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THE EFFECTS OF LINES ON THE MEAN OPACITIES IN NOVAE, SUPERNOVAE, AND ACCRETION DISKS

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RESUMEN. Para la solución de las ecuaciones de hidrodinámica en medios con movimiento diferencial se requiere el cálculo del flujo de radiación, del gradiente de la presión de radiación y del balance energético. Después de discutir brevemente las deficiencias de las aproximaciones que generalmente se usan, sugerimos un método preciso para el cálculo de estas cantidades por medio de tres coeficientes de opacidad media que sustituyen a la media de Rosseland del caso estático. Este enfoque permite incluir consistentemente la importante contribución de las líneas espectrales al campo de radiación.

ABSTRACT. For the solution of the hydrodynamic equations in differentially moving media the calculation of the radiation flux, the gradient of the radiation pressure, and the energy balance is required. After a brief discussion of the shortcomings of the approximations which are frequently made, we suggest a fast method for the accurate calculation of these quantities by means of three mean opacity coefficients which replace the Rosseland mean of the static case. This method allows a consistent inclusion of the important contribution of spectral lines to the radiation field.

Key words: ACCRETION – OPACITIES – STARS: NOVAE – STARS: SUPERNOVAE

1. INTRODUCTION

Modeling of expanding supernova and nova atmospheres and of accretion disks involves considerable numerical effort, in particular since the lines which significantly contribute to the opacity are affected by Doppler shifts. The direct inclusion of a very large number of lines is often prohibitive, and an approximate treatment of the absorption coefficients by introducing appropriate mean values is therefore required. However, in differentially moving media concepts adopted from static media do not seem to be valid and hence may result in rather inaccurate models.

The importance of the line absorption in a differentially expanding atmosphere can be illustrated by the modeling of the nova U Sco by Starrfield et al. (1988): by adopting the Rosseland mean opacity no explosion could be achieved. If, however, the Doppler shifts of the lines are incorporated in terms of an "expansion opacity" (Karp et al., 1977) the effective mean opacity is increased by about a factor of two which then leads to an exploding nova model. Expansion opacities have also been applied in supernova models, e.g. by Höflich et al. (1991) and by Chilukuri and Wagoner (1988).

Similar requirements are also found in the modeling of accretion disks (Wehrse and Shaviv, 1991). Here even the coupling of the temperatures and pressures to the opacities is much closer than in stellar atmospheres so that lines are expected to influence significantly the structure, which implies that the effects of the differential Keplerean rotation on the lines require careful treatment.

In this contribution we discuss some problems inherent to the concepts of mean opacities for differentially moving media. For the solution of the hydrodynamic equations a calculation of the radiation flux, the radiation pressure, and the energy balance is required. We suggest a fast method for the accurate calculation of the corresponding mean flux and mean opacities.

2. APPROXIMATION PROBLEMS FOR MEAN OPACITIES IN MOVING MEDIA

In this Section we briefly comment on three specific approximations that are commonly made in the radiation-hydrodynamic modeling of novae, supernovae and accretion disks but appear quite questionable to us:

(i) The approximation of the *radiation pressure* $P_{\rm rad}$

$$\frac{dP_{\rm rad}}{dz} = \frac{d}{dz}\left(\frac{a}{3}T^4\right) \qquad (1)$$

which is frequently used in hydrodynamical calculations of nova and supernova atmospheres is fairly crude. It should be replaced by

$$\frac{dP_{\rm rad}}{dz} = \frac{1}{c}\int(\kappa_\nu + \sigma_\nu)F_\nu d\nu \ . \qquad (2)$$

Here F_ν is the monochromatic radiation flux, κ_ν the absorption, and σ_ν the scattering coefficient. a is the radiation constant and c the speed of light. In this paper all integrals over frequency ν are to be understood as integrals over the entire range (0–∞).

(ii) The equation of *energy transport* may be written as

$$\frac{dF}{dz} = \frac{d}{dz}\left(-\frac{4ac}{3}\frac{T^3}{\bar\kappa}\frac{dT}{dz}\right) \qquad (3)$$

where $F = \int F_\nu d\nu$. The mean opacity $\bar\kappa$ is certainly *not* equal to the Rosseland mean opacity when lines in a differentially moving configuration are important since in such a medium the assumption, which enters the derivation of the Rosseland mean,

$$\int \mu^2 I_\nu d\mu \simeq \frac{1}{3}\int I_\nu d\mu \qquad (4)$$

is *not* a good approximation. ($\mu = \cos\theta$, I_ν is the specific intensity, and the μ integral is to extend from -1 to $+1$.)

(iii) Due to their Doppler shifts in a differentially expanding atmosphere, the strong and medium strong lines can amplify the opacity. This *expansion opacity*, introduced by Karp et al. (1977) upon the assumption that the diffusion approximation (eq. 3) holds, was further expanded by Wagoner et al. (1991) for more general conditions. They find that the contribution of the lines integrated over small frequeny intervals acts like a "coarse grained" continuum, which depends on the mean line separation and on the velocity gradient. In these calculations the line profiles are, however, approximated by δ functions. In particular for optically thin cases this simplification leads to an overestimate of the line contribution since the excess absorption by the "expansion wing" is partly compensated by a decrease of the line peak.

3. CONSISTENT TREATMENT OF MEAN OPACITIES

For a consistent treatment it has to be realized that the radiation field enters hydrodynamic calculations in two ways. (i) In the momentum (Euler or Navier-Stokes) equation the gradient of the radiation pressure occurs as a force term. (ii) In the energy equation the difference of the energy absorbed locally per unit time and the energy emitted locally per unit time has to be taken into account. In addition, the luminosity or the total radiation flux are always of interest. Ideally, all these quantities are derived from the monochromatic specific intensities I_ν by integration over frequency and angle:

$$A_{\rm rad} = \frac{dP_{\rm rad}}{dz} = \frac{1}{c}\int(\kappa_\nu + \sigma_\nu)F_\nu d\nu$$
$$Q = \int \kappa_\nu (B_\nu - \frac{1}{2}\int I_\nu d\mu)d\nu \tag{5}$$
$$F = \int\int \mu I_\nu d\mu d\nu \ .$$

The monochromatic intensities are in turn obtained from the radiative transfer equation in the comoving frame (Mihalas and Mihalas, 1984)

$$\begin{aligned}\gamma(\mu+\beta)\frac{\partial I_\nu}{\partial z} &+ \frac{\partial}{\partial\mu}\{-\gamma(1-\mu^2)\gamma^2(\mu+\beta)\frac{\partial\beta}{\partial z}I_\nu\} \\ &- \frac{\partial}{\partial\nu}\{\gamma^3\mu(\mu+\beta)\frac{\partial\beta}{\partial z}\nu I_\nu\} + \gamma^3(1+\mu^2-2\beta\mu)\frac{\partial\beta}{\partial z}I_\nu \\ &= -(\kappa_\nu+\sigma_\nu)I_\nu + \frac{1}{2}\sigma\int I_\nu d\mu + \kappa_\nu B_\nu\end{aligned} \tag{6}$$

(β is the ratio of matter velocity to the velocity of light, $\gamma = 1/\sqrt{1-\beta^2}$, and B_ν is the monochromatic Planck function). However, such a way is by far too time consuming and one has to regress to a direct calculation of the integrated quantities by means of suitably averaged absorption coefficients as in the static case. Since—as has been explained above—the approximations made in the derivation of the Rosseland mean break down for differentially moving configurations, and since we were not able to find a generalized coefficient which leads at the same time by "cheap computations" to correct values of A, Q, and F, we propose here to introduce three coefficients $\tilde{\kappa}^A$, $\tilde{\kappa}^Q$ and $\tilde{\kappa}^F$ which fulfil this condition. They are functions of the chemical composition, pressure, temperature and the velocity gradient, but they should not depend on the temperature or pressure gradient or the velocity.

For the calculations of these averaged absorption coefficients we propose now a scheme which is a generalisation of the one described by Unsöld (1955) for the derivation of the Rosseland mean. For this purpose we consider a plane-parallel medium with a velocity gradient in the z direction. The monochromatic transfer equation for unpolarized radiation is then given by eq. (6). When now the velocity field, the absorption and scattering coefficients, the run of the temperature and the total depth as well as the boundary conditions are given, this equation can easily be solved by means of several methods such as the discrete-ordinate-matrix-exponential (DOME) method (Hauschildt and Wehrse, 1991) or the short characteristic method of Hauschildt (1991).

We now assume that the scattering coefficient is frequency independent (Thomson scattering) and that the transfer equation integrated over frequency can be written in the form

$$\begin{aligned}\gamma(\mu+\beta)\frac{\partial\tilde{I}}{\partial z} &+ \frac{\partial}{\partial\mu}\{-\gamma(1-\mu^2)\gamma^2(\mu+\beta)\frac{\partial\beta}{\partial z}\tilde{I}\} + \gamma^3(1+\mu^2-2\beta\mu)\frac{\partial\beta}{\partial z}\tilde{I} \\ &= \tilde{\kappa}B + \frac{1}{2}\sigma\int\tilde{I}d\mu - (\tilde{\kappa}+\sigma)\tilde{I}\end{aligned} \tag{7}$$

From the frequency integrated intensities $\tilde{I}$ we obtain the following quantities which are analogous to those defined above

$$\tilde{A}_{\rm rad} = \frac{1}{c}(\tilde{\kappa} + \sigma)\tilde{F}$$

$$\tilde{Q} = \tilde{\kappa}(B - \frac{1}{2}\int \tilde{I} d\mu) \tag{8}$$

$$\tilde{F} = \int \mu \tilde{I} d\mu$$

Note that $\tilde{A}_{\rm rad}$, $\tilde{Q}$, and $\tilde{F}$ are again functions of depth.

We now require that the use of eq. (7) with the average absorption coefficients leads to fluxes, gradients of the radiative pressure, and net output of volume elements that are identical with those obtained from eq. (6) with a subsequent frequency integration, i.e. we require that equations

$$A_{\rm rad} = \tilde{A}_{\rm rad}$$

$$Q = \tilde{Q} \tag{9}$$

$$F = \tilde{F}$$

hold. It implies that these equations can now be considered as implicit expressions for the calculation of the the average absorption coefficients $\tilde{\kappa}^A$, $\tilde{\kappa}^Q$, and $\tilde{\kappa}^F$!

Note that by using this procedure the determination of the average opacities is quite involved and takes considerable computing times, but the data can be precalculated once for all as in the static case and can then be used in many hydrodynamic calculations.

In the actual calculations we use slabs of large optical depths with prescribed densities, temperatures, and velocity gradients which for simplicity are assumed to be constant. The frequency dependent equations are solved with standard LTE continuum cross-sections and line data from the list of Kurucz (1988) to obtain $A_{\rm rad}$, Q, and F. We then solve eq. (7) with estimated mean absorption values. From the differences

$$\Delta A = A_{\rm rad} - \tilde{A}_{\rm rad}$$

$$\Delta Q = Q - \tilde{Q} \tag{10}$$

$$\Delta F = F - \tilde{F}$$

we then obtain improved average opacities by means of a solver for non-linear systems which avoids the explicit calculation of the Jacobian matrix. The iteration is stopped when the relative error is below approximately 0.001.

When the tables with these coefficients are available, in actual model calculations they can be interpolated; and then by means of the solution of eq. (7) accurate values for the flux, the gradient of the radiation pressure as well as for the difference of absorbed and emitted energy are obtained, and the approximations discussed in Section 2 can be avoided. This method now seems feasible since on a modern computer the solution of eq. (7) takes only about 0.1 s, and can therefore be performed at every time step in dynamical calculations with only a moderate increase in the total computing time.

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