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RADIATIVE OPACITIES AND STELLAR PULSATIONS

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RESUMEN. Se revisan algunos problemas actuales en pulsaciones estelares en los cuales las opacidades radiativas juegan un papel crucial. Se le presta atención particular a las opacidades ya publicadas de OPAL, que parecen estar a punto de resolver las perennes dificultades de los modelos pulsacionales. También se le da una primera y breve mirada a los resultados preliminares de las opacidades provenientes del Proyecto de la Opacidad.

ABSTRACT. We review some current problems in stellar pulsations in which the radiative opacities play a crucial role. Particular attention is paid to the already-published OPAL opacities, which seem to go a long way toward resolving long-standing difficulties in pulsational models. A brief, first look is also given to a preliminary version of opacities from the Opacity Project.

Key words: OPACITIES – STARS: PULSATION

I. INTRODUCTION

The equations governing the envelope of a pulsating star are conceptually simple. There is an equation of motion for each layer, where the only forces playing a role (we hope) are gravity and pressure. There is an energy balance equation, including gravitational and thermal energy, and taking account of any sources or sinks. And there is an equation describing the transport of radiation, either by photon diffusion or by convection. This set must be supplemented by auxilliary relations detailing physical properties of the system. The most important of these concern the opacity and equation of state of the stellar gas.

The above scheme leads to a coupled set of nonlinear partial differential equations, which, subject to boundary conditions at both surface and interior, does not constitute an initial value problem. While there is no proof of the existence (not to mention uniqueness) of solutions to this system, the equations may nonetheless be solved in numerical form. Numerical solutions of this type constitute *pulsation models.*

Generally speaking, two levels of model have been constructed: linear and hydrodynamic. In the former, small perturbations are treated and the stellar structure equations linearized. The solutions yield an infinite set of normal modes, with a period and growth rate for each. Here and in what follows we consider radial pulsations only, although nonradial solutions may also be found, both on the computer and in nature itself.

The hydrodynamic pulsation models consist of brute-force numerical solutions to the full-blown set of nonlinear equations. These give rise not only to theoretical periods but also to light and radial velocity curves (periodic variation with time of luminosity and radial velocity, the latter measured by Doppler shifts in the spectral lines) which can be compared with observations.

There are uncertainties associated with both sorts of pulsation model. The linear models are simpler and more straight-forward. The theoretical periods depend upon global (rather than detailed) properties, and, in particular, upon the mean density. This dependence has the sense which agrees with intuition: the more compact the star, the shorter the period. One might think that the linear approximation could induce errors in the calculated periods, and, indeed, this is sometimes the case. However, for the stars which shall interest us here, the linear and nonlinear (hydrodynamic) periods seem to agree very closely (Kovács and Buchler 1988; Simon 1990a). Finally, we note that the linear growth rates, which are influenced by numerical niceties such as boundary conditions and zoning in the models, are more uncertain than the periods. Fortunately, the growth rates shall not concern us in the discussions which follow.

The hydrodynamic calculations yield much more information but also more grief. Here, questions involving numerical methods become even more important (see the review by Kovács 1989), in particular the treatment of ionization fronts and shocks in the outer layers. There is evidence that artificially imposed shock dissipation plays a crucial role in limiting the growth of pulsation in the calculations and thus determining the final limit cycle. Clearly, a new class of improved models is needed to remedy this unacceptable shortcoming.

In addition to the uncertainties associated with modelling, there are also those connected with the input physics. Perhaps the best example is our present focus, the opacity and equation of state of the gas in the stellar envelope. It must be emphasized here that when we compare pulsation models with observations and find the former wanting, it is not clear which of the uncertainties might be at fault, and in what combination. The disentanglement of these various threads will require progress on many fronts.

II. PERIODS AND LINEAR MODELS

The simplest observation one can make is that of the period itself, a quantity which is, for our purposes, constant and repetitive. For the stars we shall treat, only the lowest modes come into play – say, the fundamental and first and second overtones. The linear models yield expressions for the periods, of the form

$$\log P_0 = f_0\,(M,R), \tag{1}$$

$$\log P_1 = f_1\,(M,R), \tag{2}$$

etc., where P_0 and P_1 and the fundamental and first overtone periods, and M and R are the stellar mass and radius.

Equations (1) and (2) are schematic renditions of the "pulsation equation" or "period/mean-density law," the fundamental relation of pulsation theory. An actual version (Simon 1990b), obtained from models of RR Lyrae stars and written in terms of the luminosity and effective temperature instead of the radius, is given in the following:

$$\log P_1 = 0.802 \log L - 0.604 \log M - 3.346 \log T_e + 10.933. \tag{3}$$

As indicated above, the period is seen to depend on global properties of the model, increasing with luminosity and decreasing as the mass or temperature rises.

It is clear from Eq. (3) that the measurement of a period alone does little to constrain the models. Independent measurements of R, T_e or L are also necessary, but these are difficult to obtain. However, there do exist friendly stars which pulsate simultaneously in two modes, the fundamental and first overtone. These are the so-called double mode or "beat" pulsators. The two periods can be disentangled and extracted from the observations, whereupon Eqs. (1) and (2) together yield the mass and radius of the star. However, the pulsation equation is model dependent and, in particular, will always refer to a given opacity law. For example, Eq. (3) was obtained with standard Los Alamos opacities. A change in the opacity law would result in a change in the coefficients.

Let us now attempt to apply the theory to observations of actual pulsating stars, beginning with the classical Cepheids. These are young stars, located in the thin Galactic disk and with a heavy element abundance approximately solar, i. e., $Z \cong 0.02$. About a dozen "beat" Cepheids are known, with fundamental periods between 2 and

6 days. Various versions of the theory of stellar evolution agree that these stars ought to have masses somewhere in the range $4 \lesssim M/M_{\odot} \lesssim 7$ (Chiosi 1990; Simon 1990c).

The double mode pulsation problem is traditionally analyzed by use of the "Petersen diagram", which is a plot for each star of the period ratio P_1/P_0 vs. one of the individual periods, say P_0. Figure 1 is a Petersen diagram for beat Cepheids, borrowed from Moskalik, Buchler and Marom 1991 (hereafter MBM). The dots are the observed stars while the curves give loci for the theoretical models with masses as indicated. The models denoted by dashes were calculated with Los Alamos opacities. Their loci lie far above the observed points, implying that the masses of the beat Cepheids are much lower than the evolution calculations indicate. However, when the same models are recalculated with OPAL opacities (Iglesias and Rogers 1991), the pulsation loci (solid lines) intersect the observations at masses which agree with stellar evolution. The present authors have independently performed the calculations with OPAL opacities, and have obtained results almost identical to those of MBM.

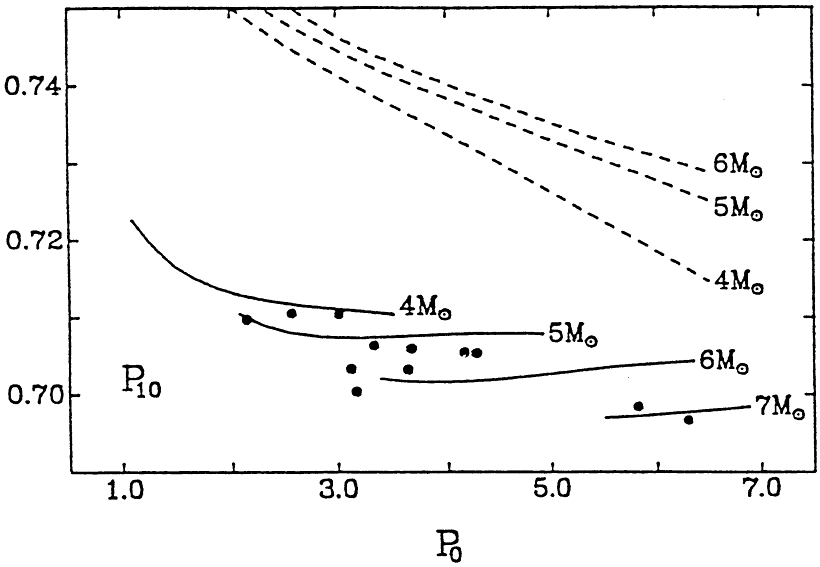

FIG. 1. Petersen diagram for beat Cepheids (from MBM). *Solid lines*: OPAL models; *dashed lines*: Los Alamos models; *dots*: observed points.

We turn now to the RR Lyrae stars. These are (generally) metal-poor pulsators with periods of around half a day. They are found all over the Galaxy, but particularly in the halo, both as individual objects and contained in globular clusters. The clusters which have RR Lyrae stars seem to separate into distinct types, called Oosterhoff I (Oo I) and Oosterhoff II (Oo II). In the former (with metallicity $Z \lesssim 10^{-3}$) about 75% of the RR Lyrae stars are fundamental mode pulsators with a mean period of about 0.55d, while in the latter ($Z \sim 10^{-4}$) the corresponding numbers are 50% and 0.65d (e.g., Rood and Crocker 1989). Double mode RR Lyrae stars (RR*d* stars) are found in both Oo I and Oo II clusters, though by no means in all Oosterhoff clusters.

The RR*d* stars are also treated by means of the Petersen diagram. Such a plot is shown, for example, in Figure 10 of Clement, *et al.* (1986). There it is seen that the two Oosterhoff groups separate remarkably, with the Oo II stars lying toward the upper right (longer periods and higher period ratios) and the Oo I stars toward the lower left. Model loci indicate masses $M \cong 0.55\ M_{\odot}$ for the Oo I RR*d* stars, and $M \cong 0.65\ M_{\odot}$ for the Oo II RR*d* stars. These values are obtained with Los Alamos opacities.

However, when OPAL opacities are employed in this problem, the results change dramatically (Cox 1991; Kovács, Buchler and Marom 1991). Under these conditions, the inferred RR*d* masses are seen to exhibit an extremely strong metallicity dependence. For a reasonable range of metallicities, these masses vary from about 0.65 to 0.80 $M_{\odot}$ in Oo I clusters and from 0.75 to 0.80 $M_{\odot}$ in Oo II clusters (Kovács, Buchler and Marom 1991). These new values seem to be more in accord with those obtained from the theory of stellar evolution (e.g. Lee, Demarque and Zinn 1990).

Once again, the present authors have independently performed calculations with OPAL opacities, and have reproduced the Kovács, *et al.* results.

III. FOURIER PARAMETERS AND HYDRODYNAMIC MODELS

The light and/or radial velocity curves of a pulsating star may be usefully summarized by fitting the time variations with a Fourier series, e.g., for the observed magnitudes,

$$\text{mag} = A_0 + \sum_{j=1}^{n} A_j \cos(j\omega t + \phi_j) \; . \tag{4}$$

For most single-mode classical Cepheids and RR Lyrae stars, an excellent fit to the light variations is obtained with a series truncated at n = 4. The structural properties (shape) of the light curve may then be addressed in terms of combinations of the lower order Fourier coefficients, viz.,

$$R_{ij} \equiv A_i / A_j \quad \text{and} \quad \phi_{ij} \equiv \phi_i - i\phi_j \; . \tag{5}$$

An extensive discussion of the Fourier decomposition technique is given by Simon (1988).

We shall first employ this technique to discuss the classical Cepheids. Simon and Moffett (1985) give plots, for a large Cepheid sample, of the observational Fourier parameters R_{21}, ϕ_{21}, ϕ_{31} and ϕ_{41}, all versus period. In each of these plots a sharp break is seen to occur at a period of about 10d. It turns out that this is the result of an accidental resonance between the fundamental mode and the second overtone (Simon and Schmidt 1976; Buchler, Moskalik and Kovács 1990):

$$P_2/P_0 = \tfrac{1}{2} \quad \text{at} \quad P_0 \cong 10\text{d} \; . \tag{6}$$

Figure 2, borrowed, once again, from MBM, shows a plot of P_2/P_0 vs. P_0 for middle-period classical Cepheid models (known as "bump" Cepheids due to the shape of their light curves) with Z = 0.02. The cross indicates

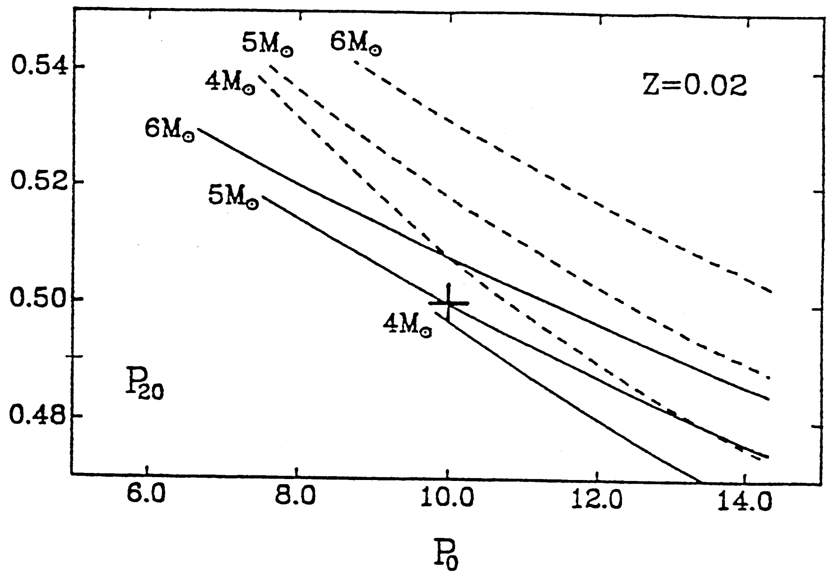

FIG. 2. Petersen diagram for bump Cepheids (from MBM). *Solid lines*: OPAL models; *dashed lines*: Los Alamos models; *cross*: observed point, according to resonance explanation.

the point demanded by the resonance and given in Eq. (6). We note that models calculated with Los Alamos opacities (dashed curves) lie above the cross, implying Cepheid masses < $4M_{\odot}$ whereas the OPAL opacities (solid curves) yield masses of about $5M_{\odot}$. One suspects, from the stellar evolution calculations, that a 10-day Cepheid ought to have a somewhat higher mass, say, $6M_{\odot}$, and MBM show that if one raises the metallicity of the models to Z = 0.03, this result is obtained. However, there is little evidence for Cepheid metallicities higher than solar.

The Fourier parameters, given in Eq. (5), find their most important use in comparing theoretical and observed oscillations. The theoretical models have a mixed record in matching the observations (Simon 1988), with the phase parameter ϕ_{21} turning out the most problematical. Buchler, Moskalik and Kovács (1990) subjected a large grid of hydrodynamic light curves to Fourier decomposition and compared the theoretical values of ϕ_{21} to those extracted from observed fundamental mode classical Cepheids. The agreement was generally poor, with the model-derived values of ϕ_{21} found to be too high at all periods. These models were calculated with Los Alamos opacities. Recently, MBM redid the calculations using OPAL opacities, and found significant improvement in the theoretical ϕ_{21}'s. The amount of this improvement was perhaps underemphasized by MBM themselves. While the new hydrodynamic light curves still have some shortcomings, they nonetheless represent a significant plus for the OPAL opacities compared with the traditional calculations from Los Alamos.

Turning to the RR Lyrae stars, a similar situation exists at present, in which theoretical values of ϕ_{21} (and also ϕ_{31}), obtained from models with Los Alamos opacities, significantly exceed the observed values in F-mode pulsators (Simon and Aikawa 1986). If OPAL-based models were able to lessen or eliminate this discrepancy (these calculations have not yet been done), this would constitute yet another piece of evidence favoring the new opacities.

IV. A REMARK ON THE "OP" OPACITIES

Yu Yan (1992, this volume) has compared the OPAL opacities with a preliminary calculation by the Opacity Project (Seaton, *et al.* 1992, this volume) along various loci where the quantity $R = \rho/T_6^3$ is held constant. In Figure 3, we show

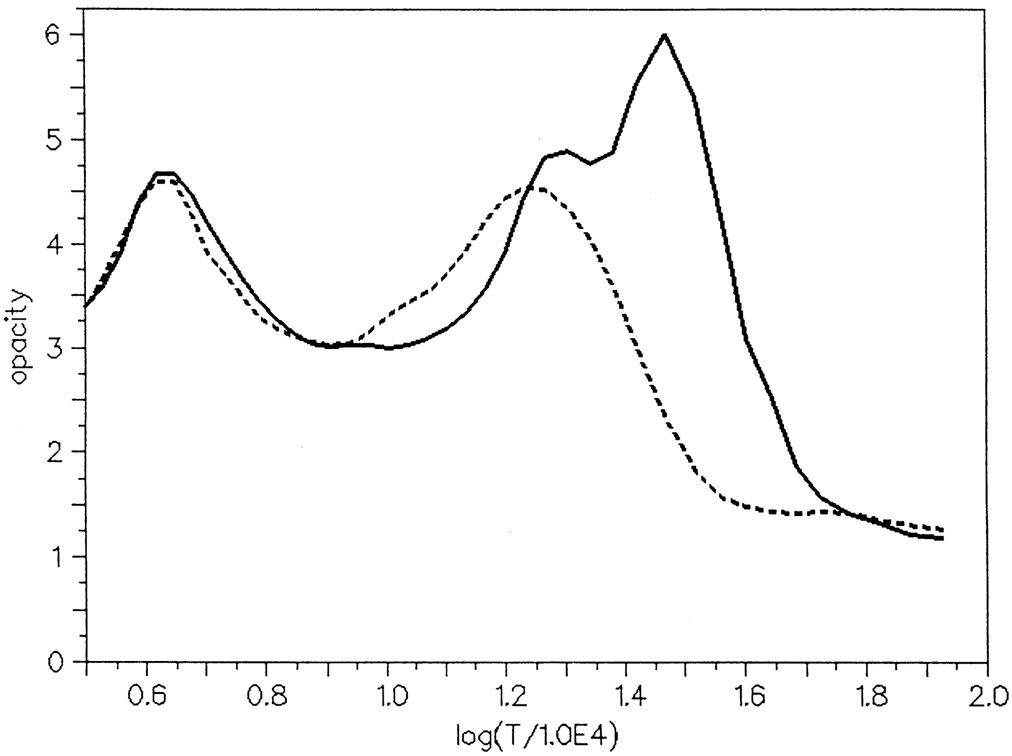

FIG. 3. Rosseland mean opacity vs. log $(T/10^4)$ along a density-temperature locus in a bump Cepheid model. *Dashed line*: OPAL; *solid line*: OP.

the run of opacity along a locus of temperature and density points corresponding to an actual stellar model, in this case a bump Cepheid model. The ordinate is the Rosseland mean opacity, while, along the abscissa, the (T,ρ) locus is given in terms of the temperature, in the form log $(T/10^4)$. The dashed curve shows the OPAL opacities, while the solid curve corresponds to the preliminary version of OP. (The jagged appearance of the latter is caused by a crude interpolation scheme.) In the range below 30,000 K (not shown here), the two opacities are virtually identical.

One notes that, along the given locus, the OPAL opacities are slightly larger in the range, $1.0 \times 10^5 \lesssim T \lesssim 1.5 \times 10^5$ K, while the OP opacities are substantially bigger in the range, $2.5 \times 10^5 \lesssim T \lesssim 4.0 \times 10^5$K. Although both opacities show a large enhancement over the Los Alamos version in the domain between 1.0 and 4.0×10^5 K, the differences remarked here could have interesting ramifications in the study of some of the pulsation problems discussed above. These will need to be worked out in future investigations.

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