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A SIMPLE MODEL TO FIT ROTATION CURVES

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RESUMEN

Hemos estudiado analíticamente un modelo muy simple de la distribución de masa en galaxias planas, el cual consiste en un esferoide aplanado y un bulbo esférico, siendo ambos componentes inhomogéneos. Se muestran ajustes directos de las curvas de rotación derivados analíticamente con algunos tipos de curvas de rotación obtenidos observacionalmente. Hemos hecho una comparación con la solución de disco máximo sin un halo obtenido por Kent (1986).

ABSTRACT

We study analytically a very simple model of mass distribution in flat galaxies, consisting of a flattened spheroid and a spherical bulge, both components being inhomogeneous. We show direct fits of the analytically derived rotation curves with some types of observationally obtained rotation curves. A comparison is made with the maximum-disk solution without a halo obtained by Kent (1986).

Key Words: **GALAXIES - STRUCTURE**

1. INTRODUCTION

A usual procedure to construct galactic models consists in proposing specific mass density functions for each possible component (disk, bulge, halo), and with these functions obtaining the contribution of each component by means of numerical fits to the observed rotation curve. These fits give the values of the parameters appearing in the density functions.

It is of practical and theoretical interest to make use of analytic procedures to carry out the fits, because in this way the analytic relations between the parameters and the intrinsic properties of a rotation curve (position of the maxima and minima, velocity of rotation at these points, etc.) give a clear idea of how the mass is allotted; thus it is possible to sort out all the different kinds of rotation curves arising from the proposed mass distribution model.

The analytical treatment is possible if we smooth out the complexity of the density functions, while staying within the representative set of functions for a galactic system.

In this work we consider this analytic procedure in a flattened galaxy consisting of two inhomogeneous components: a plane spheroid and a spherical bulge. Some fits are made on several observed rotation curves using data of Rubin et al. (1985), and the best fits are compared with the maximum-disk solution without a halo discussed by Kent (1986).

2. PROPERTIES OF THE MASS DISTRIBUTION

As a first approximation we propose two components to represent the mass distribution in plane galaxies: (1) a Schmidt-type (1965), high-eccentricity inhomogeneous oblate spheroid (called the disk) with similar strata, and with density law

$$\rho_D(a) = qa^{-1} + pa$$

where a is the length of the major semi-axis of any similar spheroidal surface within the distribution; (2) an inhomogeneous spherical bulge at the center of the system, with density law

$$\rho_B(a) = r + sa.$$

Both densities are decreasing functions of a; the parameters p, q, r, s are to be obtained analytically from the overall properties of this mass distribution and from the characteristics of an observed rotation curve.

The spatial extensions of both components are constrained with the requirement that ρ_D and ρ_B be positive. In particular at the boundary of the bulge $\rho_B = 0$; then introducing the radius of the bulge, R_B, the number of independent parameters to be determined still remains four.

Using potential theory one easily obtains the analytical expression for the circular rotation on the plane of symmetry of the mass distribution. With this expression we find the position of the maxima and minima and the scaled rotation velocity reached at these points. It turns out that within the bulge there is a maximum, at a position we shall call R_0, and outside this bulge, but still within the disk, there are two extreme points: a minimum at a position designated R_1 and a maximun at R_2 ($R_1 < R_2$).

For an observed rotation curve showing the above-mentioned characteristics, we enter the values of the four quantities R_0, R_1, R_2, and the rotation at R_0, $\Theta(R_0)$, to obtain the four independent parameters of our model.

With some manipulation of all the equations at hand we obtain the expressions of the four parameters, and then the expression for the rotation in terms of R_0, R_1, R_2 and $\Theta(R_0)$, which is needed to obtain a fit.

Let

$$\mu = R_0/R_B \, , \alpha_1 = R_1/R_0 \, , \alpha_2 = R_2/R_0 \, ,$$

and

$$\xi = \Theta(R)/\Theta(R_0) \, .$$

Also the following constant is important in the discussion:

$$C = \frac{2}{3}\left[2 + \frac{\alpha_1^2 + \alpha_2^2}{\mu^3 \alpha_1^2 \alpha_2^2}\right] \, . \qquad (1)$$

Then with $\tau = R/R_0$ and $\tau_D = R_D/R_0$ (the boundary of the disk, with no constraint at the start other than $\rho(\tau_D) \geq 0$) we obtain the following expressions for the quadratic scaled circular rotation at a position τ:

$\tau \leq 1/\mu$:

$$\xi^2(\tau) = -\frac{2\tau}{C}\left[1 - \frac{3}{4}C - 2\tau + \left(1 + \frac{C}{4}\right)\tau^2\right] \, , \qquad (2)$$

$1/\mu \leq \tau \leq \tau_D$:

$$\xi^2(\tau) = -\frac{2\tau}{C}\left[1 - \frac{3}{4}C + \frac{1}{4}(4 + C - 6\mu)\tau^2 - \frac{1}{2\tau^2\mu^3}\right] \, .$$

To use these equations we must first find the value of μ. This is obtained from the solution of a quartic equation; the steps are the following:

$$F_1 = \frac{\alpha_1^2 + \alpha_2^2 - 1}{\alpha_1^2 \alpha_2^2} \, ,$$

$$F_2 = \sqrt{1 + \frac{27}{16}F_1} \, ,$$

$$F_3 = \frac{9}{4}\left(\frac{1}{2}F_1\right)^{1/3}\left[(F_2 - 1)^{1/3} - (F_2 + 1)^{1/3}\right] \, , \qquad (3)$$

$$F_4 = \sqrt{1 + F_3} \, ,$$

$$F_5 = F_4 + \left[3 - F_4^2 + \frac{2}{F_4}\right]^{1/2} \, ,$$

$$\mu = \frac{2}{9}(1 + F_5) \, .$$

With the value of μ we obtain the radius of the bulge, R_B, and the constant C in equation (1). The mass of the bulge (in units where the gravitational constant is equal to one) will be given by:

$$M_B = \frac{R_0 \Theta^2(R_0)}{\mu^3 C} \, . \qquad (4)$$

Up to this point the expressions are independent of the disk's eccentricity, e; i.e., a given rotation curve may arise from different mass distributions. For a fixed eccentricity the position at which $\rho_D = 0$ is given by

$$(\tau_D)_L = \left[\frac{3E_1}{E_2}(\alpha_1^2 + \alpha_2^2)\right]^{1/2} \, ; \qquad (5)$$

with

$$E_1 = \frac{2 - (2 + e^2)\sqrt{1 - e^2}}{3e} \, ,$$

$$E_2 = e\left(1 - \sqrt{1 - e^2}\right) \, .$$

The boundary of the disk, $\tau_D = R_D/R_0$, must be such that $\tau_D \leq (\tau_D)_L$. The mass of the disk is obtained as follows

$$M_D = \frac{M_B e^3 \tau_D^4}{12 E_1 \alpha_1^2 \alpha_2^2}\left[2\left[\frac{(\tau_D)_L}{\tau_D}\right]^2 - 1\right] \, . \qquad (6)$$

3. APPLICATIONS OF THE MODEL

In Figure 1 we show some fits on twelve of several observed rotation curves of flattened galaxies obtained by Rubin et al. (1985). Table 1 shows the values of R_0, $\Theta(R_0)$, α_1, α_2 used in these fits. When $\alpha_1 = 1$ the minimum at R_1 coincides with the maximum at R_0 (in this case $R_0 = R_B$, i.e., $\mu = 1$), giving rise to an inflection point in the rotation curve.

Table 1 also lists the radius, R_B, and mass, M_B, of the spherical bulge. As to the disk, we can estimate the eccentricity e of the spheroid with similar strata representing this disk for a constant mass-luminosity ratio, and using the inclination i at which the system is observed (see Rubin et al. 1985 for references giving i) and the isophotal ellipticity ε (measured by Kent 1986).

The equation relating these quantities is

$$e = \frac{\sqrt{\varepsilon(2-\varepsilon)}}{\sin i}, \; i \neq 0 . \qquad (7)$$

Using this equation we obtain $e > 1$ for some galaxies in Table 1; this means that there are inconsistencies and/or errors in the values i, ε and the assumption of similarity of the mass distribution of the disk.

We have taken $e = \sqrt{1-(0.1)^2}$ to re-calculate the inclination i, the quantity supposedly with the largest error, and have obtained values close to the ones given in the references (Rubin et al. 1985): a mean of 3.3 degrees in the difference for the galaxies in Table 1. With the above-mentioned value of e we obtain the limiting radius and mass of the disk, $(R_D)_L$, $(M_D)_L$ (see eqs. (5) and (6)) listed in Table 1.

4. DISCUSSION AND CONCLUSIONS

Figure 1 shows that with the proposed simple model good fits can be obtained in a number of cases and this is easily done using the analytical expressions emerging from the model. In particular we have applied this procedure in the study of the central region of NGC 4736 (Pişmiş & Moreno, 1993*a*,*b*).

Of course, this simple model is not well suited for every observed rotation curve; at present we are carrying out the analytical study of a model consisting of three components: bulge, disk, and halo, and expect to be able to handle some complicated rotation curves given by Rubin et al. (1985).

Kent (1986) has analyzed some rotation curves of Rubin et al. (1985), based on photometry. A

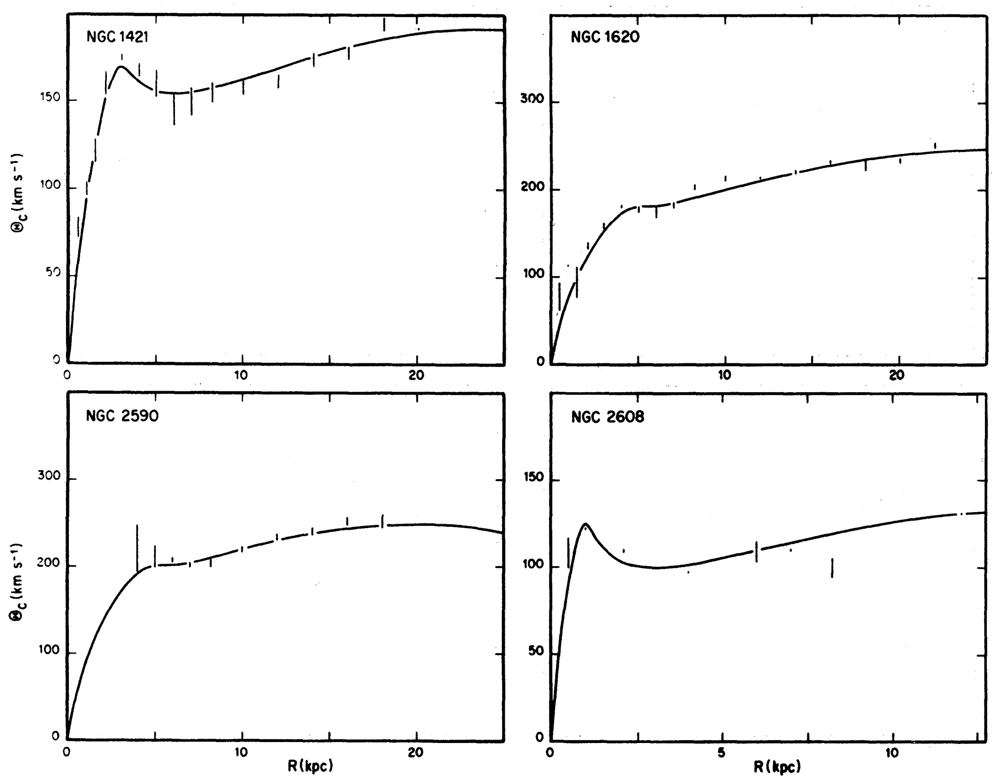

Fig. 1*a*. Some fits with our model (continuous line) of observational rotation data obtained by Rubin et al. (1985).

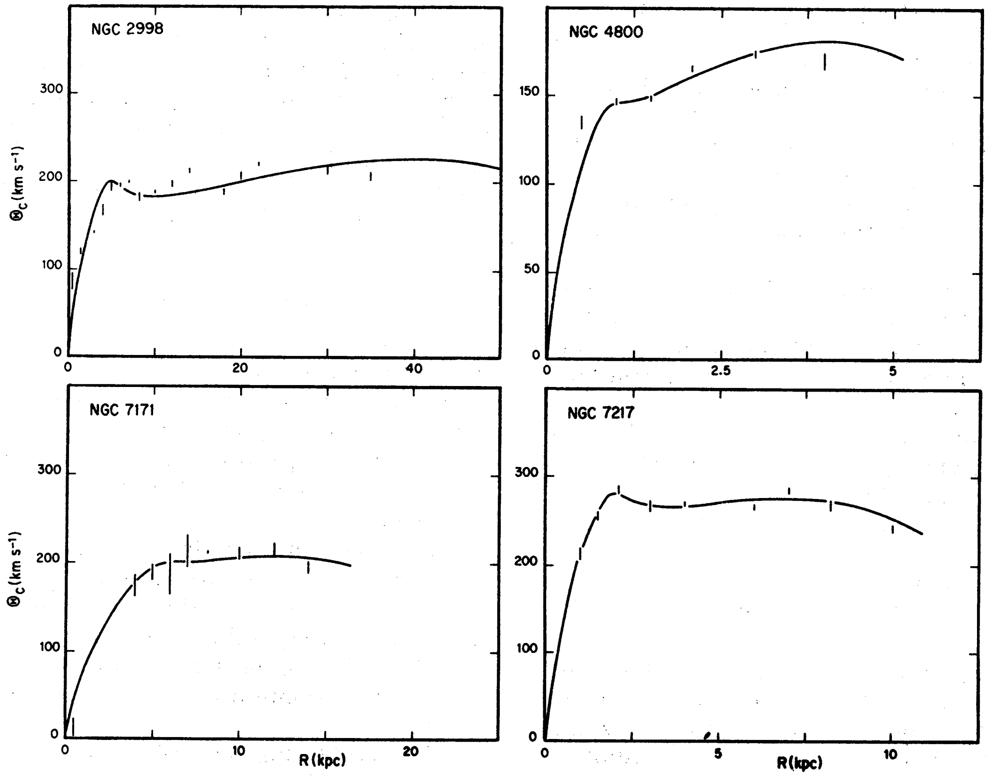

Fig. 1*b*. Same as Figure 1*a*.

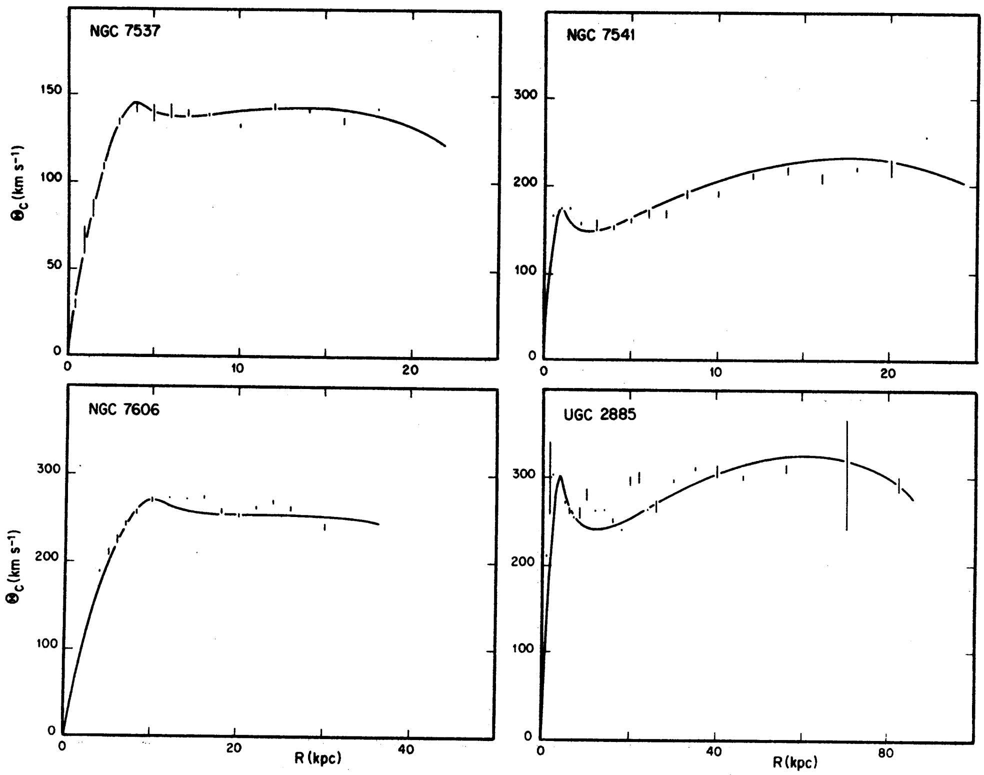

Fig. 1*c*. Same as Figure 1*a*.

TABLE 1

SOME PROPERTIES OF OUR FITS

Object	R_0 (kpc)	$\Theta(R_0)$ (km s^{-1})	α_1	α_2	R_B (kpc)	$(R_D)_L$ (kpc)	M_B (10^{10} $M_\odot$)	$(M_D)_L$ (10^{11} $M_\odot$)	R_{opt} (kpc)	$(M_D)_{opt}$ (10^{11} $M_\odot$)	$(M_T)_{opt}$ (10^{11} $M_\odot$)	$(M_T)^K_{opt}$ (10^{11} $M_\odot$)
NGC 1421	3	170	2	8	3.24	34.18	1.629	1.545	24.7	1.192	1.355	1.27
NGC 1620	5	180	1	5	5.00	35.23	1.854	2.633	30.9	2.492	2.677	2.64
NGC 2590	5	200	1	4	5.00	28.48	2.273	2.155	32.6	2.155	2.382	2.58
NGC 2608	1	125	3	15	1.10	21.14	0.339	0.481	14.1	0.333	0.367	0.22
NGC 2998	5	200	2	8	5.40	56.97	3.759	3.565	39.4	2.595	2.970	1.93
NGC 4800	1	145	1	4	1.00	5.7	0.239	0.226	8.0	0.226	0.245	0.20
NGC 7171	6	200	1	2	6.00	18.54	2.570	0.843	19.6	0.843	1.100	1.47
NGC 7217	2	280	1.75	3.5	2.13	10.81	2.637	0.865	13.7	0.865	1.129	1.57
NGC 7537	4	145	1.75	3.5	4.25	21.63	1.414	0.464	19.5	0.448	0.589	0.44
NGC 7541	1	175	2.5	17.5	1.10	24.43	0.633	1.695	28.3	1.695	1.758	1.66
NGC 7606	10	270	2	2.5	10.65	44.24	12.293	2.712	40.1	2.626	3.855	3.61
UGC 2885	4	300	3	15	4.41	84.54	7.814	11.093	90.4	11.093	11.874	9.81

possibility he considers is that the luminous matter dominates the system, the observed luminosity being associated with a bulge and a disk, where the bulge is of finite extent. To obtain the rotation curve one first needs to deproject the observed brightness to obtain the mass densities of both components, which are in general complicated functions of position. This model of Kent, called the maximum-disk solution, is similar to the one proposed here as to the number of components and the shape and finite extension of the bulge; it is therefore of interest a comparison of both models. Kent shows in his Figure 2 some fits to observed rotation curves, using the maximum-disk solution. In particular for the galaxies listed in Table 1, his fits are somewhat similar to the fits in Figure 1 shown in this paper.

The last two columns in Table 1 present a comparison of the integrated total mass up to a distance of R_{opt}. $(M_T)_{opt}$ is for our model, and $(M_T)^K_{opt}$ corresponds to the maximum-disk solution of Kent. As far as this integrated total mass is concerned, we can see that our model gives a good approximation to the more elaborate model of Kent, giving at the same time a good fit to the rotation curve.

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