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ON THE TWO TEMPERATURE CONDITIONS FOR ACCRETION DISKS

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RESUMEN

Hemos analizado las restricciones que deben cumplir el número de Mach turbulento (α), la tasa de acreción ($\dot{M}$) y masa (M) del objecto compacto, para satisfacer la solución de un modelo de disco de dos temperaturas para ser consistente. Encontramos que la consistencia se encuentra cuando

$$9.13 \times 10^{-3}/\alpha < [\, \dot{M}_{17} S/\alpha\, M_{34}\, r^{3/2}\,]^{1/2} < r^{-1/2} .$$

Esto impone severas restricciones en el sistema. En particular, Cygnus X-1 satisface este criterio sólo en el regimen de turbulencia ligeramente subsónica ($\alpha > 0.5$). Se muestra también que, aún para tan alto valor de α, la conducción es el principal proceso de transporte de energía.

ABSTRACT

We have analysed the constraints that the turbulent Mach number (α), accretion rate ($\dot{M}$) and mass (M) of the compact object should satisfy in order that the solution for the two temperature disk model to be self-consistent with the requirements of the model. We found that consistency is met whenever

$$9.13 \times 10^{-3}/\alpha < [\, \dot{M}_{17} S/\alpha\, M_{34}\, r^{3/2}\,]^{1/2} < r^{-1/2} .$$

This imposes severe constraints in the system. In particular, Cygnus X-1 satisfies this criterion just for mildly subsonic turbulence ($\alpha > 0.5$). It is shown that, even for such a high value of α, conduction is the main energy transport process.

Key words: ACCRETION DISKS – STARS-BINARIES – TURBULENCE

I. INTRODUCTION

Most of the models proposed for Cygnus X-1 invoke non-thermal processes (Apparao and Chitre 1976; Galéev, Rosner and Vaiana 1979; Ipser and Price 1982; Liang and Nolan 1984; Liang and Price 1977; Liang and Thomson 1979; Mészaros 1976; Mészaros 1983). The present stage of our knowledge on these processes constitutes in the major difficulty in making reliable predictions for physical conditions based on these alternatives. Among the models based on thermal processes, the inverse comptonization of soft photons from a very copious source, in an optically thin region with electronic temperature $T_e \sim 10^9$ K and electron scattering depth $\tau_{es} \sim 0.2$–5, seems to be, at present, the best candidate for explaining the observe spectrum. This model leads naturally to a power law spectrum.

The first Compton model was proposed by Shapiro, Lightman and Eardley (1976) and was inspired on an observation of Thorne and Price (1975) according to which an instability develops in the thin (optically thick) disc driving the inner region to a geometrically thick and optically thin disc. It is assumed further that there is no thermal equilibrium between protons (whose temperature is T_i) and electrons (temperature T_e) and unsaturated Comptonization of an external soft source (this is equivalent to imposing the Kompaneets parameter $Y = 1$).

In this paper we will analyse the range of parameters ($\dot{M}$, M, α) for which there is a consistent solution for the two temperature disc accretion model. We have been particularly concerned with the bounds imposed, for consistency reasons, on the electronic temperature T_e and ionic temperature T_i. We show that T_i has a lower bound (imposed by gas pressure dominance) and an upper bound (imposed by thin disc approximation). These two bounds entail the condition $(\dot{M}_{17}/\alpha M_{34}) < 1$. The imposition that T_e be bounded by 6×10^9 K (the threshold for pair production) leads to a lower bound to $\dot{M}/\alpha M_{34}$.

Besides the analysis of the consistency of the model being made in terms of the bounds on the temperature T_e and T_i our paper differs from the others in the literature in the fact that in our paper we do not assume a constant Coulomb logarithm. As a matter of fact, we further show how to determine, consistently, the value of the Coulomb log.

Our conclusion is that the acceptable range of parameters (M, $\dot{M}$, α) for which there is a consistent solution to the two temperature accretion model is very narrow. For Cygnus X-1 it yields mildly subsonic turbulence, with energy transport largely dominated by conduction.

II. THE ASSUMPTIONS AND SOLUTION OF THE TWO TEMPERATURE DISC MODEL

We shall consider a gas of completely ionized hydrogen in a Newtonian disc, with protons and electrons out of thermal equilibrium. As usual, in all equations we shall make an average over z, the coordinate normal to the plane of the disc.

The usual assumptions of this model that we also make in this paper are:

1) The system has axial symmetry, the azimuthal velocity is keplerian.
2) Hydrostatic equilibrium normal to the disc surface, the z-direction.
3) Geometrically thin disc, $\ell/r \ll 1$ and ℓ and r are respectively semi-scale height of the disc and distance to the compact central object.
4) Radiative energy transport only in the z-direction.
5) The stress tensor has essentially the $r\varphi$ component given by $W_{r\varphi} = \alpha P$, P being the pressure and α the turbulent Mach number.
6) In the inner region $\sigma_T \gg K_R$, with σ_T being the Thomson electron cross section and K_R the Rosseland mean opacity for free-free absorption.
7) An instability develops in the outer optically thick disc, making the inner region optically thin and geometrically thick.
8) The main cooling mechanism in the inner region is unsaturated inverse Comptonization of soft photons from a very copious external source. The Kompaneets parameter

$$Y = \frac{4kT_e}{m_e c^2} f(\tau) \quad , \quad f(\tau) = \begin{cases} \tau \ , & \tau < 1 \\ \tau^2 \ , & \tau > 1 \end{cases} ,$$

is taken to be equal to 1 always; k, T_e, τ, m_e, c are respectively Boltzmann constant, electronic temperature, scattering optical depth, electronic mass, velocity of light.
9) The main contribution to the pressure is from the gas, the radiation pressure being negligible. It is also assumed that the ionic temperature is much bigger than the electronic temperature.
10) Turbulence is subsonic.
11) Ions and electrons are coupled by collisional energy exchange. No other instability that may provide further coupling is present.

Through this paper we shall adopt the following set of units:

a) r in units of the inner radius $R_i = 3Rs$, where Rs is the Schwarzschild radius, b) T_i, T_e in units of 10^9 K, c) M_{34}, mass of the central compact object units of 10^{34} g, d) $\dot{M}_{17}$, accretion rate in units of 10^{17} gs^{-1}.

By making these assumptions, following Shapiro *et al.* (1976), we get the following solution for the thin disk structure variables: density (ρ), pressure (P), disc half-thickness (ℓ), T_e and T_i:

$$\ell = 1.04 \times 10^5 M_{34} T_i^{1/2} r^{3/2} \text{ cm} \ , \tag{1}$$

$$\rho = 1.55 \times 10^{-2} \frac{\dot{M}_{17} S}{\alpha M_{34}^2 T^{3/2} r^3} \text{ gcm}^{-3} \ , \tag{2}$$

$$P = 1.24 \times 10^{15} \frac{\dot{M}_{17} S}{\alpha M_{34}^2 T^{1/2} r^3} \text{ dyn cm}^{-2} \tag{3}$$

$$T_e = 0.53 \left\{ \frac{M_{34}}{\alpha \dot{M}_{17} S} \right\}^{1/6} r^{1/4} (\ln \Lambda)^{1/3} \ , \tag{4}$$

and

$$T_i = \frac{216}{\alpha^2} \left\{ \frac{\alpha \dot{M}_{17} S}{M_{34}} \right\}^{5/6} \frac{1}{r^{5/4}} (\ln \Lambda)^{1/3} \ ; \tag{5}$$

where $S = 1 - \delta r^{-1/2}$. δ is the ratio of the actual angular momentum of the flow to the Keplerian one. According to Apparao and Chitre (1976); Pacheco and Steiner (1976) $\delta \simeq 0.9$.

In the following we shall frequently use the variable ξ defined as

$$\xi = \left[\frac{\dot{M}_{17} S}{\alpha M_{34} r^{3/2}} \right]^{1/2} . \tag{6}$$

In terms of the ξ variable one can write, for instance, the ratio of temperatures T_e/T_i as

$$\frac{T_e}{T_i} = \frac{2.5 \times 10^{-3}}{\xi^2} . \tag{7}$$

Assuming $\ell n \Lambda = 15$, equations (4) – (5) reduce to those of Shapiro *et al.* (1976). In our paper we look for solutions and to the question of self-consistency of the model without resorting to this approximation. This is due to the fact that, as we argue in this paper, we cannot fix the value of $\ell n \Lambda$ *a priori* since it depends on T_e and ρ that are variables we intend to determine from the model. In fact, from the definition of the Debye number we get (Golant, Zhilinsky and Sakharov 1980)

$$\Lambda = 3.3 \times 10^3 \frac{T_e}{\rho^{1/2}} . \tag{8}$$

So that, in fact equations (1) to (5) are not the final solution, but a set of equations from which the solutions can be inferred. Equations (4) – (5) are replaced by

$$T_e = 0.53\, \alpha^{-1/3}\, \xi^{-1/3} \left\{ \ln \left[\frac{3.3 \times 10^3\ T_e}{\rho^{1/2}} \right] \right\}^{1/3} , \quad (9)$$

and

$$T_i = 216\, \alpha^{-1/3}\, \xi^{-5/3} \left\{ \ln \left[\frac{3.3 \times 10^3\ T_e}{\rho^{1/2}} \right] \right\}^{1/3} . \quad (10)$$

Without the assumption of constant $\ell n\ \Lambda$ it is no longer easy to get solutions to equations (1–3) and (9–10). One possible strategy, that we use in the following, is to write an equation for $\ell n\ \Lambda$ and then to use this solution to get the solution for the relevant variables.

The equations satisfied by $\ell n\ \Lambda$ are

$$\frac{\exp \frac{12}{7}\ \ell n\ \Lambda}{\frac{12}{7}\ \ell n\ \Lambda} = 7.36 \times 10^9\, M_{34}^{6/7}\, r^{9/7}\, \xi^{-1/7}\, \alpha^{-1} =$$

$$= (\exp 22.72)\, M_{34}^{6/7}\, r^{9/7}\, \xi^{-1/7}\, \alpha^{-1} , \quad (11)$$

where ξ is defined by equation (6).

For a given value of r and for a given point the (M, $\dot{M}$, α) space one can find by using equation (11) the corresponding $\ell n\ \Lambda$ value. For this value of $\ell n\ \Lambda$ one can find the solution for equations (1) to (5).

Before continuing let us analyse the consistency of the model.

III. CONSISTENCY OF THE MODEL

In order that the model be consistent, it is necessary to ckeck if the solutions are compatible with the hypotheses made. First, we will concentrate on two requirements: the thin disk approximation and the gas pressure dominance hypothesis. The thin disk approximation imposes an upper bound to T_i, that is

$$T_i < \frac{616}{r} . \quad (12)$$

The gas pressure dominance hypothesis on the other hand, sets a lower temperature to T_i. Since

$$P_g = 1.24 \times 10^{15} \frac{\xi^2}{M_{34}\, r^{3/2}\, T_i^{1/2}} , \quad (13)$$

and

$$P_r = 3.7 \times 10^{15} \frac{\alpha}{r^{3/2}} \frac{\xi^2}{T_i\, M_{34}} ; \quad (14)$$

then the $P_r/P_g \ll 1$ hypothesis implies

$$T_i > 8.9\, \alpha^2\, \xi^4 . \quad (15)$$

The consistency of the model requires that T_i be in the range

$$8.9\, \alpha^2\, \xi^4 < T_i < 616/r . \quad (15b)$$

However, the unsaturated condition together with $\tau < 1$, places a lower bound to T_i greater than that given by (16). Definitively the T_i range is

$$617\, \xi^2 < T_i < 616/r . \quad (16)$$

If one uses equation (5), then relation (16) is equivalent to the following constraint in $\ell n\ \Lambda$

$$22.67\, \alpha\, \xi < \ln \Lambda < 23\, r^{-3} \frac{\alpha}{\xi^5} . \quad (17)$$

That is, a solution of equation (11) is consistent if the Coulomb logarithm is in the range given by relation (17). By looking at this inequality it follows our first restriction on the region of the allowed (α, M, $\dot{M}$) parameters. This restriction is, independently of the object,

$$r^{1/2}\, \xi < 1 . \quad (18)$$

This requirement just follows from the fact that the right hand side be larger than the left of the inequality. Inequality (18) implies

$$\frac{\dot{M}_{17}\ S}{M_{34}\ r^{1/2}} < \alpha . \quad (19)$$

The requirement of subsonic regime associated with the bound in expression (19) imposes a restriction on $\dot{M}_{17}/M_{34}$, that is

$$\frac{\dot{M}_{17}\ S}{M_{34}\ r^{1/2}} < 1 . \quad (20)$$

Another consequence of (18) is that, since r is larger than 1 and considering $M_{34} > 1$, it is possible to get a lower bound to $\ell n\,\Lambda$. From it follows

$$\ln \Lambda > 13.25 \quad . \tag{21}$$

From (2.1) it cam be predicted that T_e satisfies the bound

$$T_e > \frac{1.25}{(\alpha\xi)^{1/3}} \quad . \tag{22}$$

Expression (22) imposes another severe constraint on the allowed values of the ξ variable. First of all, it can be noted that, independently of the object, $T_e > 1.48$ (since $\alpha > 1$ and $\xi < 1$). Furthermore if it is required that $T_e < 6$ (which is the threshold for pair production, that is not taken into account in the model) the consistency requirement is given by

$$\xi > \frac{9.13 \times 10^{-3}}{\alpha} \quad . \tag{23}$$

From relations (18) and (23) it follows that the variable ξ should be in the range

$$\frac{9.13 \times 10^{-3}}{\alpha} < \xi < \frac{1}{r^{1/2}} \quad . \tag{24}$$

The range of allowed values for ξ is indeed very narrow. The main conclusion of our paper is that, independently of the object considered, the description of the disc in terms of the two temperature model is consistent only if the variable ξ, defined in equation (6) is in the range (24). It can be noted that condition (24) does not permit objects differing by one order of magnitude in the variable ξ.

The bounds on the M, $\dot{M}$, α parameters read

$$\frac{8.34 \times 10^{-5}}{\alpha} < \frac{\dot{M}_{17}\, S}{M_{34}\, r^{3/2}} < \frac{\alpha}{r} \quad . \tag{25}$$

Condition (25) is quite general being independent of the object. We now apply expression (25) to Cygnus X-1. Assuming that the energy above 3 KeV ($L \simeq 3 \times 10^{37}$ erg s^{-1}) is generated in the inner region, we obtain (for a maximally rotating Kerr black hole),

$$\dot{M}_{17} = \frac{0.22}{0.4 + \frac{0.6}{r_0^{3/2}} - \frac{1}{r_0}} \quad , \tag{26}$$

where r_0 is the location of the instability. However, the location of the instability is given by (Shapiro *et al.* 1976)

$$r_0^{21/8}\, S_0^{-2} = 5.95 \times 10^4\, \dot{M}_{17}^2 (\alpha/M_{34}^7)^{1/4} \quad . \tag{27}$$

Combining equations (26) and (27) we obtain

$$\alpha = \frac{7.96 \times 10^{-14}}{M_{34}}\, r_0^{5/2} \left\{ M_{34}\, r_0 \left[\frac{0.4 + \frac{0.6}{r_0^{3/2}} - \frac{1}{r_0}}{1 - 0.9\, r_0^{-1/2}} \right] \right\}^8 \tag{28}$$

which inserted into the right hand side of expression (25) gives

$$1 < 3.25 \times 10^{-14} \frac{r_0^2}{M_{34}} \left\{ M_{34}\, r_0 \left[\frac{0.4 + \frac{0.6}{r_0^{3/2}} - \frac{1}{r_0}}{1 - 0.9\, r_0^{-1/2}} \right] \right\}^9 \quad . \tag{29}$$

For $M_{34} = 3$, we obtain

$$r_0 > 14.7 \quad ,$$

$$\alpha > 0.51 \quad ,$$

and

$$\ln \Lambda = 16.8 \quad . \tag{30}$$

How does this conclusion change if we assume, instead, a black hole with negligible angular momentum and null boundary condition for the torque at the inner radius?

In that case, equations (26), (27) and (28) change to

$$\dot{M}_{17} = \frac{4.02}{1 + 2r_0^{-3/2} - 3r_0^{-1}} \tag{31}$$

$$r_0^{21/8}\, S_0^{-2} = 5.41 \times 10^2\, \dot{M}_{17}^2 (\alpha/M_{34}^7)^{1/4} \tag{32}$$

$$\alpha = \frac{1.72 \times 10^{-16}}{M_{34}}\, r_0^{5/2} \left\{ M_{34}\, r_0 \left[\frac{1 + 2r_0^{-3/2} - 3r_0^{-1}}{1 - r^{-1/2}} \right] \right\}^8 \quad . \tag{33}$$

Again, applying the expression for α into the right hand side of inequality (25) we get

$$1 < 4.28 \times 10^{-17} \frac{r_0^2}{M_{34}} \left\{ M_{34}\, r_0 \left[\frac{1 + 2r_0^{-3/2} - 3r_0^{-1}}{1 - r^{-1/2}} \right] \right\}^9 . \quad (34)$$

The solution of which is

$$r_0 > 12.6 \quad ,$$

$$\alpha > 0.34 \quad ,$$

and

$$\ln \Lambda = 18.38 \quad . \quad (35)$$

As it is required that $\alpha < 1$ to avoid collisionless shock heating, which would rapidly equalize the electron and ion temperatures, we must conclude that the range of α is very narrow, a result independent of the boundary condition for the torque at the inner radius.

Furthermore, a crude estimate of the ratio of conductive energly flux (for a classical conductive constant) to radiative energy flux yields

$$q = \frac{7.61 \times 10^5}{\alpha^{49/12}} \left[\frac{\dot{M}_{17} S}{M_{34}} \right]^{23/12} \frac{(\ln \Lambda)^{7/6}}{r_0^{19/8}} \quad (36)$$

(corresponding to a maximally rotating hole).

With the values given by (30), we obtain

$$q \simeq 1.46 \times 10^4 \quad (37)$$

a result that makes conduction a non negligible energy transport process in a two temperature accretion disk, even for such a high value of the turbulent Mach number.

Therefore, a two temperature accretion disk is very unlikely to occur around Cygnus X-1.

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