Rev. Mexicana Astron. Astrof., 24, 129-133 (1992)

DOES DYNAMICAL FRICTION PRODUCE DYNAMICAL HEATING?

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Received 1992 February 25

RESUMEN

Utilizamos experimentos numéricos para investigar la posibilidad de que la fricción dinámica pueda producir calentamiento dinámico. Consideramos un sistema de partículas de prueba que orbitan en torno a una partícula masiva; este sistema está inmerso en un medio de partículas masivas que tienen movimientos tanto al azar como sistemáticos. Los últimos provocan el movimiento del sistema de partículas de prueba por fricción dinámica, pero hallamos que los cambios en la energía interna del sistema, si existen, deben ser muy pequeños.

ABSTRACT

We use numerical experiments to investigate the possibility that dynamical friction could produce dynamical heating. We consider a system of test particles orbiting around a massive particle; this system is immersed in a sea of massive particles that have both random and systematic motions. The latter forces the system of test particles to move through dynamical friction, but we found that changes in the internal energy of the system, if present, should be very small.

Key words: **STARS-STELLAR DYNAMICS**

I. INTRODUCTION

In a preprint that circulated a few years ago Miller & Smith (1985) indicated that, when a system of separate bodies (say, a galaxy or a globular cluster) is braked through dynamical friction, part of the energy released by the braking goes into the system which, as a result, gains energy and loses particles. The preprint was never published and, some time later, Miller (1988) indicated to one of us that numerical problems might have affected their results.

While those problems could have yielded an incorrect quantitative estimate of the energy that goes into the system (between one fourth and one half of the braking energy, according to Miller and Smith 1985), the idea that dynamical friction could produce dynamical heating is extremely interesting. Moreover, if true, it could have very important consequences, because the energy released by braking is so large, compared to the internal energy of the stellar systems, that even a small fraction of it could play havoc in the structure of those systems. For example, a globular cluster that moves through a galaxy with a velocity of about 300 km s^{-1} has internal velocities of about 10 km s^{-1} only; therefore, if the cluster is braked to a stop by dynamical friction, the energy released is 1 000 times the internal energy of the cluster. Thus, if there is some process that, even with a very low efficiency, can introduce that energy into the system, the effects of the dynamical heating will be very significant. The key question is, of course, whether such a process exists.

One such possible process was investigated by Muzzio *et al.* (1988): they followed Kalnajs' (1972) approach, considering that dynamical friction is the drag caused by the gravitational attraction of the wake that follows the moving body, and computed the tidal effect of that wake on the body. Although they found such a tidal effect to be present, they concluded nevertheless that it had a negligible effect on the moving body.

In the present paper we follow a purely experimental approach. Without questioning what could be the process that transforms braking energy in in-

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2. On a Fellowship from the Comisión de Investigaciones Científicas de la Provincia de Buenos Aires.

ternal energy, we perform numerical simulations to try to establish whether such transformation can actually take place.

II. MODEL AND EXPERIMENTS

In our model we considered a sphere of "background" particles, with a "stellar system" at its center; the background particles have a systematic motion, as well as random motions, which accelerates the stellar system through dynamical friction. In other words, instead of braking the stellar system we accelerate it; we will refer however to, say, "braking energy", since the meaning is perfectly clear, no matter how the experiments are performed.

Our model includes two kinds of particles: *field particles* and *test particles*; among the former there is a privileged particle which we will call the *nucleus*. Field particles, including the nucleus, interact among themselves and act on test particles, which are considered massless. Thus, the latter are attracted by field particles but do not attract them or other test particles. The nucleus is surrounded by a cloud of test particles, and the whole ensemble of nucleus plus test particles represents the *stellar system* that is affected by dynamical friction and is our probe to check whether it absorbs or not braking energy. The field particles make up the background which exerts the dynamical friction on the stellar system.

We intend to represent an infinite and homogeneous background, but the need to use a finite number of particles demands some approximations to be made. The background is, therefore, limited to a sphere which has the stellar system at its center. Now, if this sphere were actually part of an infinite homogeneous background, there would be no force on a particle immersed in it (the *Jeans swindle*) while, in our experiments, the field particles distributed in the sphere would exert a net force that, on average, attracts the particle toward the center of the sphere. We compensated for this effect adding in all cases a radial *repulsive* force equal to the attractive force that would have exerted the homogeneous sphere at the point in question. The presence of the stellar system produces a concentration of the background particles, so that the background density used to compute the force is the density of the outermost shell of the sphere.

The use of a sphere of background particles has the additional problem that, due to their random and systematic motions, particles escape from the sphere as the experiment proceeds. Thus, every time a field particle left the sphere, we had to create at random a new particle entering the sphere; this method can be tricky, and other researchers (Bouvier & Janin 1970; Cruz-González & Poveda 1971) failed to introduce the new particles truly at random, without changing the initial distribution. As Hénon (1972) showed, the particles have to b created in numbers proportional to $f(v)\ v\ \cos$ i where $f(v)$ is the velocity distribution function, the velocity, and i the angle between the velocit and the radius vector. We were careful to take int account Hénon's analysis in our own experiment and the new particles were created as follows: 1 Their positions on the limiting sphere were chose at random. 2) Their velocities were chosen from spherical Gaussian distribution (with a mean valu in the X direction equal to the mean velocity chose for the experiment in question), and using the dc product of the velocity and the position vectors t model the distribution according to Hénon's recipe

To check the preservation of the space distr bution we used a case with no systematic velocit whose distribution function has a very simple ana lytical solution, since it is spherically symmetric an function of the energy only. In our case, where th background is homogeneous and with a Gaussia velocity distribution, the number of particles withi spherical shells can then be numerically evaluate in terms of the error function. We used the χ^2 te to check the preservation of the space distributio as field particles left the sphere and new ones wer created, using a central sphere and five spheric shells of equal volume. The minimum and ma imum χ^2 values obtained out of 14 cases (after equ librium had been reached) were 0.656 and 7.90 respectively, while the theoretical values for five de grees of freedom and probabilities of 99% and 1 are 0.554 and 15.086, respectively. We also checke the mean velocity of the field particles for the sam experiment, which should remain zero for symme try reasons. For each of the 14 cases, the three con ponents of the mean velocity were all less than 2 (i.e., essentially zero).

Except for the fact that new particles are create as other particles leave the sphere, the numeric integration is performed with the same code use in previous papers of our group (Muzzio, Martíne & Rabolli 1984; Muzzio, Dessaunet, & Vergn 1987). The field particles, as well as the nucleu are regarded as Schuster (or Plummer) sphere so that the forces they exert on the test particle are the usual forces due to the Schuster potenti (e.g., Binney & Tremaine 1987), while the force be tween two such spheres can be approximated by th asymptotic formula of Muzzio & Martínez (1982 The integrator is based on the method of Wiele (1967) and uses interpolating polynomials, but the are always of second degree; to start the integratio procedure, an integration is performed with zer degree polynomials to obtain the accelerations a two instants different from the initial one. Th absolute error and the relative error (in Wielen notation) were adopted as 2.5×10^{-12} and 2.

10^{-3}, respectively. In previous checks of the ethod, considering a self-gravitating system (i.e., ith no field particles leaving or being created), the nter of mass, the total angular momentum and e total energy were conserved with an accuracy etter than 10^{-4}. Being massless, the test particles not enter in these computations, but we are articularly interested on how well they conserve eir energies per unit mass, because their changes e the main objective of the present study. Models at included just the nucleus and the test particles ere therefore also run, and we found that the tegrals of motion per unit mass of the massless articles were conserved again with an accuracy of e order of 10^{-4}.

We chose initial conditions that, within the nitations of our approach (not the least of which that we used a mini – rather than a super – mputer!), resembled those of Miller & Smith 985). Since they had limited the background with cube of 64 units of length on its side, we chose our here as circumscribing a cube of the same side, ., a sphere of radius 55.43 units. Similarly, we ose a softening parameter of 0.5 units, as they d, and the mean mass density of our background here is initially the same as the mean mass density their cube; we performed different experiments ing different numbers of background particles, amely, 250, 177 and 125, so that the mass of one article was in each case, respectively, 0.212, 0.299 d 0.424 (Newton's constant was taken as G = 0). The velocity distribution of the field particles as taken as Gaussian with a dispersion of $3^{-1/2}$ in ch direction; mean velocity values were zero for e Y and Z components and, depending on the periment, taken initially as zero or as 3.5 for the X mponent. The chosen values are such that the dimeter of the limiting sphere is 6% shorter than the ans wavelength for the system of field particles, us avoiding the Jeans instability. Moreover, the eed to limit the background to a region the size the Jeans wavelength when investigating dynamal friction appears clearly in analytical studies, hether one follows the traditional approach (e.g., inney & Tremaine 1987) where the limit is the "size the system" (which, according to the virial eorem, turns out to be of the order of the Jeans avelength), or Kalnajs' (1972) method where avenumbers shorter than the Jeans wavenumber e rejected. The size limit is needed because, in l these studies, the wake of background particles tends to infinity and the force it exerts on the oving body is also infinite. The reason for this parlox is, of course, that all these investigations start om a system that cannot exist (an infinite homoeneous stellar system), and this is the price to pay r the Jeans swindle.

The test particles were distributed initially around the nucleus following the Schuster law with a half-mass radius of 7 units and a total mass of 18.22; the latter value (i.e., the total mass of the stellar system) is used only to compute the attractive force that causes the nucleus, and was chosen so as to have a mean spatial velocity dispersion of 1.0 for the Shuster sphere when Newton's constant G = 1.0. The Schuster sphere has infinite radius, so that we truncated our distribution at $R_{max} = 28.76$, which would have included 95% of the total mass if the sphere had extended to infinity. Although the sphere of background cannot be larger than the Jeans wavelength, as explained above, we followed the approach of Mulder (1983) to derive from numerical integrations the position of the density peak of the wake and check that, at least that peak, was well inside the region considered. It turned out that the peak is located at 14.6 and 10.9 units from the center of the sphere for mean background velocities of 2.9 and 2.0, respectively, as we used (see below). Thus, the peak density of the wake lies not only well inside the background sphere, but inside the limiting radius of the stellar system as well.

The velocity distribution for the Schuster sphere is well known (e.g., Binney & Tremaine, 1987) but, since we had limited the radius, we used it with the additional constraint that, when the initial conditions were generated, velocities that would have carried the particle beyond the limiting radius were rejected. Part of the experiments were performed using this distribution but, for others, we simply chose a spherically simmetric Gaussian velocity distribution, with different velocity dispersions at different radii computed so as to fulfill the condition of hydrostatic equilibrium (Ogorodnikov 1965) and, again, velocities exceeding the escape velocity were rejected. In any case, the particles were allowed to evolve for a few crossing times, before using them in the experiments, in order to ensure that truly stable distributions were being used.

Three different cases were considered for the velocity of the stellar system relative to the background. First, as comparison, we considered the case of zero relative velocity. We used a random number generator to create initial conditions where the field particles were distributed in the chosen sphere which had the nucleus at its center. In order to make sure that an equilibrium condition had been reached, this configuration was allowed to evolve for 800 time units before adding the test particles (which, in turn, had been allowed to evolve around the nucleus as explained above). The cases where the stellar system was in motion relative to the background were created in a similar fashion, but this time the field particles had also a systematic velocity of 3.5 units. The nucleus and the field particles were allowed to evolve for 200 time units, to get a relative velocity of 3.1, and for 460 time units, to get a relative velocity of 2.4; in this way, we get the field particles moving with two different velocities relative to the nucleus and, at the same time, we ensure that an equilibrium condition has been reached. Again, the test particles (which had already evolved around the nucleus alone) were added only after this initial stabilization period. We will use the roman numerals I, II and III to indicate, respectively, the cases with velocity 0, 3.1, and 2.4 units. In each case, the subsequent evolution was followed for another 200 time units. The crossing time of the stellar system is 18.2 time units, so that the first half of those 200 time units is an interval long enough to ensure that an equilibrium condition has been reached, and only the remaining 100 units interval was used for our analysis (after the initial 100 time units evolution the relative velocities for cases II and III were, respectively, 2.9 and 2.0). The reason for being so careful in letting evolve a few crossing times all the tested configurations before using or analyzing them was that, as White (1986) suggested, it was possible that the effect seen by Miller & Smith (1985) had arisen from a transient phenomenon.

There is still another effect to be taken into account. As indicated above, when the stellar system is added at the center of the sphere, the field particles tend to concentrate toward that same center (when there is no relative motion) or toward a "wake" (when there is relative motion). Since we use a fixed number of particles, the result is that the particle density diminishes in the outer parts of the sphere, which are the ones that we take as representative of the mean density of an infinite homogeneous background; now, the larger the relative motion, the smaller the concentration, so that two experiments performed using the same number of field particles but different relative velocities are not stricly equivalent, because each one of them corresponds to a different background density. Thus, although we performed experiments keeping the same number of particles in cases with different relative velocities, we also included a few experiments where, in the cases with non-zero relative velocity we reduced the number of particles so as to have for them the same background density as for the cases with the stellar system at rest relative to the field particles.

III. RESULTS AND ANALYSIS

As indicated above, the whole ensemble (nucleus, plus test particles, plus field particles) was allowed to evolve for 200 time units and, to ensure that an equilibrium condition had been reached, only the last 100-unit interval was considered for our investigation.

To measure how much energy had entered the stellar system, we used the total energy of th whole ensemble computed as if it were an isolate system, which is essentially the same approach o our previous work (e.g., Muzzio *et al.* 1984, an Muzzio *et al.* 1987). Thus, we assigned to eacl test particle a mass equal to the total mass of th stellar system divided by the total number of tes particles; we used its velocity relative to the nucleu to compute its kinetic energy, and its distance t the nucleus to compute its potential energy in th field of the nucleus. The energy of each particle obtained adding its kinetic and potential energies was used to decide whether the particle had escape (zero or positive energy), or not (negative energy from the stellar system. The energy of the whol ensemble was then obtained adding the energies o all the particles that had not escaped.

Similarly, we computed the kinetic energy of th stellar system using the velocity difference betwee the nucleus and the mean velocity of the fiel particles, together with the total mass of the stella system; the difference between the initial and fina kinetic energy is the braking energy.

For every experiment we obtained over the tim interval considered: a) the number of test particle that had escaped, i.e., whose energy relative to th nucleus had turned positive; b) the total change i the energy of the whole ensemble of test particles c) in the cases where the initial relative velocity wa not zero, the change in the kinetic energy of th nucleus, i.e., the braking energy. The results ar presented in Table 1. The first column identifie the models as follows: Models where the velocit distribution was initially the one corresponding t the Schuster law are identified with the letter *S* while those with a Gaussian distribution are denote with *G*; The roman numeral I characterizes model with zero relative velocity, while II and III denot

TABLE 1

NUMERICAL EXPERIMENTS

Case	# particles escaped	ΔE_{int}	ΔE_{brk}
S I 250	0	1.73	...
S III 250	0	0.78	16.80
G I 250	2	0.89	...
G I 177	0	2.00	...
G I 125	1	4.76	...
G II 250	0	0.09	11.69
G II 125	1	1.13	12.82
G II 159	0	0.33	11.61
G III 250	0	0.64	16.58
G III 125	0	1.13	17.43
G III 157	0	0.42	8.98

models with initial velocity 3.5 that were allowed to evolve, before adding the test particles, for 200 and 460 time units, respectively. Finally, the arabic numerals give the total number of field particles included in the models (cases 157 and 159 are those where the number of particles was reduced so as to match the background density of the equivalent case with 250 particles and zero relative velocity). The second column of Table 1 gives the number of particles that escape, the third column gives the internal energy change for the stellar system, and the last column gives the kinetic energy it gained due to dynamical friction (i.e., the braking energy).

It is immediately obvious from the table that both the number of escapes and the binding energy changes are much smaller than the values obtained by Miller & Smith (1985). Moreover, while the escapees are too few to show any clear relation to the number of field particles, it is very clear that the internal energy changes increase as the number of field particles decreases (and their individual masses increase, in order to keep constant the mass density); therefore, those energy changes are *upper limits* only because, in addition to any possible dynamical heating, they include the effect of the variable field due to the discreet number of particles (see, e.g., Hernquist & Barnes 1990). Moreover, the largest energy changes are found for the cases where the relative velocity is zero, i.e., dynamical friction cannot be causing dynamical heating because the largest possible "heating" is found when there is no "friction".

IV. CONCLUSION

Although our experiments were not sensitive enough to reject the possibility that very small fractions of the braking energy could enter the stellar system and, as indicated in the introduction, play havoc in its internal constitution, they clearly show that fractions as large as those claimed by Miller & Smith (1985) can be ruled out. If we neglect the experiments performed with small numbers of particles, we see that the energy absorbed by the stellar system cannot be larger than, at most, 5%. Moreover, part, perhaps all, of that energy arises just from the fluctuations in the gravitational field due to the limited number of field particles used, and not from "heating" caused by dynamical friction. This is confirmed by the fact that the largest energy changes were found in the experiments where the stellar system was at rest relative to the background.

Let us end with a word of caution about the application of our results to the real world. As indicated in the Introduction, the case of a globular cluster being braked within a galaxy implies a braking energy 1 000 times larger than the binding energy of the cluster, while the limitations of the numerical experiments forced us to consider cases where that energy ratio was two orders of magnitude smaller. It can be reasonably argued that, if dynamical heating actually exists, its effect should have been enhanced at the low speeds considered in our investigation because it is at those speeds that dynamical friction effects are largest. Alternatively, even an efficiency much smaller than our 5% limit could result in significant effects with braking to binding energy ratios closer to those of real cases. More refined numerical experiments with larger computers or, perhaps, analytical investigations along the lines of the one of Muzzio *et al.* (1988), might help to settle the issue in the future. For the time being, we may conclude that if dynamical heating actually exists, its efficiency is very low.

We are very grateful to L.A. Aguilar, R.H. Miller, B.F. Smith, and S.D.M. White for discussions on this subject; the comments of an anonymous referee were very useful to improve the original manuscript. This work was supported by grants from the Comisión de Investigaciones Científicas de la Provincia de Buenos Aires and the Consejo Nacional de Investigaciones Científicas y Técnicas de la República Argentina.

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