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NEWTONIAN VERSION OF THE VARIABLE MASS THEORY OF GRAVITY

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RESUMEN. Se presenta una versión Newtoniana de los modelos cosmológicos espacialmente homogéneos e isotrópicos con masa variable. La influencia de la variación de masa en la evolución de la función de escala está establecida para el caso de un Universo lleno de polvo bajo la suposición de que esta variación es un efecto estrictamente cosmológico. Se muestra que el carácter hiperbólico, parabólico o elíptico del movimiento de fluído puede ser modificado a lo largo de la expansión.

ABSTRACT. This paper presents a Newtonian version of the spatially homogeneous and isotropic cosmological models with variable mass. The influence of the mass variation on the evolution of the scale function is established for the case of a dust-filled Universe under the assumption that this variation is a strict cosmological effect. It is shown that the hyperbolic, parabolic or elliptic character of the fluid motion can be modified along the expansion.

Key words: COSMOLOGY

I - Introduction

Different theories of gravity, alternatives to Einstein's general theory of relativity, have been proposed in which either the gravitational constant G and/or the rest masses of the objects vary with time. Recently, Wesson (1983, 1984) discussed the difficulties encountered by these different approaches and proposed a variable mass theory of gravity where the mass is regarded as a geometrical coordinate in a continuum 5D space-time-mass. In some sense, the usual 4D Einstein's theory would be embedded in it. McCrea (1978) pointed out that in a theory with time dependent gravitation it seems more plausible to let the mass change with time rather than the gravitational constant G, since m is the source of the field. This requirement is fulfilled by Wesson's theory.

Here we intend to investigate cosmological models generated by the newtonian theory of gravity with variable mass as suggested by McCrea. In principle, one should expect to find cosmological models which are close to those furnished by the correct relativistic theory with variable mass, as it happens with Einstein's and the standard newtonian theory.

II - Newtonian Models with Variable Mass

We shall follow the same procedure used by McCrea and Milne (1934) for constant mass but considering that in the present case the mass is a function of time. Of course, it will be necessary to modify the continuity equation. We consider an homogeneous and isotropic pressureless material medium (dust). In this fluid, let us imagine a particle with velocity v at a distance r from an observer. If its mass is m and ρ is the density of the medium, the particle equation of motion ($\vec{F} = d\vec{p}/dt$) reads

$$\frac{1}{m}\frac{dm}{dt} v + \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{4}{3}\pi G \rho r \qquad (1)$$

We suppose that m and ρ are functions of time t alone. Since the mass is no longer constant but varies with time, the continuity equation takes the form of a balance equation with a source term

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v) = \frac{1}{h}\frac{dh}{dt} \qquad (2)$$

where h is a function of t. Since we assume homogeneity each term of Equation (2) is independent of r. In particular, the second term

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v) = \frac{1}{h}\frac{dh}{dt} - \frac{1}{\rho}\frac{d\rho}{dt} = 3F(t) \qquad (3)$$

can be integrated with respect to t and leads to

$$v(r,t) = r F(t) . \qquad (4)$$

Equation (1) then becomes

$$\dot{F} + F^2 + \frac{\dot{m}}{m} F = -\frac{4}{3}\pi G \rho , \qquad (5)$$

Integrating (4) we obtain $r = r_o R(t)/R_o$, where r_o is the radius for R(t)=R and R(t) satisfy

$$\frac{\dot{R}}{R} = F(t) . \qquad (6)$$

Using (3) we have

$$\frac{\dot{R}}{R} = \frac{1}{3}\frac{\dot{h}}{h} - \frac{1}{3}\frac{\dot{\rho}}{\rho} , \qquad (7)$$

which after integration gives the following expression for the density

$$\rho = \rho_o \left[\frac{R_o}{R}\right]^3 \frac{h}{h_o} . \qquad (8)$$

On the other hand, since the mass inside the sphere of radius r is M = 4/3 $\pi \rho r^3$, we can write, by using Equations (6) and (8),

$$M(t) = M(t_o) \frac{h}{h_o} . \qquad (9)$$

Thus, if $h(t)$ varies with time, the mass in a comoving volume is not constant.

Now we examine the functional form of $h(t)$. The mass should change very slowly with time so that $(dh/dt)/h \le 10^{-10}$ yr^{-1} as suggested by Wesson (1983). Also, we make the assumption that the variation of mass is a cosmological effect and put $\dot{h}/h$ proportional to the relevant time scale of the problem, that is, the Hubble time:

$$\frac{\dot{h}}{h} = \alpha \frac{\dot{R}}{R} \qquad (10)$$

Integrating this we have

$$h(t) = h_o \left[\frac{R}{R_o}\right]^{\alpha} \qquad (11)$$

Inserting the above expression, together with Equation (6) and (9) into (5) we obtain

$$R\ddot{R} + \alpha \dot{R}^2 + A R^{\alpha-1} = 0 \qquad (12)$$

with the constant $A = 4\pi G\rho_o R_o^2/3$.

One can compare (12) with the equation found by McCrea and Milne (1934) for constant mass

$$R\ddot{R} + A R^{-1} = 0 \qquad (13)$$

We see that Equation (12) is identical to (13) if $\alpha = 0$, which corresponds to the limit of constant mass.

The first integral of (12) is given by

$$\dot{R}^2 + k R^{-2\alpha} = \frac{2A}{(1-3\alpha)} R^{\alpha-1} \qquad (14)$$

if $\alpha \neq 1/3$, and

$$\dot{R}^2 + k R^{-2/3} = A R^{-2/3} \ln(1/R) \qquad (15)$$

if $\alpha = 1/3$. Here k is an arbitrary constant. As may be easily checked, if $\alpha = 0$, Equation (14) reduces to the first integral of (13).

Particularly, if we take $\alpha < 1/3$ and $k=0$, Equation (14) reduces to

$$\dot{R}^2 = \frac{2A}{(1-3\alpha)} R^{\alpha-1}$$

and therefore

$$R = \left[\frac{(3-\alpha)^2 A}{2(1-3\alpha)}\right]^{\frac{1}{3-\alpha}} t^{\frac{2}{3-\alpha}}$$

For $\alpha=0$, that is, no mass variation, this gives the standard result $R \propto t^{2/3}$ for the parabolic dust model.

It is remarkable that for this model the deceleration parameter is given by

$$q = -\frac{\ddot{R}\,R}{\dot{R}^2} = \frac{(1-\alpha)}{2}$$

Thus, if the mass decreases in the course of time, we can take, for instance, $\alpha=-1$ to obtain $q=1$. This is just about the present value of the deceleration parameter furnished by the luminosity distance versus red-shift relation. In what follows we discuss the physical meaning of the above equations. For simplicity we consider only the case $\alpha\neq 1/3$.

III - Discussion and Conclusion

Let us compute the escape velocity of a test particle with variable rest mass. Combining Equations (6) and (14) we can write the kinetic energy of the particle by unit mass as

$$\frac{1}{2}\,v^2 = \frac{1}{2}\,r^2\,F(t)^2 = \frac{G\,M}{r} + E(t) \qquad (16)$$

where

$$E(t) = -\frac{1}{2}\,k\,R^{-2\alpha} + \frac{3\,\alpha\,A'}{(1-3\alpha)}\,R^{\alpha-1} \qquad (17)$$

and

$$A' = A\left(\frac{r_o}{R_o}\right)^2 = \frac{G\,M_o}{r_o} \qquad (18)$$

The positive constant A' is the same for all particles of the fluid. The integration constant k may assume negative, null or positive values. It is just minus twice the constant value of the total energy of the particle for $\alpha=0$. In this particular case if k is greater, smaller or equals zero it follows that v is smaller, greater or equals to the escape velocity. For $\alpha\neq 0$ such an interpretation does not remain valid since the effective total energy in (17) is now a time dependent function. As a matter of fact, in the framework of classical mechanics, the sum of kinetic and potential energy is constant only if the forces acting in the particle are conservative. Of course this is no longer the case since the mass is an explicit function of time. As a consequence, in the standard model ($\alpha=0$), there is no inversion of motion (recolapse) if $k\leq 0$ (parabolic or hyperbolic cases). The models will expand forever. However, for α non-zero and greater than 1/3 and $k<0$, there is a finite value $R^*(R_o)$ for which $\dot{R}(R^*)=0$ and $\ddot{R}<0$ (see (14) and (12)). Thus, in these cases we must have a recolapse of the fluid system. This result may be interpreted by saying that the growth of mass compels the test particle to diminish its velocity below to the instantaneous escape velocity.

Note also that if k=0, Equation (14) shows that the solutions are expansionist for all values of α equal or less than 1/3. In this case, we have from (17) that the net energy may be positive if $0<\alpha<1/3$ or even negative if $\alpha<0$. However, the "open" character of the models may be easily understood from (17) if one takes into account that the energy goes to zero for large value of the cosmological time, independently of the finite value assumed by the parameter α.

The analogy between the solutions presented here and a full relativistic model with variable mass could, in principle, be anticipated by deriving a "Friedmann type equation". For that, we add Equation (12) with one half of (14) to obtain

$$R\ddot{R} + \left(\alpha + \frac{1}{2}\right)\dot{R}^2 + \frac{1}{2}kR^{-2\alpha} - \frac{3\alpha A}{(1-3\alpha)}R^{\alpha-1} = 0\,, \qquad (19)$$

$$R\ddot{R} + \left(\alpha + \frac{1}{2}\right)\dot{R}^2 + \frac{1}{2}k^* = 0 \qquad (20)$$

where the effective curvature parameter k^* is nothing but minus twice the total energy of a fluid particle (see Equation (17)). Equation (20) has the same appearance of the Friedmann differential equation obtained from the General Relativity. Apart from the α factor multiplying $\dot{R}^2$ the main difference is that the curvature parameter is now a time dependent quantity.

Note that if $\alpha=0$, then k^* reduces to the constant value k of the usual Friedmann model. This result indicates that in a consistent cosmological relativistic model with variable mass, the curvature parameter must be a function of time. Accordingly, the open, close or flat character of the models must vary along the cosmic evolution.

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