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PULSATION, MASS LOSS AND THE UPPER MASS LIMIT

Jaime Klapp

Instituto Nacional de Investigaciones Nucleares and
Departamento de Física, Universidad Autónoma Metropolitana-Iztapalapa

Manuel G. Corona-Galindo[1]

Centro de Investigación en Física, Universidad de Sonora

RESUMEN. La existencia de estrellas con masas en exceso de $\sim$ 100 $M_\odot$ ha sido cuestionada por mucho tiempo. Límites superiores para la masa de $\sim$ 100 $M_\odot$ han sido obtenidos de teorías de pulsación y formación estelar. En este trabajo nosotros primero investigamos la estabilidad radial de estrellas masivas utilizando la aproximación clásica cuasiadiabática de Ledoux, la aproximación cuasiadiábatica de Castor y un cálculo completamente no-adiabático. Hemos encontrado que los tres métodos de cálculo dan resultados similares siempre y cuando una pequeña región de las capas externas de la estrella sea despreciada para la aproximación clásica. La masa crítica para estabilidad de estrellas masivas ha sido encontrada en acuerdo a trabajos anteriores. Explicamos la discrepancia entre este y trabajos anteriores por uno de los autores. Discutimos cálculos no-lineales y pérdida de masa con respecto al límite superior de masa.

ABSTRACT. The existence of stars with masses in excess of $\sim$ 100 $M_\odot$ has been questioned for a very long time. Upper mass limits of $\sim$ 100 $M_\odot$ have been obtained from pulsation and star formation theories. In this work we first investigate the radial stability of massive stars using the classical Ledoux's quasiadiabatic approximation, the Castor quasiadiabatic approximation and a fully nonadiabatic calculation. We have found that the three methods of calculation give similar results provided that a small region in the outer layers of the star be neglected for the classical approximation. The critical mass for stability of massive stars is found to be in agreement with previous work. We explain the reason for the discrepancy between this and previous work by one of the authors. We discuss non-linear calculations and mass loss with regard to the upper mass limit.

Key words: **STARS-MASS FUNCTION – STARS-MASS LOSS – STARS-PULSATION**

In recent years growing attention has been paid to the potentially important role played by very massive I > 100 $M_\odot$) stars in various areas of astrophysics. The potential importance of very massive stars remains in contrast the fact that the feasibility of these stars both from the point of view of their formation as of their stability against sruptive processes still faces many uncertainties. The theoretical results for the upper mass limit of star formation ffer widely from 60 $M_\odot$ to several 100 $M_\odot$.

The existence of stars with masses in excess of $\sim$ 100 $M_\odot$ has been questioned on grounds of their ulsational instability alone (Ledoux 1941 and Schwarzschild and Härm 1959). According to these results, in such ars the nuclear energizing of pulsations in the stellar core - the ϵ-mechanism - overcomes the damping effect of the ıvelope. The resulting pulsations are believed to eventually disrupt the stars or cause strong mass loss which reduces ıeir lifetimes effectively. In order to investigate this possibility, direct nonlinear calculations have been performed by :veral workers. Osaki (1966) suggested that the amplitude of oscillation could be limited by mass loss if the gain of echanical energy of pulsation is all lost in the form of mass ejection from the surface. He estimated that the time scale ‚er which most of the star's mass is lost is comparable to the Helmholtz - Kelvin time scale. The main criticism to his ork is that he assumes that mass loss is the only dissipation mechanism.

[1] On sabbatical leave from Instituto Nacional de Astrofísica, Optica y Electrónica, Puebla, México.

Papaloizou (1973*a,b*) has studied non-linear pulsations of 70-170 $M_\odot$ stars. He found that resonances cause systematic modulation of the velocity curve and showed that the oscillation amplitude is probably limited to fairly low values. The hydrodynamical calculation of Ziebarth (1970) showed that stars above the stability limit could reach a limiting amplitude before mass loss occurs (see, however Appenzeller 1970*a,b*). On the background of these investigations the existence of high-mass stars beyond 100 $M_\odot$ has been reconciled with their being unstable linearly and in some cases nonlinearly by referring to the limiting effect of the outer layers which might permit finite lifetimes depending on the presence and intensity of mass-loss associated with the oscillations. The significance of the existing nonlinear results for the fate of VMS remains, however, uncertain. In particular, it is not clear that the linear instability of VMS implies that these stars cannot exist. The non-linear calculations we have described suggests that non-linear effects can limit the linear instabilities.

In a previous work by one of the authors (Klapp, Langer and Fricke 1988, hereafter referred to as Paper I) we obtained that the fundamental mode becomes unstable at a mass greater than the accepted value of $\sim$ 100 $M_\odot$. The results obtained for the overtones were similar to those reported by other authors. In this work we reinvestigate the linear radial stability of massive stars using the three most common methods of calculation, which are the classical Ledoux's quasiadiabatic approximation, the Castor quasiadiabatic approximation and a fully nonadiabatic calculation. Part of the motivation for this work is to understand the discrepancy between our previous result and those obtained by other authors.

For the linear stability analysis we have used a modified version of the Los Alamos Linear Nonadiabatic code (Pesnell 1983). In Castor's (1971) notation we can write the linearized momentum and energy equations as

$$\omega^2 X = G_1 X + G_2 Y, \tag{1}$$

$$i\omega Y = K_1 X + K_2 Y, \tag{2}$$

where $X = (X_i, i = 2, N+1)$, $Y = (Y_i, i = 1, N)$, N is the number of zones in the star and the X_i and Y_i are defined by $X_i = (DM2_i)^{1/2}\delta r_i$, $Y_i = T_i \delta S_i$, δr_i and δS_i are the radius and entropy perturbations and $DM2_i$ is the mass of zone i. From Castor 1971 the quasiadiabatic eigenfrequency ω is given by

$$\omega^2 - \omega_0^2 = \frac{\int_0^{M_*} (\delta r_i^T G_2 (i\omega_0 I - K_2)^{-1} K_1 \delta r_i) dm}{\int_0^{M_*} (\delta r_i)^2 dm}, \tag{3}$$

where ω_0 and δr_i are the adiabatic eigenfrequency and radial eigenvector, I is the identity matrix and T denotes the transpose operation. We shall call this the Castor quasiadiabatic approximation (CQAD approximation).

Ledoux's (1941) quasiadiabatic approximation (LQAD approximation) for the imaginary part of the eigenfrequency is calculated using the well-known expression

$$Im(\omega) = \frac{-\frac{1}{2}\int_0^{M_*} \frac{\delta T}{T} (\delta\epsilon - \frac{d\delta L}{dm}) dm}{\omega_0^2 \int_0^{M_*} (\delta r_i)^2 dm}. \tag{4}$$

The fully nonadiabatic calculation (NAD) is obtained by solving equations (1) and (2) (see Castor 1971 for details of the method).

For our computations, the equilibrium models were constructed in two different ways. The first with the Göttingen stellar evolution code (Langer, El Eid and Fricke 1985) and the second by an inward only integration for a given set of surface parameters (mass, effective temperature, luminosity and composition), which is the usual method of construction of the equilibrium model in the Los Alamos code. In a forthcoming communication we shall describe the advantages and disadvantages of each method in relation to the linear stability problem.

In Table 1 we show the physical parameters for the stellar models using the Göttingen stellar evolution code. The physical parameters for the second method are very similar to the ones shown in the table.

The linear stability results that we now describe have been obtained with the second set of models (for a complete description of the results see Klapp and Corona-Galindo 1990a). In general, we have found reasonable agreement between the three methods of calculation provided that the contribution from the outermost nonadiabatic layers be neglected for the LQAD approximation. The LQAD values depend somewhat on the cutoff assumption introduced in the evaluation of the damping integral (eq. (4)). Authors differ widely on the cutoff assumption: Ledoux's (1941) cutoff is at a temperature $T = 6T_{eff}$. Stothers and Simon (1970) neglected the nonadiabatic layers entirely for the hydrogen-burning models. Maeder (1985), for hydrogen-burning models, and Noels and Masereel (1982), for helium-burning models, introduced the cutoff at a point where the nonadiabatic terms are 10% of the adiabatic ones. For the evaluation of our LQAD values we have adopted an approach similar to that of Maeder (1985) and Noels and Masereel (1982).

TABLE 1[a]. Physical Parameters for the Stellar Models.

$M(M_\odot)$	$Log\ \frac{L}{L_\odot}$	$Log\ T_{eff}$	$\frac{R}{R_\odot}$	$Log\ \rho_c$	$Log\ T_c$	q_{cc}
130	6.298	4.747	15.63	0.165	7.644	0.862
208	6.610	4.761	20.26	0.040	7.651	0.911
300	6.828	4.772	24.81	−0.034	7.661	0.935
400	6.991	4.778	29.07	−0.091	7.668	0.952
500	7.113	4.782	32.86	−0.135	7.673	0.962
750	7.327	4.787	41.02	−0.213	7.681	0.974
1000	7.473	4.790	47.80	−0.267	7.687	0.979
2000	7.813	4.796	68.95	−0.391	7.700	0.989
5000	8.244	4.801	110.45	−0.564	7.716	0.903

a. For all models the composition is (X,Z) = (0.687,0.043); qcc is the mass fraction of the convective core, and the index c designates central quantities. The remaining symbols have their usual meaning.

In Table 2 we show the periods and ϵ-folding times for the first three modes. All numbers correspond to the nonadiabatic values. The differences between the LQAD approximation and the nonadiabatic values are always less than 10% and in some cases even less than 5%. The differences between the CQAD approximation and the nonadiabatic values are always less than 1%. The CQAD approximation is so good that unless we require the nonadiabatic eigenvectors there is no real need for calculating the fully nonadiabatic eigenvalues. The real advantage of the CQAD approximation is that there is no need for introducing cutoff assumptions.

The values obtained for the imaginary part of the eigenfrequency of the fundamental mode are similar to those obtained by other authors. The fundamental mode becomes unstable at about 100 $M_\odot$. The somewhat larger value obtained for the critical mass in paper I was due to slight differences between the stellar structure code and the pulsation code which produced enhanced damping of the outer layers (or too little driving from the stellar core) pushing up the critical mass to a few hundred solar masses (see Klapp and Corona-Galindo 1990*a* for details). This suggest that although the fundamental mode becomes unstable at about 100 $M_\odot$, it's instability is rather marginal. Effects such as running waves at the surface that may destroy the perfect reflection condition could have some effect on the stability of these stars.

Despite the fact that stars are linearly unstable for masses that are high enough, more work is required to settle the question of whether pulsation reaches a limiting amplitude or if the star is completely disrupted.

It has been suggested by several authors that the non-existence of stars with masses >>100 $M_\odot$ could be the effect of mass loss. The mass loss mechanism in early type stars is not completely understood which makes it neccesary to rely on observations. Although there has been a great number of observations,the mass loss rate has been accurately estimated only for stars with high mass loss rates. By analysing the infrared excess of 34 OBA supergiants and 10 Of and Oe stars, Barlow and Cohen (1977) have found that the mass loss rate is related to the luminosity by the equation

$$\dot{M} = aL^b, \tag{5}$$

TABLE 2[a]. Periods P_0, P_1, P_2 and ϵ-folding Times τ_0, τ_1, τ_2, for the Fundamental Mode, First and Second Overtones, Respectively.

$M(M_\odot)$	P_0	P_1	P_2	$\frac{\tau_0}{10^{10}}$	$\frac{\tau_1}{10^6}$	$\frac{\tau_2}{10^6}$
130	0.393	0.178	0.132	−5.6	0.965	4.43
208	0.504	0.211	0.156	−3.2	1.034	4.33
300	0.611	0.241	0.178	−2.3	1.053	4.01
400	0.710	0.268	0.197	−2.1	1.122	3.88
500	0.797	0.289	0.213	−1.9	1.145	3.88
750	0.985	0.334	0.246	−1.8	1.161	3.71
1000	1.165	0.381	0.276	−1.4	1.236	2.74
2000	1.760	0.490	0.350	−1.2	1.355	2.69
5000	4.189	0.698	0.486	−0.6	1.519	2.37

a. Periods are in days and ϵ-folding times in seconds for all modes.

where $a = 5 \times 10^{13}$ $M_\odot$ yr^{-1}, $b = 1.1 \pm 0.06$ or 1.2 ± 0.08 for O stars and B-A supergiants, respectively. L is given in solar luminosities. In Klapp (1982, 1983, 1984) we computed evolutionary sequences for population III VMS with the Barlow and Cohen (1977) empirical law that can be written in the form

$$\dot{M} = N\frac{L}{c^2}, \tag{6}$$

where N is a constant in the range (50–500).

In Table 3 we show the age and fraction of total mass lost toward the end of the main sequence and toward the end of the helium burning phase. The fraction of mass lost depends mainly on N and only increases slightly in the very high mass range. For high enough N (≥ 300) the mass is reduced considerable, in some cases even below ~ 100 $M_\odot$. However, values of N near ~ 100 are more consistent with observations. In this case stars stay in the VMS range even though large amounts of mass are lost. The results obtained for population I and II compositions are very similar with regard to mass loss (Klapp and Corona-Galindo 1990*b*). The reason for this is that our mass loss algorithm depends only on the luminosity and this is rather insensitive upon composition.

Thus, these calculations suggest that mass loss is effective in reducing the mass of the star but unable to inhibit the passage of VMS through the hydrogen and helium burning phases.

TABLE 3[a]. Age and Fracation of Total Mass Lost toward the End of the Main Sequence ($\tau_6^{MS}, (\Delta M/M_i)^{MS}$, Respectively), and Toward the End of the Helium Burning Phase ($\tau_6^{He}, (\Delta M/M_i)^{He}$, Respectively).

$M(M_\odot)$	N	τ_6^{MS}	$\left(\frac{\Delta M}{M_i}\right)^{MS}$	τ_6^{He}	$\left(\frac{\Delta M}{M_i}\right)^{He}$	$M_f(M_\odot)$
500	100	2.33	0.37	2.59	0.41	295
500	300	2.52	0.74	2.90	0.79	105
500	500	2.57	0.89	2.77	0.90	50
1000	100	2.24	0.36	2.48	0.42	580
1000	300	2.27	0.75	2.56	0.80	200
1000	500	2.52	0.92	3.07	0.94	60
10000	100	1.87	0.43	2.12	0.48	5200
10000	300	2.04	0.83	2.27	0.87	1300
10000	500	2.28	0.95	2.36	0.97	300

a. M_f is the mass at the end of the helium burning phase. τ_6 denotes time in units of 10^6 yr.

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Ianuel G. Corona-Galindo: CIF, Universidad de Sonora, Apartado Postal A-088, 83190 Hermosillo, Son., México.
ıime Klapp: Instituto Nacional de Investigaciones Nucleares, UNAM, and Departamento De Física, Universidad Autónoma Metropolitana-Iztapalapa, Apartado Postal 55-534, 09340 México, D.F., México.