

NUMERICAL RESULTS FOR A POLYTROPIC COSMOLOGY INTERPRETED AS A DUST UNIVERSE PRODUCING GRAVITATIONAL WAVES

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RESUMEN. A nivel cosmológico pensamos que se ha estado produciendo radiación gravitacional en cantidades considerables dentro de las galaxias. Si los eventos productores de radiación gravitatoria han venido ocurriendo desde la época de la formación de las galaxias, cuando menos, sus efectos cosmológicos pueden ser tomados en cuenta con simplicidad y elegancia al representar la producción de radiación y, por consiguiente, su interacción con materia ordinaria fenomenológicamente a través de una ecuación de estado politrópica, como lo hemos mostrado en otros trabajos. Presentamos en este artículo resultados numéricos de este modelo.

ABSTRACT. A common believe in cosmology is that gravitational radiation in considerable quantities is being produced within the galaxies. If gravitational radiation production has been running since the galaxy formation epoch, at least, its cosmological effects can be assessed with simplicity and elegance by representing the production of radiation and, therefore, its interaction with ordinary matter phenomenologically through a polytropic equation of state as shown already elsewhere. We present in this paper the numerical results of such a model.

Key words: COSMOLOGY – GRAVITATION

INTRODUCTION

Gravitational radiation is believed to be produced by various astrophysical events taking place within the galaxies, for example, during supernova explosions. If this and other gravitational radiation producing events have been going on since early times, they may have contributed in some way to conform the present state of our universe.

The cosmological consequences of its existence has been studied, using different models, by several authors (Rees 1971; Jackson 1972; Bertotti and Cavaliere 1971; Isaacson and Winicour 1973; Swinerd 1977b).

In this paper we present numerical results of a cosmological model that takes into account the production of gravitational waves. Such a model can be assessed in a rather simple way through an isotropic cosmological model that contains matter producing gravitational radiation both considered as perfect fluids. In the geometrical optics limit of General Relativity (i.e. when the scale factor $R \gg \lambda$, the gravitational radiation wavelength), the gravitational radiation can be treated as a perfect fluid with equation of state given by $p_r = \frac{1}{3}\rho_r c^2$ (Isaacson 1968a,b; Swinerd 1977a).

We have investigated (Chauvet, Cervantes-Cota and Klapp 1990a,b) the possibility of introducing of a polytropic equation of state to represent phenomenologically at a cosmological level, the interaction between matter and gravitational radiation that may give an adequate gravitational radiation time behaviour.

In this paper we present the range of model parameters giving reasonable model universes and its predictions.

II. PHYSICS BEHIND THE MODELS

To describe a cosmological model with matter and gravitational radiation, we have considered that the gravitational radiation field $h_{\mu\nu}$, can be taken as perturbations to a cosmological background space-time $\gamma_{\mu\nu}$, given by a Robertson-Walker line element. The total metric $g_{\mu\nu}$ is then

$$g_{\mu\nu} = \gamma_{\mu\nu} + \epsilon h_{\mu\nu}, \quad 0 < \epsilon \ll 1. \quad (1)$$

In the geometrical optics limit of General Relativity (high frequency waves), when the total metric substituted in the Einstein field equations, and after expanding in power of ϵ , one obtains

$$R_{\mu\nu}^{(o)} - \frac{1}{2}\gamma_{\mu\nu}R^{(o)} = -8\pi G(T_{\mu\nu}^{(o)} + E_{\mu\nu}^{eff}), \quad (2)$$

where both, the matter-energy tensor $T_{\mu\nu}^{(o)}$ and the “effective” tensor $E_{\mu\nu}^{eff}$ associated with the gravitational field, can be considered as perfect fluids. Specifically, the radiation field has an equation of state given by $p_r = \frac{1}{3}\rho_r c^2$ (Isaacson 1968a,b; Swinerd 1977a).

From equations (2) the model dynamics is given by

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) - \frac{\kappa c^2}{R^2}, \quad (3)$$

where ρ_m, ρ_r are the matter and gravitational radiation densities, κ is the curvature constant, c is the speed of light and a dot denotes time derivative.

The energy conservation equation is

$$(\rho R^3)' + 3(\ln R)' \frac{p R^3}{c^2} = 0, \quad (4)$$

where $\rho (= \rho_m + \rho_r)$ and $p (= p_m + p_r)$ are the total density and pressure, respectively.

We then have a two fluid model, gravitational radiation and matter, with equations of state given by

$$p_r = \frac{1}{3}\rho_r c^2 \quad (\text{high frequency radiation}) \quad (5)$$

and

$$p_m = 0 \quad (\text{dust}). \quad (6)$$

Due to the present ignorance of the precise mechanisms by which gravitational waves are generated by matter we have introduced (Chauvet, Cervantes-Cota and Klapp 1990a,b) a polytropic equation of state given by $p = \beta \rho^{1+n}$, with β and n (the polytropic index) constants, leading to an adequate time profile for the densities of matter and radiation: ρ_m and ρ_r are decreasing and increasing functions of time, respectively. We have obtained that

$$\rho_m = \frac{F-3}{\beta^n F^{n+1}} \quad \text{with} \quad F \equiv \frac{\alpha}{\beta} R^{3/n} - 1 \quad (7)$$

and

$$\rho_r = \frac{3}{\beta^n F^{n+1}}, \quad \text{where } \alpha \text{ is an integration constant.} \quad (8)$$

Equation (8) fulfils the “galactic hypothesis” ansatz that implies that the gravitational radiation is postgalactic nature. This means that the polytropic index must be in the range $-1 < n < 0$. In this case $\rho_r \rightarrow 0$ when $R \rightarrow 0$, that is, there is no gravitational radiation at the beginning of the time.

III. NUMERICAL INTEGRATION

To obtain the dynamics of the model we have integrated equation (3) together with equations (7) and (8). However, from equation (7) one can see that if $R \geq R_i \equiv (4\beta/\alpha)^{n/3}$ one gets a negative matter density, which is unphysical. To avoid this condition we numerically integrate our equations up to $(t = t_c, R = R_c < R_i)$ and then join

a decoupled model of matter and gravitational radiation, that is, without interaction. Thus, the final model has to be as: a "generating era" ($t \leq t_c$) and the decoupled era ($t > t_c$). To join the interacting model with the non interacting one, we used the Lichnerowicz (1955) conditions (see Chauvet, Cervantes-Cota and Klapp 1990b for more details).

$$\frac{\rho_m}{\rho_{mc}} = \left(\frac{R_c}{R}\right)^3 \quad \text{and} \quad \frac{\rho_r}{\rho_{rc}} = \left(\frac{R_c}{R}\right)^4, \quad (9)$$

here ρ_{mc} and ρ_{rc} are appropriate constants determined by the junction conditions.

In some closed models the scale factor reaches its maximum before ρ_m becomes negative, still, it is necessary to stop the radiation production. The reason is that, since the densities depend solely on the scale factor they invert their roles after that point, that is, instead of gravity waves production one would have absorption which is not permitted, according to Hawking (1966).

One of the above models, including a standard dust Friedmann model for comparison is shown in figure (1).

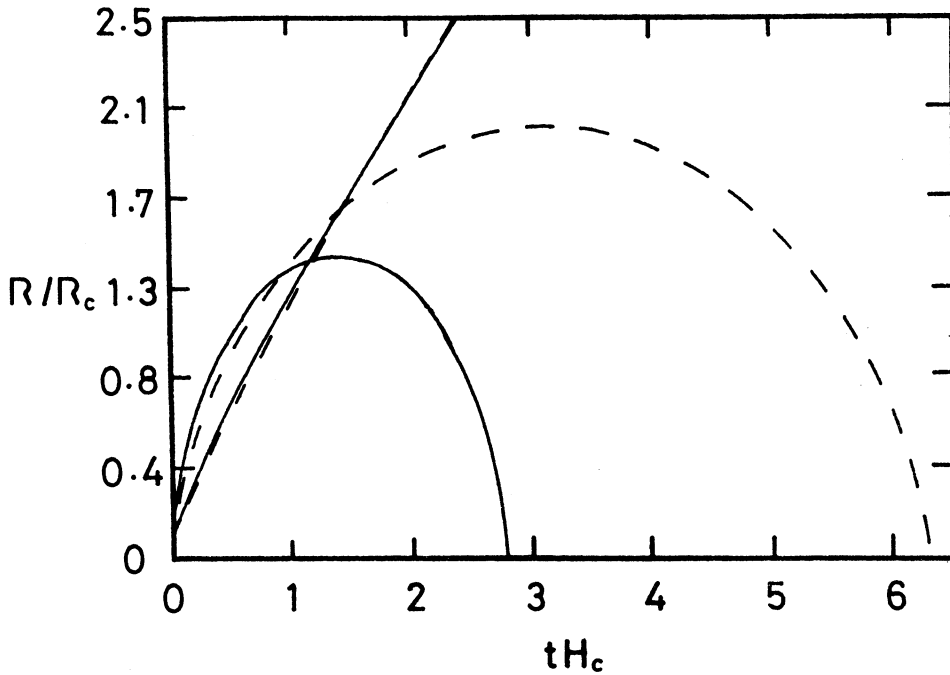


Fig. 1. The scale factor as a function of time for the parameters: $\epsilon = 0.1$, $n = -0.9$, $f_o = 10^{-10}$, $H_o = 50$ km per megaparsec. Open models have $\sigma_{mo} = 0.1$. Closed models have $\sigma_{mo} = 1.0$. The standard Friedmann dust models are depicted with a dashed line.

V. NUMERICAL CALCULATION AND CONCLUSIONS

To carry out the numerical integrations and calculations we use the cosmological parameters

$$\sigma_{mo} = \frac{4\pi G}{3H_o^2} \rho_{mo}, \quad H_o = \frac{\dot{R}_o}{R_o} \quad \text{and} \quad f_o = -(\ln \rho_m R^3)'|_o \quad (10)$$

here f_o is the present fractional conversion rate of matter into radiation. These parameters are in the range: KV

$$30 \leq H_o \leq 100 \quad (\text{km per megaparsec})$$

$$.1 \leq \sigma_{mo} \leq 1$$

$$10^{-12} \leq f_o \leq 10^{-8} \quad (\text{year}^{-1}),$$

(11)

while polytropic constants are in the range

$$-1 < n < 0, \quad 0 < \varepsilon < 1/3, \quad (12)$$

where ε is a constant defined in terms of α and β .

With these values we derive the range of other cosmological parameters such as the actual gravitational radiation energy density σ_{ro} and the actual deceleration parameter q_o :

$$\sigma_{ro} = \frac{4\pi G}{3H_o^2} \rho_{ro} \quad \text{and} \quad q_o = -\frac{\ddot{R}_o R_o}{\dot{R}_o^2}, \quad (13)$$

that in terms of our quantities are given by

$$\sigma_{ro} = \frac{n\sigma_{mo}}{8} \left[1 - \frac{3}{n} - \frac{f_o}{H_o} - \sqrt{\left(1 - \frac{3}{n} - \frac{f_o}{H_o} \right)^2 - \frac{16}{n} \frac{f_o}{H_o}} \right] \quad \text{and} \quad (14)$$

$$q_o = \sigma_{mo} + 2\sigma_{ro} \quad (15)$$

Furthermore, to compare the magnitude of today quantities with those at the junction point, we have calculated the ratio of cosmological parameters evaluated at $t = t_o$ with those at $t = t_c$ (see Chauvet, Cervantes-Cota and Klapp 1990b).

From Table (1) one can see that the present density of gravitational radiation, σ_{ro} , for n fixed, is an increasing function of f_o . On the other hand, σ_{ro} also increases when $n \rightarrow -1$. Since the gravitational radiation contribution to the deceleration parameter is twice that of matter (see equation 15), q_o increases when f_o increases or when $n \rightarrow -1$.

We demand $(R_o/R_c) < 1$ because we assume that nowadays gravitational waves are still being produced within the galaxies. We note that (R_o/R_c) diminishes with decreasing f_o or when $n \rightarrow -1$. As expected, from table (1)

Table 1. Cosmological parameters σ_{ro} , q_o , R_o/R_c and H_o/H_c as a function of n and f_o for $\varepsilon = 0.1$, $\sigma_{mo} = 0.1$ and $H_o = 50$ km per megaparsec.

n	f_o	σ_{ro}	q_o	R_o/R_c	H_o/H_c
-0.9	10E-09	4.30E-1	0.96	1.29	3.99
-0.9	10E-10	4.36E-2	0.19	1.00	0.99
-0.9	10E-11	4.39E-3	0.11	0.57	1.94
-0.9	10E-12	4.39E-4	0.10	0.29	3.86
-0.5	10E-09	2.47E-1	0.59	1.13	2.03
-0.5	10E-10	2.64E-2	0.15	0.95	1.09
-0.5	10E-11	2.71E-3	0.11	0.68	1.61
-0.5	10E-12	2.72E-4	0.10	0.46	2.39
-0.1	10E-09	5.56E-2	0.21	1.01	0.99
-0.1	10E-10	6.04E-3	0.11	0.95	1.08
-0.1	10E-11	6.13E-4	0.10	0.88	1.18
-0.1	10E-12	6.14E-5	0.10	0.82	1.29
-0.01	10E-08	5.65E-2	0.21	1.00	0.99
-0.01	10E-09	6.20E-3	0.11	0.99	1.01
-0.01	10E-10	6.31E-4	0.10	0.99	1.02
-0.01	10E-11	6.32E-5	0.10	0.98	1.03
-0.01	10E-12	6.32E-6	0.10	0.97	1.04

Our model predicts that the smaller the present density parameter σ_{r_0} is, the later in time the matter and gravitational radiation can be decoupled.

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