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ON THE ROLE OF SUPERSONIC TURBULENCE AND HOMOGENEITY IN THE STRUCTURE OF ACCRETION DISCS IN ACTIVE GALACTIC NUCLEI

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RESUMEN

En este trabajo se señala que las pulsaciones de presión turbulenta deben de ser tomadas en consideración en la ecuación de estado para la presión en regímenes supersónicos. A la luz del modelo de disco de acreción estándar se muestra que la estructura y emisión de la región ópticamente delgada de discos de acreción estacionarios en núcleos activos de galaxias, pueden ser alterados drásticamente. Se muestra que la homogeneidad en z es una suposición muy fuerte que nos puede llevar a una solución en la que la materia fluya hacia afuera del objeto compacto. Las soluciones obtenidas son en todas partes térmicamente inestables. También se muestra que las temperaturas en el rango (1-5) $\times 10^3$ °K dan masas bastante diferentes de los valores usualmente supuestos para los cuasares, a menos de que sea usada una relación dada entre M y $\dot{M}$.

ABSTRACT

In this paper it is pointed out that turbulent pressure pulsations should be taken into account in the equation of state for the pressure, in supersonic regimes. In the light of the celebrated α standard accretion disc model, it is shown that the structure and emission of the optically thin region of steady accretion discs in Active Galactic Nuclei (AGN) may be drastically altered. It is shown that z-homogeneity is a very strong assumption that may lead to a solution with matter flowing back from the compact object. The solutions obtained are everywhere thermally unstable. It is also shown that temperatures in the range (1-5) $\times 10^3$ °K yield masses quite different from the values usually assumed for Quasars, unless a given relation between M and $\dot{M}$ is used.

Key words: ACCRETION DISCS – GALAXIES-ACTIVE – QUASARS – TURBULENCE

I. INTRODUCTION

The purpose of this paper is to point out some aspects concerning the role of turbulent pressure and z-nonhomogeneity; their account in the hydrostatic equilibrium and angular momentum conservation equations may lead to criticism concerning the applicability of the α standard accretion disc model to Active Galactic Nuclei (AGN). In a recent paper by Collin-Souffrin (1987) the structure and emission of the optically thin region of steady accretion discs in Active Galactic Nuclei (AGN) is obtained in the light of the celebrated α accretion disc model (Shakura and Sunyaev 1973, 1976). Some of the main results obtained by the author, such as how far this region extends from the center, temperature, density and scale height of the disc as well as the region of gas pressure dominance and the effects of self gravity are highly dependent on the value of the turbulent Mach number. From the assumption of z-homogeneity one should expect the constancy of the scale-height of the disc, a result that, if not confirmed, may lead to matter flowing from the compact object in the case of ℓ growing with the radial distance (Urpkin 1983).

Though not questioning the idea of accretion discs to explain some features (or all of them) of the emission spectrum, the applicability of the model to these objects, as it stands in the paper, should be seen with reticence.

II. THE INCLUSION OF THE TURBULENT PRESSURE PULSATIONS IN THE HYDROSTATIC EQUILIBRIUM EQUATION

It is usual to separate the velocity field V in the disk into an average velocity field, here denoted U and a stochastic one represented by u. Clearly,

$$V_i = U_i(\bar{x}) + u_i(\bar{x},t) \tag{1}$$

For the continuity equation we have, in the stationary case,

$$\frac{\partial}{\partial x_i}(\rho\, U_i) = 0 \tag{2}$$

and

$$\frac{\partial}{\partial x_i} u_i = 0 \quad . \tag{3}$$

We have neglected pulsations in the density ρ and assumed homogeneous turbulence.

Assuming stationarity for the turbulence and taking the time average of the pressure P, u_i and U_i the force equation reads

$$U_j \frac{\partial}{\partial x_j} U_i + \frac{1}{\rho} \frac{\partial}{\partial x_i} <P> +$$

$$+ \frac{\partial}{\partial x_j} <u_i u_j> - \nu \frac{\partial^2}{\partial x_j^2} U_i = \frac{\partial}{\partial x_i} \psi \quad . \tag{4}$$

Linear terms in u_i have disappeared with the averaging. ψ stands for the gravitational potential and ν for the molecular viscosity. Compared with the other terms, the term due to the molecular viscosity is of the order of 1/Re (Re ≡ Reynolds number $\gg$ 1) and can be neglected.

Following Favre *et al.* (1976) we make the assumption of proportionality between the Reynolds stresses and the turbulent kinetic energy

$$u_i u_j = a \sum_{k=1}^{3} u_k^2 = au^2 \tag{5}$$

a is for the order unity,and will be set equal to 1.

Then, the z component of equation (4) is, assuming thin disk and $U_z = 0$,

$$\frac{\partial}{\partial z} P + \rho \frac{\partial}{\partial z} <u^2> = -\rho\, \Omega^2 z \tag{6}$$

where P is the sum of gas and radiation pressures and Ω is the keplerian angular velocity.

If matter in the disk is uniform along z and falls abruptly to zero at z = ℓ (ℓ = semi-scale height of the disk), Shakura and Sunyaev (1976),

$$-\frac{\partial}{\partial z}\left\{\left[1 + \frac{m_t^2}{\gamma}\right] P\right\} = \rho\, \Omega^2 z \quad , \tag{7}$$

where m_t is the turbulent Mach number and γ is the adiabatic index. We shall set γ equal to 1 instead of the correct value 4/3.

We now obtain the relation between the viscocity parameter α and the turbulent Mach number m_t. According to Shakura and Sunyaev (1973, 1976), setting magnetic fields to zero,

$$\alpha = \frac{\ell_t\, u}{\ell\, v_s} \quad , \tag{8}$$

where ℓ_t ≡ turbulent scale length, and v_s ≡ sound velocity. However, $m_t = u/v_s$ so

$$m_t = \alpha \frac{\ell}{\ell_t} \quad , \tag{9}$$

with $\ell/\ell_t \geqslant 1$.

Using equation (9), we may write

$$P_t = m_t^2 \rho\, v_s^2 = \alpha^2 \left[\frac{\ell}{\ell_t}\right]^2 \rho\, v_s^2 \quad . \tag{10}$$

The calculation of ℓ_t is very difficult and usually one sets $\ell_t = \ell$. This implies $\alpha = m_t$.

It should be noticed that equation (7) is quite general and is similar to the expression used by Shakura, Sunyaev and Zilitinkevich (1978), White and Holt (1981, preprint) and Kriz and Hubeny (1986).

III. THE STRUCTURE OF THE ACCRETION DISK

In the following we shall obtain the structure of accretion disk in the optically thin and gas pressure dominated region. As in Collin-Souffrin's (1987) paper, we shall assume homogeneity in z for the density and the temperature and, with the exception of the inclusion of the turbulent pressure pulsations in the hydrostatic equilibrium equation, our equations are identical to hers. The consequences of the z homogeneity will be analyzed later on this paper. Let us briefly review the equations.

The continuity equation (2) is easily integrated to give

$$\dot{M} = -4\, \pi\, r\, \ell\, V_r\, \rho \tag{11}$$

$\dot{M}$ is the accretion rate, r the radial distance and V_r the radial velocity. In an inhomogeneous medium, V_r and ρ in equation (11) should be interpreted as averaged values over z.

From the φ component of equation (4), we obtain through the definition of α

$$V_r = -\alpha\, v_s \frac{\ell}{r} \quad . \tag{12}$$

Neglecting pressure gradients along the r direction, the heat generation function is given by

$$D(r) = \frac{3}{8\pi} \Omega^2 \dot{M} \quad , \tag{13}$$

and the radiative flux is

$$F(r) = \pi \int_0^\infty B_\nu(t) \left[1 - e^{-\tau_\nu} \right] d\nu \; , \qquad (14)$$

where B_ν is the Planck function and τ_ν, the optical depth, is given by

$$\tau_\nu = 2 \rho \ell x_\nu \qquad (15)$$

where x_ν is the exponential mass absorption coefficient ($cm^2\ g^{-1}$).

Integrating equation (7) over z, with the boundary condition $P(z = \ell) = 0$, averaging over z and equating this to the gas pressure, we obtain

$$\ell = 1.56 \times 10^4 \frac{(1+\alpha)^{1/2}}{\Omega} T^{1/2} \text{ cm} \qquad (16)$$

where T is the temperature. Using (11), (12) and (16) we obtain for the density

$$\rho = \frac{6.29 \times 10^{-14}\, \dot{M} \Omega^2}{\alpha (1+\alpha^2)^{3/2}\, T^{3/2}} \text{ g cm}^{-3} \; . \qquad (17)$$

In the stationary regime heat generation equals radiative cooling so, using equations (13), (14), (15) and (16) we obtain

$$x_\nu = 6.8 \times 10^{11} \frac{\alpha(1+\alpha^2)\Omega}{T^3} \; . \qquad (18)$$

For x_ν we shall adopt the values by Alexander, Johnson and Rypme (1983), as given by Collin-Souffrin (1987). From now on we shall express the radial distance in units of parsec (r), the mass in units of 10^9 solar masses (M_9), the scale height in units of 10^{15} cm (ℓ), the accretion rate in units of 1 solar mass per year ($\dot{M}_1$) and the density in g cm^{-3}.

The solutions for the physical variables will be labelled by a subscript i, running from 1 to 5, corresponding to different expressions of the absorption coefficient.

From Table 1 in Collin-Souffrin's paper, the temperature at r_L, the radius at which the disk becomes optically thin, never exceeds 2400 °K. So, we shall confine our attention to temperatures lower than 3200 °K and shall neglect bound-free, free-free, H, H^- opacities and electron scattering.

In the following, we shall give the expressions for T, ℓ and ρ for different expressions of the absorption coefficient. We shall also give expressions for r_L and T_L, the temperature at which the disk becomes optically thin.

1) Solution for log T < 3.2:

$$x_1 = 63\, \rho^{0.625} \; ,$$

$$T_1 = 2.24 \times 10^2 \frac{\alpha^{26/33} (1+\alpha^2)^{31/33}\, r^{6/33}}{\dot{M}_1^{10/33} M_9^{2/33}} \; ,$$

$$\ell_1 = 6.2 \frac{\alpha^{13/33} (1+\alpha^2)^{32/33}\, r^{35/22}}{\dot{M}_1^{5/33} M_9^{35/66}} \; ,$$

$$\rho_1 = \frac{5.42 \times 10^{-12}\, \dot{M}_1^{16/11} M_9^{12/11}}{\alpha^{24/11} (1+\alpha^2)^{32/11}\, r^{36/11}} \qquad (19)$$

$$T_{1L} = 1.97 \times 10^2\, \dot{M}_1^{-0.195}\, \alpha^{26/41} (1+\alpha^2)^{3/4}$$

$$r_{1L} = \frac{0.5\, \dot{M}_1^{73/123} M_9^{41/123}}{\alpha^{104/123} (1+\alpha^2)} \; .$$

2) Solution for 3.2 < log T < 3.35:

$$x_2 = 10^{-27}\, T^9 \rho^{0.625}$$

$$T_2 = 1.2 \times 10^3 \frac{\alpha^{26/177} (1+\alpha^2)^{31/177}\, r^{2/59}}{\dot{M}_1^{10/177} M_9^{2/177}}$$

$$\ell_2 = 11.3 \frac{\alpha^{13/177} (1+\alpha^2)^{104/177}\, r^{89/59}}{\dot{M}_1^{5/177}}$$

$$\rho_2 = \frac{4.37 \times 10^{-13}\, \dot{M}_1^{64/59} M_9^{60/59}}{\alpha^{72/59} (1+\alpha^2)^{52/27}\, r^{180/59}} \qquad (20)$$

$$T_{2L} = 1.09 \times 10^3 \frac{\alpha^{13/118} (1+\alpha^2)^{31/236}}{\dot{M}_1^{0.043}}$$

$$r_{2L} = 5.19 \times 10^{-2} \frac{\dot{M}_1^{217/555} M_9^{1/3}}{\alpha^{26/177} (1+\alpha^2)^{31/177}} \; .$$

3) For 3.35 < log T < 3.5, and log ρ > −7

$$x_3 = 7.41 \times 10^{-7}\, T^{1.47}$$

$$T_3 = 64.72 \frac{\alpha^{0.22} (1+\alpha^2)^{0.22} M_9^{0.11}}{r^{0.34}}$$

$$\ell_3 = 2.62 \frac{\alpha^{0.11} (1+\alpha^2)^{0.61}}{\dot{M}_9^{0.44}} r^{1.33}$$

$$\rho_3 = \frac{3.5 \times 10^{-11} \dot{M}_1 M_9^{0.83}}{\alpha^{1.34} (1+\alpha^2)^{1.84} r^{2.49}} \qquad (21)$$

$$T_{3L} = 39.94 \frac{\alpha^{0.41} (1+\alpha^2)^{0.41}}{\dot{M}^{0.223}}$$

$$r_{3L} = 4.24 \frac{\dot{M}^{0.631} M^{1/3}}{\alpha^{0.55} (1+\alpha^2)^{0.55}}$$

4) For $-3.35 < \log T < 3.5, \log \rho < -10$:

$$x_4 = 10^{17} T^{-6}$$

$$T_4 = \frac{1.03 \times 10^5 r^{1/2}}{M_9^{1/6} \alpha^{1/3} (1+\alpha^2)^{1/3}}$$

$$\ell_4 = 1.05 \times 10^2 \frac{(1+\alpha^2)^{1/3} r^{7/4}}{M_9^{0.5} \alpha^{1/6}} \qquad (22)$$

$$\rho_4 = \frac{5.5 \times 10^{-16} \dot{M}_1 M_9^{5/4}}{\alpha^{1/2} (1+\alpha^2) r^{9/4}}$$

$$T_{4L} = \frac{6.86 \times 10^3 \dot{M}^{1/10}}{\alpha^{1/5} (1+\alpha^2)^{1/5}}$$

$$r_{4L} = 4.43 \times 10^{-3} \dot{M}^{1/5} M^{1/3} \alpha^{4/15} (1+\alpha^2)^{4/15} .$$

5) For $3.35 < \log T < 3.5, -10 < \log \rho < -7$:

$\log x_5 = (2.49 \log \rho + 18.9) \log T - 7.71 \log \rho - 60.1$.
Now T and T_L are given, respectively, as the solutions of

$$3.74 (\log T)^2 - (\log T) \log \left\{ \frac{1.2 \times 10^{13} (M_9 \dot{M}_1)^{2.49}}{\alpha^{2.49} (1+\alpha^2)^{3.74} r^{7.47}} \right\} + \log \left\{ 8.35 \times 10^{-2} \frac{\dot{M}_1^{7.71} M_9^{8.21}}{\alpha^{7.71} (1+\alpha^2)^{11.57} r^{26.13}} \right\} = 0 \qquad (23)$$

and

$$6.22 (\log T_L)^2 + (\log T_L) \log \left\{ \frac{7.23 \times 10^{-44}}{\alpha^{2.49} (1+\alpha^2)^{3.74}} \right\} - \log \left\{ \frac{1.36 \times 10^{-76}}{\alpha^{7.71} (1+\alpha^2)^{11.57} \dot{M}_1 M_9^{1/2}} \right\} = 0 . \qquad (24)$$

The correct solutions of equations (23) and (24) are those that yield T in the appropriate range.

IV. CAN THIS MODEL BE APPLIED TO QUASARS?

Clearly, the answer to that question is highly dependent upon the picture we assume for a Quasar, i.e., the masss, the accretion rate and how they are related to each other. For that we shall assume the evolutionary scheme described by Luminet (1981), according to which the Quasar phase is fueled by stellar collisions which begin to dominate over other supplying gas processes at a core (nucleus) mass of about $M_9 \simeq 0.035$ and lasts till the mass of the nucleus reaches $M_9 \simeq 1$. In that phase the mass and the accretion rate are related by

$$\dot{M}_1 = 10^2 M_9^3 . \qquad (25)$$

The results obtained show that temperatures lower than 3200 °K are only produced in the disk, for masses and accretion rates in the Quasar range, if turbulence is supersonic. To clear this point, we shall find out the relations M_9 and α should satisfy in order to have T_L and ρ_L in the prescribed value for each region (x_i)

1) $$1.19 \times 10^{-2} \alpha^{1.08} (1+\alpha^2)^{1.28} < M_9 < 2.6 \times 10^{-2} \alpha^{1.08} (1+\alpha^2)^{1.28} ; \qquad (26)$$

2) $$8.34 \times 10^{-4} \alpha^{0.85} (1+\alpha^2)^{1.01} < M_9 < 1.23 \times 10^{-2} \alpha^{0.85} (1+\alpha^2)^{1.01} ; \qquad (27)$$

3) $$3.13 \times 10^{-4} \alpha^{0.62} (1+\alpha^2)^{0.62} < M_9 < 5.24 \times 10^{-4} \alpha^{0.62} (1+\alpha^2)^{0.62} ,$$
$$M_9 < 2.5 \times 10^{-4} \alpha^{0.01} (1+\alpha^2)^{-0.28} ; \qquad (28)$$

4) $5.17 \times 10^{-3}\ \alpha^{2/3}\ (1+\alpha^2)^{2/3}$

$$< M_9 < 1.63 \times 10^{-2}\ \alpha^{2/3}\ (1+\alpha^2)^{2/3} \quad \alpha < 4.8 \quad ; \qquad (29)$$

5) $0.13\ \alpha^{0.18}\ (1+\alpha^2)^{0.28} < M_9 < 0.13\ \alpha^{0.29}\ (1+\alpha^2)^{0.43}$,

$3.88 \times 10^2\ \alpha^{2/5}\ (1+\alpha^2)^{3/5}$

$$< T_L < 6.16 \times 10^3\ \alpha^{2/5}\ (1+\alpha^2)^{3/5} \qquad (30)$$

It is readily seen that only relations (26), (27) and (30) are compatible with masses usually assumed for quasars. It is also seen that this compatibility is only achieved for $\alpha > 1$. If turbulence pressure pulsations were not included α would be much greater.

It is worth remarking that only for region 4 temperature is a decreasing function of the radius. As the density is everywhere decreasing with radius, region 4 will only occur in Quasars for r_L belonging to it. However, as seen from relation (28), it only happens for masses below 2.5×10^{-4}, which is definitely outside the quasar mass range. For relations (26), (27), (29) and (30), temperature is a monotonically increasing function of the radius, while the flux behaves differently. Therefore, the disk is thermally unstable everywhere.

Finally, it should be noted that with the inclusion of turbulent pressure pulsations, density will decrease in the disk (for Quasars), therefore the effects of self-gravity will be highly attenuated.

V. IS DISK ACCRETION COMPATIBLE WITH z-HOMOGENEITY?

According to the α standard disk model of Shakura and Sunyaev (1976) the density, the kinematic viscosity coefficient, K, and the stress (not integrated over z), assuming Keplerian velocity, are given by

$$\rho = \rho_c(r)\, H(\ell - z) \quad ,$$

$$K = \frac{\alpha\Omega\ell^2}{3} \quad , \qquad (31)$$

and $$\omega_{r\varphi} = \frac{3}{2} K\rho\Omega \quad ;$$

where H is the Heaviside function and ρ_c is the density (constant over z) given by

$$\rho_c = \frac{3}{4\pi} \frac{\dot{M}\ S}{\ell^3\Omega} \quad , \qquad (32)$$

with

$$S = 1 - \delta\left[\frac{r_*}{r}\right]^{1/2} .$$

In this expression r_* is the inner radius of the disk and δ is the ratio of the actual angular momentum of the flow to the Keplerian one at $r = r_*$.

It should be remarked from equation (11) that accretion onto the compact object means $\dot{M} > 0$, which implies $V_r < 0$, i.e., the radial velocity is directed toward the compact object.

Using equation (31) we may write the angular momentum conservation equation as

$$\frac{\rho V_r V_k r}{2} = -\frac{\partial}{\partial r}\left\{\frac{\alpha\rho\ell^2\ \Omega^2 r^2}{3}\right\} \quad , \qquad (33)$$

V_k being the keplerian velocity. Using now equations (11) and (32) we obtain

$$\frac{r\Omega}{2\ell} = \frac{\partial}{\partial r}\left\{\frac{S\Omega r^2}{\ell}\right\} \quad . \qquad (34)$$

Making $y = \frac{S\Omega r^2}{\ell}$, equation (34) changes to

$$\frac{\partial}{\partial r} y = \frac{1}{2}\frac{y}{Sr} \quad , \qquad (35)$$

the solution of this equation is

$$y = C\, r^{1/2} S \quad , \qquad (36)$$

where C is an integration constant.

Comparing the definition of y with equation (36) we conclude that for the α standard accretion disk model the disk scale height ℓ is constant.

Expressing V_r by means of equations (33) and (32) gives

$$V_r = -\frac{1}{2\pi}\frac{\dot{M}}{\rho\, r^2\Omega}\frac{\partial}{\partial r}\left[\frac{r\Omega S}{\ell}\right] \quad , \qquad (37)$$

in the region we are concerned $S \simeq 1$. We see from equation (37) that accretion means $\partial/\partial r\ (r\Omega/\ell) > 0$. However, in every solution we obtained ℓ varying faster than $r^{1.3}$, which clearly implies $V_r > 0$.

If we now abandon the assumption of equality between the scale disk height and the scale length of turbulence and use the correct hydrostatic equilibrium equation

$$P = \frac{\rho_c \Omega^2 \ell^2}{2(1+m^2)} (1-x^2) \tag{38}$$

with $x = z/\ell$, we obtain

$$V_r \simeq -\frac{\alpha \dot{M}}{\Omega \rho r^2 (1+m^2)} \times$$

$$\left\{ \left[\frac{1}{2} \frac{r\Omega}{\ell} - \frac{r^2\Omega}{\ell^2} \frac{\partial \ell}{\partial r} \right] (1-x^2)^{1/2} + \frac{r^2 \Omega x^2}{\ell (1-x^2)^{1/2}} \frac{\partial \ell}{\partial r} \right\} \tag{39}$$

From this equation it follows that if

$$\frac{\partial \ell}{\partial r} \neq 0 \,, \quad \text{as} \quad x \to 1 \,, \quad |V_r| \to \infty \,.$$

Therefore, the only solution compatible with accretion, if the disk is homogeneous over z, is ℓ constant.

IV. CONCLUSIONS

Restricting our results to Quasars, we may say that inclusion of turbulent pressure pulsations will make the optically thin region more extended, i.e., closer to the compact object. We have shown that the Quasar mass regime is only achieved for supersonic turbulence. For this regime the structure of the optically thin region are affected because the temperature and the scale height of the disk increase, while the density decreases in a very pronounced way. This makes the disk less sensitive to the effects of self-gravity. In this regime, the optically thin region is everywhere thermally unstable. It is worth remarking that temperatures in the optically thin region in range (1-5) 10^3 °K imply masses completely outside the quasar mass range, unless we use equation (25).

Finally, it should be remarked that homogeneity is compatible with accretion only if ℓ is constant. Otherwise, we may find regions in the disk where matter is flowing outwards.

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