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KINEMATIC DYNAMO ACTION IN THE PRESENCE OF A LARGE SCALE VELOCITY

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RESUMEN. Se investiga la influencia de un campo de velocidades de gran escala sobre la acción del dínamo cinemático turbulento. Usando un proceso de expansión, las soluciones se encuentran en el límite del movimiento global y de cizalla pequeño y para números grandes de Reynolds. Se calcula la regeneración magnética hasta un segundo orden en el parámetro de expansión usando células convectivas ciclotrónicas para el campo turbulento de velocidad.

ABSTRACT. The influence a large scale velocity field upon the kinematic turbulent dynamo action is investigated. Using an expansion process, the solutions are found in the limit of small bulk motion and shear, and for large Reynolds number. The magnetic regeneration is calculated up to second order in the expansion parameter using cyclonic convective cells for the turbulent velocity field.

Key words: HYDROMAGNETICS

I - INTRODUCTION

The kinematic theory of turbulent dynamo proposed by Parker (1955) has been able to explain how astrophysical magnetic fields are generated and sustained. The difficulties encountered in solving the dynamo equations however, have led many authors to employ different approximations and simplifications. The most common approximation is the so called "first-order smoothing" approximation and is used on almost every work (Moffatt, 1970, 1974; Lerche, 1971b; Levy, 1978). It neglects terms which containing products of the turbulent component of the velocity and magnetic field.

Frequently two different limits are used: in a high conducting medium the magnetic Reynolds number R_m may be taken as being infinite and the diffusion can be neglected (Parker, 1955, 1970). On the other hand, for a poorly conducting medium, where R_m is low, the field variation is slow and time derivatives are negligible compared with the diffusion term (Moffatt, 1970; Lerche, 1971b). Improvements to these limits have been made by Levy (1978) who calculated the regeneration term of the dynamo equation for finite but high R_m in the particular case of the cyclonic convective eddies whereas Carvalho and Pires (1986) used the same velocity field to calculate a general solution for the turbulent component of the magnetic field which is valid for arbitrary values of the magnetic Reynolds number.

Usually only small scale turbulent velocity field are taken into account. The inclusion of a large scale velocity field makes the dynamo equations quite complex and only when one considers small bulk velocity and shear the mathematical difficulties can be overcome. Some authors (see, e.g., Lerche 1971a, 1971b; Krause and Roberts, 1973) have examined this problem although no concrete example has yet been worked out. Specific calculations of the regeneration term (Levy, 1978, Carvalho and Pires, 1986) often neglect large scale motion.

Here, we study the effect of a small bulk motion on the regeneration term of turbulent dynamo employing the high magnetic Reynolds number approximation. The cyclonic convective turbulent cells of Parker (1970) are used in order to allow comparison with early work.

II - The Effect of Small Bulk Velocities

Suppose a magnetic field $\vec{B}$ and a velocity field $\vec{V}$ wich can be separated into a large scale and a small scale field such that $\vec{B} = \vec{B}_o + \vec{b}$ and $\vec{V} = \vec{V}_o + \vec{v}$, where $\vec{B}_o$ and $\vec{V}_o$ are the large scale component and $\vec{b}$ and $\vec{v}$ the turbulent component of the magnetic and velocity field respectively. The ensemble averages $\langle\vec{b}\rangle$ and $\langle\vec{v}\rangle$ of the random components of the fields vanish and the mean fields $\vec{B}_o$ and $\vec{V}_o$ are essentially constant over the small scale ℓ of variation of $\vec{v}$, although $\vec{V}_o$ may vary on a large scale $\mathcal{L}\gg\ell$. The dynamo equations are

$$\frac{\partial \vec{B}_o}{\partial t} - D \nabla^2 \vec{B}_o = \vec{\nabla} \times (\vec{V}_o \times \vec{B}_o) + \vec{\nabla} \times \langle \vec{v} \times \vec{b} \rangle \qquad (1)$$

$$\frac{\partial \vec{b}}{\partial t} - D \nabla^2 \vec{b} = \vec{\nabla} \times (\vec{V}_o \times \vec{b}) + \vec{\nabla} \times (\vec{v} \times \vec{B}_o) . \qquad (2)$$

where the magnetic diffusivity is $D=c^2/4\pi\sigma$, with σ the electrical conductivity. Here we have used the usual smoothing approximation and neglected the term $\vec{\nabla} \times [(\vec{v} \times \vec{b}) - \langle \vec{v} \times \vec{b} \rangle]$. The solution of Equation (2) for $\vec{b}$ together with $\vec{v}$ can be used to calculate the regeneration term $\langle \vec{v} \times \vec{b} \rangle$ appearing in Equation (1). Let us write Equation (2) in terms of dimensionless variables. We shall use two scales ℓ and $\mathcal{L}$ so that $\vec{r} = \ell\, \vec{r}_1 + \mathcal{L}\, \vec{r}_2$, where $\vec{r}_1$ and $\vec{r}_2$ are dimensionless. The scales ℓ and $\mathcal{L}$ are the characteristic length of v and of $\vec{V}_o$ respectively. We also define the characteristic time τ and velocity u of the turbulent velocity field $\vec{v}$ and write $t = \tau\, t'$, $\vec{v} = u\, \vec{v}'$ and $\vec{V}_o = u\, \vec{V}_o{}'$, while the quantities ℓ, $\mathcal{L}$, u, τ are related through $u \simeq \ell\, \tau^{-1}$ and $\ell \simeq \varepsilon\, \mathcal{L}$. Here ε is a dimensionless parameter and $\varepsilon \ll 1$.

Keeping only terms to second order in ε Equation (2) becomes

$$\frac{\partial \vec{b}}{\partial t'} - \varepsilon\nabla_1^2 \vec{b} - 2\varepsilon^2 \nabla_3^2 \vec{b} = \varepsilon(\vec{V}_o'.\vec{\nabla}_2)\vec{b} + (\vec{V}_o'.\vec{\nabla}_1)\vec{b} + \varepsilon(\vec{b}.\vec{\nabla}_2)\vec{v}' + (\vec{B}_o.\vec{\nabla}_1)\vec{v}' \qquad (3)$$

where we suppose that the fluid is incompressible, that is, $\vec{\nabla}.\vec{V}_o{}'$ and $\vec{\nabla}.\vec{v}$ are equal to zero and $\vec{\nabla}.\vec{B}_o = 0$. The gradients with respect to the variables

$\vec{r}_1'$ and $\vec{r}_2'$ are respectively $\vec{\nabla}_1'$ and $\vec{\nabla}_2'$, and $\nabla_3^{2\prime}$ is given by $\vec{\nabla}_1'.\vec{\nabla}_2'$. In deriving Equation (3) the high magnetic Reynolds number approximation has been used. If the fluid possesses a high electrical conductivity, R_m=uℓ/D is large (small magnetic diffusivity) and of the order of $1/\varepsilon$.

The turbulent component of $\vec{B}$ can be asymptotically expanded near ε=0 to give $\vec{B} = \vec{B}_o + \varepsilon\,\vec{B}_1 + \varepsilon^2\vec{B}_2 + \ldots$. Our analysis is restricted to the small bulk velocity limit. If we suppose that $\vec{V}_o$ is sufficiently small we can make the substitution $\vec{V}_o' = \varepsilon\vec{U}_o'$ in (3). Using the above expression for $\vec{B}$ and equating coefficients of equal powers of ε in (3) we obtain:

$$\frac{\partial \vec{B}_o}{\partial t'} = (\vec{B}_o.\vec{\nabla}_1)\vec{v}' \quad (5a) \qquad \frac{\partial \vec{B}_1}{\partial t'} = \nabla_1^2\,\vec{B}_o + (\vec{U}_o'.\vec{\nabla}_1)\vec{B}_o \quad (5b)$$

$$\frac{\partial \vec{B}_2}{\partial t'} = \nabla_1^2\,\vec{B}_1 + (\vec{U}_o'.\vec{\nabla}_1)\vec{B}_1 + (\vec{U}_o'.\vec{\nabla}_2)\vec{B}_o + (\vec{B}_o.\vec{\nabla}_2)\vec{U}_o' + 2\nabla_3^2\,\vec{B}_o \quad (5c)$$

The turbulent component of $\vec{B}$ can now be calculated up to second order of approximation, as long as expressions for $\vec{v}'$ and $\vec{V}_o'$ are given explicitly.

As one should expect Equation (5a) which corresponds to the infinite conductivity approximation already derived by Parker (1970), is identical to that found by Levy (1978) (his equation 12a for the vector potential). The discussion made by Parker of the effect of velocity shear on the large scale component of the magnetic field does not include the bulk motion in the calculation of the regeneration term. This is justified only in the present case since the large scale velocity is much smaller than the turbulent velocity and $\vec{U}_o$ does not appears in (5a). Equation (5b) corresponds to Levy's first order finite conductivity limit with an extra term accounting for the presence of bulk velocities $(\vec{U}_o'.\vec{\nabla}_1)\vec{B}_o$. Large scale shear does not contribute in first order but only in second order in ε as we can see from Equation (5c). In fact, only the forth term on the right hand side of (5c) contains derivatives of $\vec{U}_o$ with respect to $\vec{r}_2$. Integrating Equation (5) we obtain $\vec{B}$ which is then used in the next section to calculate the regeneration term $\langle\vec{v} \times \vec{B}\rangle$ (Parker, 1970).

III - Results and Conclusion

We use the same form of the turbulent motion adopted by Parker (1970) for cyclonic convective cells. This allows comparison with previous results and to evaluate the effects of large scale motion on the regeneration term. The expression of the cartesian components of $\vec{v}$ are given by Carvalho and Pires (1986) and we do not reproduce them here. The time dependence of each convective eddy as follow: $\vec{v}\,(\vec{r},t) = \vec{v}\,(\vec{r})$ if $0 \le t \le \delta t$ and equals to zero otherwise. Here δt is the average lifetime of the cyclonic turbulent cells.

At this point, in order to proceed the integration of Equation (5), it is necessary to give the explicit form of the spatial dependence of $\vec{V}_o$ which we approximate as follows:

$$\frac{\partial \vec{V}_o}{\partial x_i} = \mathcal{L}^{-1} \frac{\partial \vec{V}_o}{\partial x_i'} = q_i' \mathcal{L}^{-1} \vec{V}_o \qquad i=1,2,3$$

where $x_1=x$, $x_2=y$ and $x_3=z$. Moreover, the null-divergence condition gives $q_x V_x + q_y V_y + q_z V_z = 0$. q_i is a measure of the large scale ($\mathcal{L}$) variation of $\vec{V}_o$ in the direction x_i and is supposed to be constant. Although the above approximation is rather crude, it represents a compromise if one wishes to produce a specific example, and at the same time avoiding rather cumbersome calculations.

The large scale magnetic field is taken to lie in the x-direction. Since the small scale motion has cylindrical symmetry there is no loss of generality in this choice as far as the x-y plane is concerned. Also, we shall only examine the particular case where the large scale velocity is on the x-y plane. The components of the regeneration term $\langle\vec{v}\times\vec{B}\rangle$ are shown below to second order in ε. Order zero:

$$\langle\vec{v}\times\vec{B}_o\rangle_x = -\frac{1}{4}\left[\frac{\pi}{2}\right]^{3/2} \nu\, V_1 V_2 ab\,(1+\alpha)\,\delta t^2\, B_x\ , \quad \langle\vec{v}\times\vec{B}_o\rangle_y = \langle\vec{v}\times\vec{B}_o\rangle_z = 0 \qquad (7)$$

First order: $$\varepsilon\langle\vec{v}\times\vec{B}_1\rangle_x = \left[\frac{\pi}{2}\right]^{3/2} \nu V_1 V_2 D\, \gamma_1\, \frac{b}{a}\, \delta t^3 B_x,\ \langle\vec{v}\times\vec{B}_1\rangle_y = \langle\vec{v}\times\vec{B}_1\rangle_z = 0 \qquad (8)$$

Second order: $$\varepsilon^2\langle\vec{v}\times\vec{B}_2\rangle_x = \frac{1}{2}\left[\frac{\pi}{2}\right]^{3/2} \nu\, V_1 V_2\, B_x \left[-D^2\, \gamma_2\, \frac{1}{ab}\, \delta t^4 + \frac{1}{8}\,\frac{b}{a}\,\delta t^4 \left(3\,V_x^2 + V_y^2 \right) + \frac{1}{12}\, ab\,(1+\alpha)\,\delta t^3\, V_x\, q_x \right]$$

$$\varepsilon^2\langle\vec{v}\times\vec{B}_2\rangle_y = \frac{1}{4}\left[\frac{\pi}{2}\right]^{3/2} \nu\, V_1 V_2 V_x\, B_x \left[\frac{1}{4}\,\frac{b}{a}\,\delta t^4\, V_y + \frac{1}{3}\, ab\,(1+\alpha)\,\delta t^3\, q_y \right]$$

$$\varepsilon^2\langle\vec{v}\times\vec{B}_2\rangle_z = \frac{1}{24}\left[\frac{\pi}{2}\right]^{3/2} \nu\, V_1 V_2 V_x\, B_x\, ab\,(1+\alpha)\,\delta t^3\, q_z \qquad (9)$$

Here ν denotes their rate of occurrence per unit volume and α, β, γ_1 and γ_2 are defined as follow

$$\alpha = 1 - \frac{a^2}{c^2}\ , \ \beta = \frac{a^2}{4b^2}\ , \ \gamma_1 = \frac{1}{3}(1+\alpha)\beta + 1,\ \gamma_2 = -\frac{1}{2}\left(\beta - \frac{1}{\beta}\right)\left(\alpha - 1\right) + \left(1 - \frac{1}{\beta}\right)$$

Equations (7) and (8) show that the results for zero and first order in ε are identical to that obtained by Levy (1978). The bulk motion and shear appears as a second order effect as it can be seen from Equation (9). The above result can be more clearly seen if we simplify Equations

(7)-(9) by making a, b and c equal to ℓ and put, following Parker (1971), $\nu \simeq 1/\ell^3\tau$ and $\ell/\tau \simeq u \simeq V_2$, with $\tau \simeq \delta t$. The regeneration term which is the sum of (7), (8) and (9) assumes the form

$$\langle \vec{v}\times\vec{b}\rangle_x = -\frac{1}{4}\left(\frac{\pi}{2}\right)^{3/2} V_1 B_x \left[1 - \frac{13}{3}\left(\frac{D}{u\,\ell}\right) - \frac{39}{4}\left(\frac{D}{u\,\ell}\right)^2 - \frac{1}{4}\,\frac{3V_x^2 + V_y^2}{u^2} - \frac{1}{3}\,\frac{V_x}{u}\,q_x\ell\right]$$

$$\langle \vec{v}\times\vec{b}\rangle_y = \frac{1}{4}\left(\frac{\pi}{2}\right)^{3/2} V_1 B_x \left[\frac{1}{4}\,\frac{V_x V_y}{u^2} + \frac{1}{3}\,\frac{V_x}{u}\,q_y\ell\right], \quad \langle v\times\vec{b}\rangle_z = \frac{1}{24}\left(\frac{\pi}{2}\right)^{3/2} V_1 B_x \frac{V_x}{u}\,q_z\ell$$

This shows the relative importance of magnetic diffusivity, convection and bulk shear to the dynamo action in the limit considered here. The turbulent dynamo action contribution to the x-component of $\vec{B}$ is entirely due to the bulk velocity and its derivatives as it can be seen by taking the above result into Equation (1). Its effect, although small, can not neglected.

We notice that the conclusion of Lerche (1971b) that, for a small bulk velocity parallel to the field $\vec{B}$, its contribution to the dynamo action is null, is only true to first order of approximation. The present results show that there exist a second order contribution from the bulk motion even when $\vec{V}$ and $\vec{B}$ are parallel. Also, by examining the terms containing the derivatives of $\vec{V}$ ($q_x V_x$ and $q_y V_x$) in (9) we can establish that there will be no dynamo regeneration due to bulk velocity shear if $\vec{V}$ is perpendicular to $\vec{B}$.

In conclusion we can say that if the bulk velocity is of order ε compared to the turbulent velocity or if the shear motion has also spatial scale of order $O(\varepsilon)$ compared to the scale of the turbulent cells, then, their contribution to the regeneration of the large scale magnetic field, for cyclonic convective eddies, is a second order $O(\varepsilon^2)$ effect. On the other hand, if this is the only contribution to the dynamo regeneration of the magnetic field, as the present example shows, it can not be neglected and must be appropriately accounted for.

REFERENCES

Carvalho, J. C. and Pires, N.: 1986, Astrophys. Space Sci. **122**, 193.
Krause, F. and Roberts, P. H.: 1973, Astrophys. J. **181**, 977.
Lerche, I.: 1971a, Astrophys. J. **166**, 627.
Lerche, I.: 1971b, Astrophys. J. **166**, 639.
Levy, E. H.: 1978, Astrophys. J. **220**, 325.
Moffatt, H. K.: 1970, J. Fluid Mech. **41**, 435.
Moffatt, H. K.: 1974, J. Fluid Mech. **65**, 1.
Parker, E. N.: 1955, Astrophys. J. **122**, 293.
Parker, E. N.: 1970, Astrophys. J. **162**, 665.
Parker, E. N.: 1971, Astrophys. J. **163**, 255.

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