot{M}_t . \qquad (62)$$

Assuming that a fraction of this energy, say g, is reflected through region 3 to region 1, we obtain

$$B = \frac{6.67 \times 10^{-3} M_{34}^8 L_{38}}{g\dot{M}_t} , \qquad (63)$$

B is the energy enhancement factor for comptonization of the soft photon source. $\dot{M}_t$ should be expressed in units of 10^{17}. If the instability is strong enough to thicken the disc in such a way as to intercept at $r = r_t$, all the matter that enters the disc at r_D, i.e.,

$$\dot{M}_1 (r_t) = \dot{M}_\infty .$$

Combining this with the approximate relation

$$\frac{M_t}{\dot{M}_\infty} = \left[\frac{r_t}{r_D}\right]^{119/112} \qquad (64)$$

gives

$$\dot{M}_t = 5.41 \times 10^{16} L_{38} , \qquad (65)$$

$$C_1 = 8.44 \times 10^{136} M_{34}^{-7} L_{38}^7 , \qquad (66)$$

$$\ell_t = 5.69 \times 10^4 M_{34}^{3/4} L_{38}^{1/8} , \qquad (67)$$

and

$$Bg = 1.23 \times 10^{-2} M_{34}^8 ; \qquad (68)$$

a rough estimate gives $g \sim 0.25$ and using $\delta = 0.9$ (Pacheco 1975) yields B = 27.

Finally we obtain the condition that must be satisfied in order to make inverse comptonization the main cooling mechanism in that region. The luminosity coming from Bremsstrahlung is

$$L_b = \frac{1.37 \times 10^{58} \dot{M}_o^{3/2} M_{34}^3}{\alpha^{3/4} \ell_o^4} , \qquad (69)$$

M in units of 10^{17}. Equating this to the luminosity yields

$$\alpha_c = \frac{7.85 \times 10^{26} \dot{M}_o^2 M_{34}^4}{\ell_o^{16/3} \quad L_{38}^{4/3}} . \qquad (70)$$

However, if $T_9 = 1$

$$\alpha = \frac{5 \times 10^{+22} \dot{M}_o^2 M_{34}^4}{\ell_o^4} . \qquad (71)$$

For $\alpha > \alpha_c$ comptonization dominates Bremsstrahlung. Comparing (71) and (67) we see that this will happen as long as

$$\ell_o > \frac{1.41 \times 10^3}{L_{38}} \qquad (72)$$

or

$$L_{38} > 1.72 \times 10^{-2} \quad . \qquad (73)$$

VIII. LINEARIZED EQUATIONS AND STABILITY

In order to see how the perturbations in the disc evolve with time, we shall linearize the disc equations, keeping only terms of the first order of the perturbed variables. We shall adopt the same procedure of Piran (1978), writing phenomenological expressions for the kinematic viscosity ν and for the energy removal Q^- . However, our procedure differs from that of Piran because we take into account the effect of turbulence pulsations in the equations of state. Specifically, we write

$$\nu = \frac{\alpha_o^{1/2}\,\Omega\,\ell_o^2}{3\sqrt{3}} \left[\frac{U}{U_o}\right]^n \left[\frac{\ell}{\ell_o}\right]^m , \qquad (74)$$

and

$$Q^- = g \left[\frac{\ell}{\ell_o}\right]^K \left[\frac{U}{U_o}\right]^S . \qquad (75)$$

The coefficients m, n, k, s describe the local behaviour of Q^- and ν around their steady state values. The above expressions are quite general and applicable for any viscosity law valid for the unperturbed flow. G depends on what is assumed for the unperturbed flow. The subscript o stands for the unperturbed flow. From the expression for the viscosity we obtain

$$\alpha = \alpha_o \left[\frac{U}{U_o}\right]^{2n} \left[\frac{\ell}{\ell_o}\right]^{2(m-2)} . \qquad (76)$$

We now define the perturbed variables in terms of the unperturbed ones

$$U = U_o\,(1+u)$$
$$\ell = \ell_o\,(1+h) \quad . \qquad (77)$$

Using equation (76) and the hydrostatic equilibrium equation (14), we obtain from the linearized equation of state

$$\frac{\delta T}{T_o} = \frac{1}{1-\beta_o-\alpha_o} \times$$

$$\times \left[2h(3-\beta_o-\alpha_o-m) - 2nu - \beta_1\right] \qquad (78)$$

where δT is the variation of the temperature and β_1 is the variation of the ratio of radiation pressure to total pressure. From equation (75) we have

$$P_r = \frac{\pi}{c}\, g\,\tau \left[\frac{\ell}{\ell_o}\right]^K \left[\frac{U}{U_o}\right]^S \qquad (79)$$

τ is the optical depth.

In the inner region electron scattering dominates free-free absorption, so we may write

$$P_r = \frac{\pi}{c}\, g \frac{\sigma_T}{M_A}\, U_o \left[\frac{\ell}{\ell_o}\right]^K \left[\frac{U}{U_o}\right]^S ; \qquad (80)$$

this gives

$$\beta = \beta_o \left[\frac{U}{U_o}\right]^S \left[\frac{\ell}{\ell_o}\right]^{K-1} \qquad (81)$$

and

$$\beta_1 = \beta_o\,\{s\,u + (k+1)\,h\} \quad . \qquad (82)$$

Using this expression, equation (78) becomes

$$\frac{\delta T}{T_o} = \{6-(3-k)\beta_o - 2\alpha_o \quad 2m\}h +$$

$$+ \{s\,\beta_o - 2n\}u \quad . \qquad (83)$$

Keeping only terms of 2nd order in ℓ and assuming $\lambda \ll \ell_o$, where λ is the wavelength of the perturbation, we obtain

$$\frac{\partial u}{\partial t} = \frac{2}{3} \alpha_o^{1/2} \Omega \ell_o^2 \frac{\partial^2}{\partial r^2} \{(n+1)u + (m-1)h\} \quad (84)$$

for the continuity equation, and

$$\frac{1}{6}\frac{\partial}{\partial t}h + \frac{1}{4}\left\{(1+\beta_o - \alpha_o)\frac{\partial}{\partial t}(u+h) + \beta_o\left[s\frac{\partial u}{\partial t} + (k-1)\frac{\partial h}{\partial t}\right] - \alpha_o\left[2n\frac{\partial u}{\partial t} + 2(m-2)\frac{\partial h}{\partial t}\right]\right\} =$$

$$= \left[\frac{5+3\beta_o - 3\alpha_o}{18}\right] \alpha_o^{1/2} \Omega \ell_o^2 \times \frac{\partial^2}{\partial r^2}\left[(n+1)u + (m-1)h\right] + \frac{\alpha_o^{1/2}\Omega}{4}\left[(n+1)u + mh - su - kh\right] \quad (85)$$

for the energy equation. To obtain equation (84) we have used

$$Q^+ = \frac{3}{4} W_{r\psi} \Omega \quad (86)$$

and the equality of heat production and energy removal by radiation in the unperturbed flow. For u and h we look for solutions of the type e^{wt} setting

$$y = (n+1)u + (m-1)h \,. \quad (87)$$

Equation of continuity (83) becomes

$$u = \frac{2}{3}\frac{\alpha_o^{1/2}\Omega}{\omega} \ell_o^2 \frac{\partial^2}{\partial r^2} y \,; \quad (88)$$

eliminating h and u from (84) we finally obtain for the energy equation

$$\{\omega^2 [5 - \alpha_o(6m-9) + 3\beta_o k] + 3\alpha_o^{1/2}\Omega\omega(k-m)\}y - \frac{2}{3}\alpha_o^{1/2}\Omega\ell_o^2 \frac{\partial^2}{\partial r^2} y\{\omega(5n+2m+3+3\beta_o \times (k(n+1) - s(m-1)) - \alpha_o[(n+1)(6m-9) - 6(m-1)n) - 3\alpha_o^{1/2} \times \Omega[(1-k)(1+n) + s(m-1)]\} = 0 \,. \quad (89)$$

As we have assumed that the perturbed variables change much more rapidly than the non perturbed ones, the neglect of variations on β_o, and Ω is justified, yielding the following dispersion relation

$$\omega^2 \{5 - \alpha_o(6m-9) + 3\beta_o k\} + \alpha_o^{1/2}\Omega\omega\{3(k-m) + \frac{2}{3}(\ell_o/\lambda)^2[5n - 2m + 3 + 3\beta_o(k(n+1) - s(m-1)) + \alpha_o^{1/2}(9 - 3n - 6m)]\} - 2(\alpha_o^{1/2}\Omega\ell_o/\lambda)^2 \times [(1-k)(1+n) + s(m-1)] = 0 \,. \quad (90)$$

The solution to equation (86) is

$$\omega + \frac{\alpha_o^{1/2}\Omega}{2z_o}\left\{-3(k-m) - \frac{2}{3}(\ell_o/\lambda)^2 Z_1 \pm \pm\left[3(k-m) + \frac{2}{3}(\ell_o/\lambda)^2 Z_1{}^2 - 8z_o(\ell_o/\lambda)^2 Z_2\right]^{1/2}\right\}, \quad (91)$$

where

$$Z_0 = 5 - \alpha_o(6m-9) + 3\beta_o k \;,$$

$$Z_1 = 5n + 2m + 3 + 3\beta_o(k(1+n) - s(m-1)) + 3\alpha_o^{1/2}(3 - n - 2m)\,,$$

$$Z_2 = (k-1)(1+n) + s(1-m)\,.$$

In the small wavelength limit equation (87) reduces to

$$\omega_+ = -\,3\, Z_2\, \alpha_o^{1/2}\, \Omega \ , \qquad (92)$$

$$\omega_- = -\,\frac{2}{3}\,(\ell_o/\lambda)^2\,\frac{Z_1}{z_o}\,\alpha_o^{1/2}\,\Omega \ . \qquad (93)$$

Thus the stability conditions will be

$$k - m > 0 \qquad (94)$$

$$5 - \alpha_o\,(6m - 9) + 3\beta_o\, k > 0 \qquad (95)$$

$$5n + 2m + 3\beta_o\,\{k(n+1) - s(m-1)\} +$$

$$+\ 3\alpha_o^{1/2}\,(3 - n - 2m) > 0, \qquad (96)$$

$$(k-1)(1+n) - s\,(1-m) > 0 \ . \qquad (97)$$

Now we apply our results to study the stability against small perturbations in the scale height and column density for a gas-pressure dominated disc in the outer region and radiation-dominated disc in the inner region. We shall assume the standard viscosity law, i.e., m = 2, n = 0, in the outer region. The (outer) region 2 emits like a black body; thus,

$$\frac{\beta_1}{\beta_o} = \left[\frac{1 - \beta_o - \alpha_o}{1 + 3\,\beta_o - \alpha_o}\right](7h - u) \qquad (98)$$

and

$$4\,\frac{T}{T_o} = h\left[\frac{8 - 4\beta_o - 8\alpha_o}{1 + 3\beta_o - \alpha_o}\right] + \frac{4\beta_o}{1 + 3\beta_o - \alpha_o}\,u \ . \qquad (99)$$

For k and s, we obtain

$$k = \frac{8 - 4\,\beta_o - 8\,\alpha_o}{1 + 3\beta_o - \alpha_o} \ , \qquad (100)$$

$$s = \frac{4\beta_o}{1 - 3\,\beta_o - \alpha_o} \ . \qquad (101)$$

Condition (90) is the most severe constraint for stability and reads

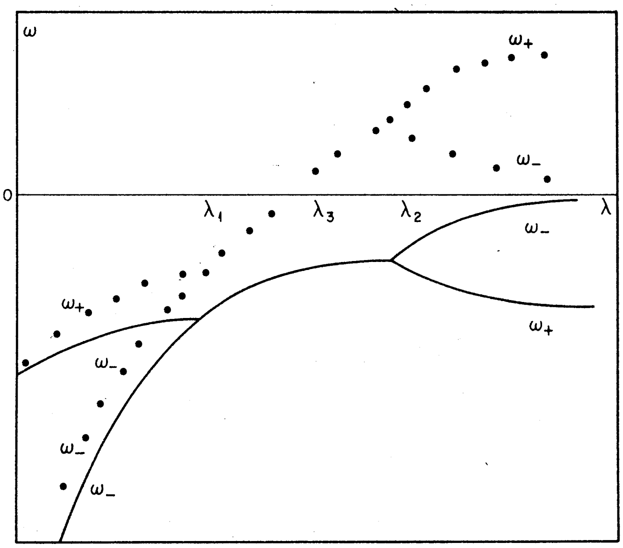

Fig. 4. ω^+ and ω^- refer to the Lightman-Eardley and to the thermal mode respectively. The lower branch applies when condition (90) is satisfied. For the solid line condition (102) is fulfilled.

$$3 - 5\,\beta_0 - 3\,\alpha_0 > 0 \ . \qquad (102)$$

In Figure 4 we plot $\omega = \omega\,(\alpha_o, \beta_o, \lambda)$ where ω^+ and ω^- refer to the Lightman-Eardley and to the thermal mode respectively. The lower branch applies when condition (90) is satisfied.

For $\lambda_1 < \lambda < \lambda_2$ perturbations grow oscillating in the upper branch and oscillate with damping in the lower branch. At $\lambda = \lambda_3$ (upper branch) perturbations just oscillate. For region 1, optically thin, assuming as Shapiro *et al.* (1976), y = 1 always, we have

$$\frac{\delta T}{T_o} = -\,u \ , \qquad (103)$$

$$\frac{\beta_1}{\beta_2} = u \ . \qquad (104)$$

Equation (94) is a direct consequence of the assumption of constant soft photon flux through region 1. Equation (94) yields k = 1 and s = 1. The expressions for m and n given by equation (78) are

$$m = \frac{1 + \alpha_0 - \beta_0}{\alpha_0} \ ,$$

$$n = \frac{1 - \alpha_0 - 2\beta_0}{2\alpha_0\,(1 - \alpha_0 - \beta_0} \ . \qquad (105)$$

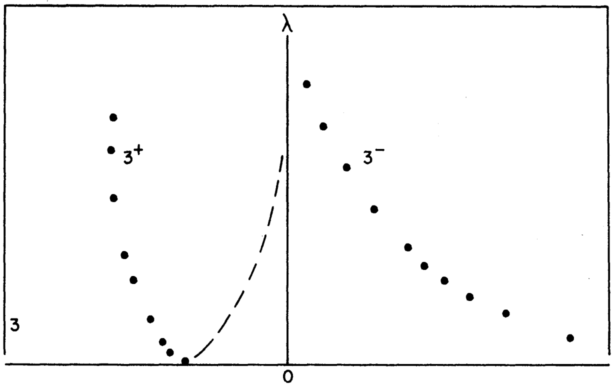

Fig. 5. $\omega = \omega(\lambda, \alpha_0, \beta_0)$ for this part of the disc. The dashed line corresponds to the case $3\alpha_0 + 9\beta_0 > 2$.

In Figure 5 we plot $\omega = \omega(\lambda, \alpha_0, \beta_0)$ for this part of the disc, where the dashed line corresponds to the case $3\alpha_0 + 9\beta_0 > 2$.

IX. CONCLUSION

The model we have constructed reproduces quite well the observational data of Cygnus X-1. This result is to a certain extent dependent on the existence of region 3. Though the physics of this region has been avoided, the condition for its existence is given as a boundary condition on the surface of the thick disc, i.e., the matter in the region 3 supplied by region 2. This region presents stronger condition for candidacy as a source of soft X-ray than the thick disc, because the latter would require a very high luminosity enhancement factor for comptonization. However, we do think that a more detailed treatment of this region is required in order to test the credibility of model.

From the stability analysis in region 2, we have concluded that the role of the turbulent pulsations should be taken into account, because they may drive the disc to instability for a lesser value of β_0, the relative radiative pressure. Concerning the viscosity, we should remark that in region 1, it is highly dependent on the soft photon flux. We also should comment that for the special viscosity law (m,n) = (0,1) ((0,0) in the analysis of Piran 1978), which leads to a stable disc, we obtain the same stable solution for the thermal and dynamical modes.

We have shown that the inner region is dynamically unstable. However, this may be due to the use of a local criterion for stability, which is independent of boundary conditions that take into account the discontinuities on passing from region 2 to region 1, as well as the existence of the corona.

Finally, we should comment that despite having a different equation to determine the point where the instability develops, the transition point, the adoption of the same value of Shapiro *et al.* 1976 does not lead to a noticeable inconsistency.

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REFERENCES

Apparao, K.M.V. and Chitre, S.M. 1976, *Space Sci. Rev.*, **19**, 281.
Bisnovatyi-Kogan, G.S. and Blinnikov, S.I. 1977, *Astr. and Ap.*, **59**, 111.
Felten, J.E. and Rees, M.J. 1972, *Astr. and Ap.*, **17**, 226.
Guilbert, P.W., Fabian, A.C., and Ross, R.R. 1980, *M.N.R.A.S.*, **199**, 763.
Icke, V. 1976, *Nature*, **266**, 699.
Illarionov, A.F. and Sunyaev, R.A. 1972, *Soviet Astr. AJ*, **16**, 45.
Landau, L. and Lifshitz, E. 1971, *Mécanique des Fluides*, (Moscow: Editions MIR).
Liang, E.P. and Nolan, P.L. 1984, *Space Sci. Rev.*, **38**, 353.
Liang, E.P. and Price, R.H. 1977, *Ap. J.*, **218**, 247.
Lightman, A.P. and Eardley, D.M. 1974, *Ap. J. (Letters)*, **187**, L1.
Lynden-Bell, D.M. and Pringle, J.E. 1974, *M.N.R.A.S.*, **168**, 603.
Lyubarskii, Yu.,E. 1984, *Soviet Astr.*, **28**, 1.
Meirelles, C. 1986, submitted to *M.N.R.A.S.*
Pacheco, J.A.F. 1975, *Ap. and Space Sci.*, **32**, 205.
Piran, T. 1978, *Ap. J.*, **221**, 652.
Pringle, J.E. and Rees, M.J. 1972, *Astr. and Ap.*, **21**, 1.
Pringle, J.E. 1981, *Ann. Rev. Astr. and Ap.*, **19**, 137.
Shakura, N.I. and Sunyaev, R.A. 1973, *Astr. and Ap.*, **24**, 337.
Shakura, N.I. and Sunyaev, R.A. 1976, *M.N.R.A.S.*, **175**, 613.
Shapiro, S.L., Lightman, A.P., and Eardley, D.M. 1976, *Ap. J.*, **204**, 187.
Stewart, J.M. 1975, *Astr. and Ap.*, **42**, 95.
Thorne, K.S. and Price, R.H. 1975, *Ap. J. (Letters)*, **195**, L101.
Zel'dovich, Ya., B. and Shakura, N.I. 1969, *Soviet Astr. AJ.*, **13**, 175.

Cesar Meirelles Filho: Instituto Astronómico e Geofísico da Universidade de São Paulo, Departamento de Astronomía, Caixa Postal 30627, 01051 São Paulo, SP, Brasil.