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THE METHOD OF EICHHORN WITH NON-STANDARD PROJECTIONS FOR A SINGLE PLATE

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RESUMEN. Se desarrollan las expresiones para el método de Eichhorn en astrometría para proyecciones diferentes a la estandar. El método se usa para obtener las coordenadas esféricas de estrellas en placas astronómicas cuando las variables contienen errores.

ABSTRACT. We develop the expressions for the Eichhorn's Method in astrometry for non-standard projections. The method is used to obtain spherical coordinates of stars in astronomical plates, when all the variables have errors.

Key words: ASTROMETRY

I. INTRODUCTION.

Eichhorn (1985) has developed a method in focal plane astrometry to obtain the spherical coordinates of stars, that uses the standard coordinates (Tan Proyection) as auxiliary quantities only. A great advantage of this method over the older methods is that the positions of the reference stars are considered as observations having errors.

For work with Schmidt telescopes or in radio astronomy one needs to consider other types of proyection as are the SIN and ARC proyections (Greisen, 1984). Therefore, we have developed the necessary formulae for the method of Eichhorn for these two proyections.

II CONDITION EQUATIONS

We consider that there are m reference stars with position estimates $((\alpha_{\nu c},\delta_{\nu c}),\nu=1,\ldots,m)$ from a catalogue,with variances $\bar{\rho}_\nu/\cos^2\delta_\nu$ and $\bar{\sigma}_\nu$ respectively (we are following the notation of Eichhorn, 1985). The measured coordinates on the plate for the νth star are X_ν and Y_ν with variances υ_ν and Φ_ν respectively. The dimensionaless coordinates (SIN or ARC) of the stars written in units of the focal length, are ζ_ν and H_ν. For each ν the measured and dimensionless coordinates are given by the model

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$$\begin{pmatrix} X_\nu \\ Y_\nu \end{pmatrix} = s \begin{pmatrix} \xi_\nu \\ \eta_\nu \end{pmatrix} + \Xi_\nu \underline{a} \qquad (1)$$

where $\underline{a}$ is the vector of plate constants, Ξ is the model matrix and s the focal length of the telescope.

The measured positions X_ν, Y_ν and the estimated coordinates$(\alpha_{\nu c},\delta_{\nu c})$ produce the condition equation $H^T = (F^T, G^T)$ where F and G are defined by Eichhorn 1985 as his equations (2) and (3), with covariance matrix

$$\sigma = \mathrm{diag}\ (\upsilon_1,\ \phi_1, \qquad ,\upsilon_m,\ \phi_m;\ \bar{\rho}_1, \bar{\sigma}_1, \ldots\ldots, \bar{\rho}_m,\ \bar{\sigma}_m) \qquad (2)$$

with

$$\sigma_{x\nu} = \begin{pmatrix} \upsilon_\nu & o \\ o & \phi_\nu \end{pmatrix} \quad \text{and } \bar{\sigma}_{a\nu} = \begin{pmatrix} \rho_\nu & o \\ o & \bar{\sigma}_\nu \end{pmatrix} \qquad (3)$$

and the residual vector is given by $H_o^T = (d^T, o)$. The adjustment parameters are the plate constants $\underline{a}$ and the correction to the positions.

$$\beta^T = (\cos\delta_1 d\alpha_1, d\delta_1, \ldots\ldots, \cos\delta_m d\alpha_m, d\delta_m)$$

III. NORMAL EQUATIONS

Eichhorn has derived the solution to the normal equations forthe corrections as

$$\underline{a} = \left[\sum_{\nu=1}^{m} \Xi_\nu^T J_\nu \Xi_\nu \right]^{-1} \sum_{\nu=1}^{m} \Xi_\nu^T J_\nu d_\nu \qquad (4)$$

and

$$\beta = \left(s^2 B \sigma_x^{T-1} B + \sigma_a^{-1} \right)^{-1} s B^T \sigma^{-1} \left(d - \Xi \underline{a} \right) \qquad (5)$$

where the weight matrix is given by

$$J_\nu = \left(\sigma_x + s^2 B \bar{\sigma}_a B^T \right)^{-1} \qquad (6)$$

and B is the blockdiagonal matrix whose 2x2 elementary block is

$$B_\nu = \left(\frac{\partial\ \xi_\nu, \eta_\nu}{\partial\ \alpha_\nu, \delta_\nu} \right) \begin{pmatrix} 1/\cos\delta_\nu & o \\ o & 1 \end{pmatrix} \qquad (7)$$

IV. COMPUTATION OF THE MATRIX B.

Our problem then reduces to the computation of the matrix B in order to obtain the weight matrix. To do that, we start considering the orthogonal matrix

$$R_1 \ -\delta_o \ R_2 \ \alpha-\alpha_o \ R_1 \ \delta = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \quad (8)$$

where (α_o,δ_o) are the spherical coordinates of the center of the plate. We evaluate B first for the SIN proyection and then for the ARC one.

a) For the SIN proyection the transformation equations are

$$\xi = \cos\delta \sin(\alpha - \alpha_o)$$

$$\eta = \sin\delta \cos\delta_o - \cos\delta \sin\delta_o \cos(\alpha-\alpha_o) \quad (9)$$

from equation (8) we have $\xi=-a_3$ and $\eta=b_3$, using the orthogonality of the matrix we obtain that

$$B = \begin{pmatrix} a_1 & -a_2 \\ -b_1 & b_2 \end{pmatrix} \quad (10)$$

and from the inverse transformations

$$\sin\delta=\eta\cos\delta_o+\sin\delta_o\sqrt{1-\xi^2-\eta^2}$$

$$\tan \alpha-\alpha_o = \frac{\xi}{\cos\delta_o\sqrt{1-\xi^2-\eta^2}-\eta\cos\delta_o} \quad (11)$$

we obtain

$$a_1 = \cos\delta_o\sqrt{1-\xi^2-\eta^2}-\eta\sin\delta_o /_T = W$$

$$a_2 = \xi \ \eta\cos\delta_o+\sin\delta_o+\sin\delta_o \sqrt{1-\xi^2-\eta^2} /T=S$$

$$b_1=-\xi \sin\delta_o/T = U$$

$$b_2= \xi^2 \cos\delta_0/T + W = V \quad (12)$$

where

$$T^2 = \xi^2 + (\cos\delta_0 + [1- \xi^2 - \eta^2 - \eta \sin\delta_0]^{1/2}) \quad (13)$$

and the determinant

$$R^3=SU+WV=1 \quad (14)$$

therefore

$$B= \begin{pmatrix} W & - S \\ U & V \end{pmatrix} \quad (15)$$

is of the same form as equation (32) of Eichhorn, and from here one can use the formalism of Eichhorn.

b) For the ARC projection the transformation equations are

$$\xi = -\frac{\theta}{\sin\theta} \cos\delta \sin(\alpha-\alpha_o)$$

$$\eta = \frac{\theta}{\sin\theta} (\cos\delta_o \sin\delta - \sin\delta_o \cos\delta (\alpha-\alpha_o)) \qquad (16)$$

from equation (8) we have $\xi = -\left(\frac{\theta}{\sin\theta}\right)a_3$, $\eta = \left(\frac{\theta}{\sin\theta}\right)b_3$.

Using the same procedure as with the SIN proyection one obtains for the elements of the B matrix, in terms of the orthogonal matrix (8), the following

$$W = -\frac{1}{\sin^2\theta}\left(c_1a_3 + \frac{\theta}{\sin\theta} c_2b_3\right),$$

$$V = -\frac{1}{\sin^2\theta}\left(c_2b_3 + \frac{\theta}{\sin\theta} c_1a_3\right),$$

$$S = \frac{1}{\sin^2\theta}\left(\frac{\theta}{\sin\theta} c_1b_3 - c_2a_3\right),$$

$$U = \frac{1}{\sin^2\theta}\left(c_1b_3 - \frac{\theta}{\sin\theta} c_3a_3\right); \qquad (17)$$

and the determinant of B is given as

$$R^3 = SU + WV = \frac{\theta}{\sin\theta} . \qquad (18)$$

The expression in (17) can be evaluated using the inverse transformations.

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$$|\theta|^2 = \xi^2 + \eta^2$$

$$\sin\delta = \left(\frac{\sin\theta}{\theta}\right)\eta \cos\delta_o + \sin\delta_o \cos\theta$$

$$\sin\alpha-\alpha_o = \left(\frac{\sin\theta}{\theta}\right)\frac{\xi}{\cos\delta} \qquad (19)$$

and the elements of the orthogonal matrix (8) to produce

$$a_3 = -\left(\frac{\sin\theta}{\theta}\right)\xi, \quad b_3 = \left(\frac{\sin\theta}{\theta}\right)\eta, \quad c_2 = \cos\theta$$

$$c_1 = \cos\delta_o\left[\left(\frac{\sin\theta}{\theta}\right)\eta \cos\delta_o + \sin\delta_o \cos\theta\right] \qquad (20)$$

$$c_2 = \left[\sin\delta_o - \left[\left(\frac{\sin\theta}{\theta}\right)\eta\cos\delta_o + \sin\delta_o\cos\theta\right]\right] \Big/ \sqrt{1-\left[\left(\frac{\sin\theta}{\theta}\right)\eta\cos\delta_o + \sin\delta_o\cos\theta\right]^2}$$

and from here again one can follow the procedure of Eichhorn.

V. CONCLUSIONS

In this work we have shown that in order to apply the method of Eichhorn in Astrometry for the SIN and ARC proyections it was only necessary to change the B matrix contained in the weight matrix. The equations for the requiered changes were derived.

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