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SMOOTHED PARTICLE HYDRODYNAMICS AND THE SIMULATION OF GALAXY AND LARGE-SCALE STRUCTURE FORMATION

Paul R. Shapiro and Hugo Martel
Department of Astronomy, The University of Texas at Austin

Jens V. Villumsen
Department of Astronomy, Ohio State University

and

Hyesung Kang
Department of Astrophysical Sciences, Princeton University

RESUMEN

A continuación describiremos el desarrollo de un nuevo método llamado Hidrodinámica de Partículas Suavizada Adaptiva (ASPH), generalizado para cosmología y acoplado al método de Malla de Particulas (PM) para resolver la ecuación de Poisson, para la simulación de formación de galaxias y estructura de gran escala. La simulación numérica precisa de fenómenos de choques y cáusticos con alto grado de no-linearidad, que ocurren generalmente en el proceso de formación de estructuras requiere un intervalo dinámico de resolución enormes. Los métodos numéricos existentes no pueden alcanzar la resolución requerida con la tecnologia computacional actual. El progeso a la resolución de este problema será presentado a continuación.

ABSTRACT

The development of a new method, called *Adaptive Smoothed Particle Hydrodynamics* (ASPH), generalized for cosmology and coupled to the Particle Mesh (PM) method for solving the Poisson Equation, for the simulation of galaxy and large-scale structure formation, will be described. The accurate numerical simulation of the highly nonlinear phenomena of shocks and caustics which occur generically in the process of structure formation requires enormous dynamic range and resolution. Existing numerical methods cannot achieve the required resolution with current computer technology. Progress toward the solution of this problem will be presented.

Key words: **COSMOLOGY: THEORY — GALAXY: FORMATION — HYDRODYNAMICS — INTERGALACTIC MEDIUM — METHODS: NUMERICAL — SHOCK WAVES**

1. INTRODUCTION

Primordial, cosmological density fluctuations and gravitational instability lead to gravitational collapse, strong shocks, and radiative cooling, occurring over an enormous range of mass and length scales. 1D pancake-like structures are a generic feature of the nonlinear growth of a 3D spectrum of density of fluctuations. 1D pancake calculations show that, in the absence of radiative cooling, a dynamic range of 10^3 in density and in length scale is required to resolve pancakes. *With* radiative cooling, the dynamic range increases to 10^5. Existing numerical hydro methods cannot achieve such high resolution within the limits of current supercomputers. In order to achieve the required dynamic range, a 3-D, Eulerian grid (i.e., fixed spatial grid, uniformly spaced) must have 10^3 cells *per dimension*, or 10^9 cells, for simulations *without* radiative cooling and 10^5 cells *per dimension* or 10^{15} cells *with* radiative cooling! Current Eulerian methods are limited by existing hardware to $\approx 10^2$ cells per dimension. Existing "Lagrangian" hydro methods such as SPH have typically used a number of particles $N_{part} \sim 32^3$ or less. As we shall demonstrate, this limitation and the requirement that artificial viscosity be used to treat shocks has kept the capabilities of this method below the level required.

2. STANDARD SMOOTHED PARTICLE HYDRODYNAMICS (SPH)

The SPH method (see Monaghan 1985, and references therein) is a Lagrangian numerical hydrodynamics method which replaces the continuous baryon-electron fluid by a set of discrete gas "particles" which carry mass and thermal energy and move with the local flow velocity. The particles are essentially moving centers of interpolation for representing the continuous flow variables. The principal virtue of this method is that, as a Lagrangian method, it has numerical resolution which dynamically adjusts so as to follow the compression, expansion, and distortion of the flow.

The resolving power of the SPH method is related to the so-called "smoothing length" h used for its interpolations, which defines a kind of spherical "zone of influence" centered on each particle. Initially this smoothing length is of order the mean interparticle spacing. In order for the resolution to adjust dynamically to accommodate a changing gas density, standard SPH varies h according to $h \propto \rho^{-1/3}$, so that particles maintain a constant number of nearest neighbors (defined as the particles located within a distance $r < 3h$). The interpolations which the SPH method performs in order to evaluate fluid quantities are based on the so-called kernel function W, which is an isotropic function of h. For a Gaussian isotropic kernel function W,

$$W(r,h) = \pi^{-3/2} h^{-3} \exp(-r^2/h^2)\,. \tag{1}$$

Artificial viscosity must be introduced in the standard SPH equations in order to prevent particle crossing and allow shocks to form. The standard forms of artificial viscosity are the bulk viscosity $\Pi_l = -\alpha\rho\ell c_s \nabla\cdot\mathbf{v}$, and the von Neumann – Richtmeyer viscosity $\Pi_q = \beta\rho\ell^2(\nabla\cdot\mathbf{v})^2$, where α and β are constants of order unity, ℓ is the length scale over which shocks are typically spread, and c_s is the sound speed. In SPH, the above expressions are computed *for each pair of approaching particles, at each time step*, with the smoothing length h replacing ℓ.

3. ADAPTIVE SMOOTHED PARTICLE HYDRODYNAMICS (ASPH)

Standard SPH suffers from two limitations which become particularly serious in the generic flows which occur in galaxy and large-scale structure formation, involving gravitational collapse through orders of magnitude of compression and strong shock waves: (1) The isotropic variable h is valid for nearly isotropic compressions or expansions, but breaks down in the presence of strongly anisotropic compression, such as in pancake collapse, where the resolution required in order to adequately describe the flow depends strongly on direction, and (2) shocks require artificial viscosity, but gravitational collapse then results in false "preheating" of supersonically infalling gas, far outside of the actual shock location. This artificially spreads out the shock-heating and can be disastrous for calculations which include radiative cooling, for example.

In ASPH, we replace h by an *anisotropic smoothing tensor* $\mathbf{H}$. In this new formalism, the spherical zone of influence of a given particle is replaced by a triaxial ellipsoid. The H-tensors are dynamically evolved by the ASPH code using the deformation tensor $\partial v_i/\partial x_j$ to follow the local deformation and vorticity of the flow. The 3D Gaussian kernel becomes:

$$W(\mathbf{r},\mathbf{H}) = \left[\pi^{-1/2}h_1^{-1}\exp(-x'^2/h_1^2)\right]\left[\pi^{-1/2}h_2^{-1}\exp(-y'^2/h_2^2)\right]\left[\pi^{-1/2}h_3^{-1}\exp(-z'^2/h_3^2)\right], \tag{2}$$

where h_1, h_2, and h_3 are the semimajor axes of the ellipsoid, and x', y', and z' are the projections of $\mathbf{r}$ along each of these axes. This technique has three strong advantages over standard SPH: (1) The resolving power of the SPH equations in regions with strong anisotropy can be increased by up to several orders of magnitudes. (2) The H-tensors track the motion of the flow, and consequently each particle keeps roughly the same set of neighbors for many timesteps. Using this, we can speed-up the algorithm by searching for the nearest neighbors only occasionally. (3) Caustics and shocks in the flow can be predicted to occur whenever one of the axes of a particle's H-tensor ellipsoid shrinks to zero. The ASPH method turns artificial viscosity on *only* for those particles for which this is about to happen. This automatically restricts artificial viscosity to just those fluid particles which are actually encountering a shock.

4. THE PANCAKE COLLAPSE TEST PROBLEM.

We focus in what follows on a tough test problem, that of the gravitational collapse of a 1D, plane wave density fluctuation in a universe comprised of baryons and collisionless dark matter. This is the cosmological pancake problem, in which an initially linear amplitude density fluctuation grows to nonlinear amplitude, forms a caustic in the dark matter distribution located in the plane of symmetry of the pancake and strong accretion shocks, one on each side of this central plane, followed by continued infall, phase-mixing of the dark matter, and radiative cooling of the shocked baryon-electron plasma. Detailed, 1D numerical solutions for this problem already exist, as do approximate analytical solutions, for comparison (Shapiro & Struck-Marcell 1985).

Our ASPH method, in principle, can automatically identify and adjust to accommodate a pancake collapse and shocks along *any* direction, not known *a priori*. To test this, we use a 2D version of the SPH and ASPH methods. In both cases the gravitational force is computed using a standard Particle-Mesh (PM) method. We consider a box of size $L_{box} \times L_{box}$ with periodic boundary conditions and one edge-on pancake ($\lambda_p = L_{box}/\sqrt{2}$) oriented with symmetry plane (*i.e.* density maximum) along the box diagonal. This will test that ASPH can adjust to follow the anisotropic collapse of the pancake regardless of its orientation. We take $N_G = N_{DM} = 64 \times 64 = 4096$ particles, each, of gas and dark matter, (equal mass particles), with PM grid of 128×128 cells. Notice that given the 45° angle of tilt and the periodic boundary conditions, there are only 22 gas particles per row perpendicular to the pancake to resolve the flow for each side of the central plane. The simulations start at redshift $z_i = 27$, with initial amplitude adjusted so that the shock forms at $z_c = 6$. We evolve the system up to $z_{final} = 3.666$. Radiative cooling is neglected.

Figure 1 shows the results of this test. The ability of ASPH to match the analytical solution with *no preheating* and with the slope of the postshock profiles in agreement over orders of magnitude of position and density variation is in contrast to the standard SPH results which show preheating, shock spreading, and poor postshock profile fitting.

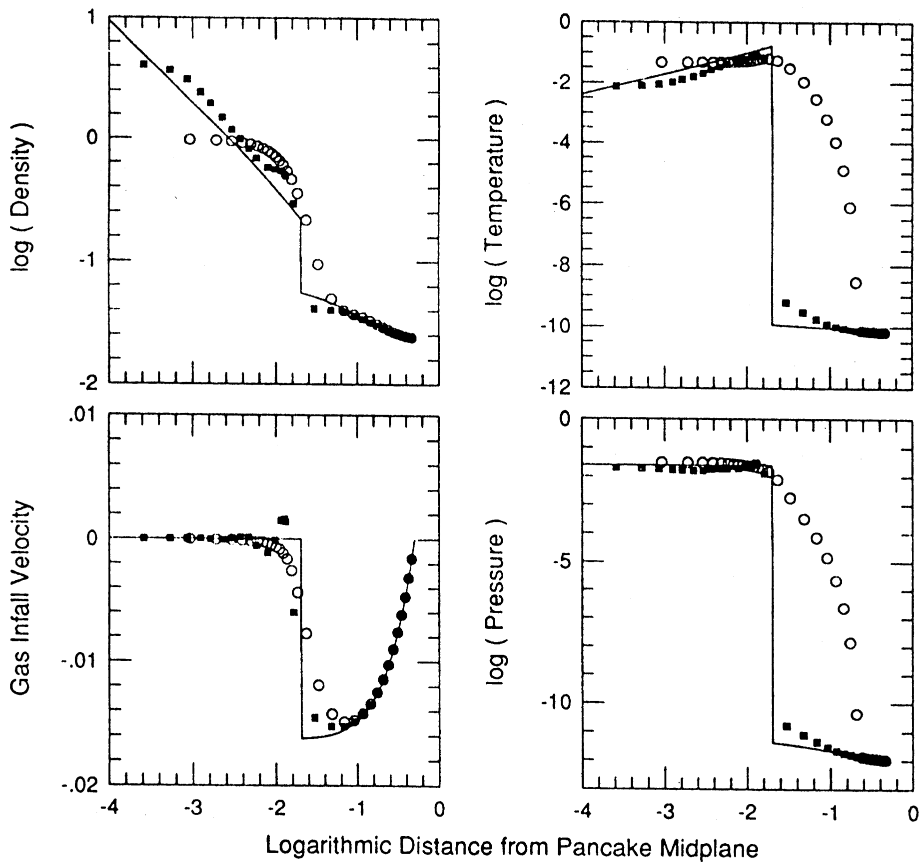

Fig. 1. — Tilted pancake results at $z = z_{final}$ for SPH (open circles), ASPH (filled squares), and analytical solution (solid line). Dimensionless variables versus position (length unit= λ_p).

5. 2D SIMULATION USING ASPH/PM: THE EXAMPLE OF AN HDM MODEL.

We apply our 2D, ASPH/PM method to simulate the growth of a spectrum of primordial density fluctuations corresponding to an HDM model, with $N_G + N_{DM} = 256 \times 256 = 65,536$ equal mass particles, with $N_G = N_{DM}/7$, in a box with 512×512 PM cells, $\lambda_{damping} \approx L_{box}/3$, and amplitude so that density fluctuation modes with $\lambda = \lambda_{damping}$ become nonlinear at $z = 9.5$.

Figure 2 shows the gas distribution at $z = 6$. As expected, the typical first structures to form are pancake-like, with denser concentrations at the intersections of pancakes. The second panel shows a blow-up of a tiny part of the computational grid, a box $(L_{box}/25) \times (L_{box}/25)$ centered on one such pancake (seen edge-on). *This demonstrates the remarkably high resolution of the simulation, with shocked pancake gas particles occupying a layer as thin as* 10^{-3} *of* L_{box}*!*

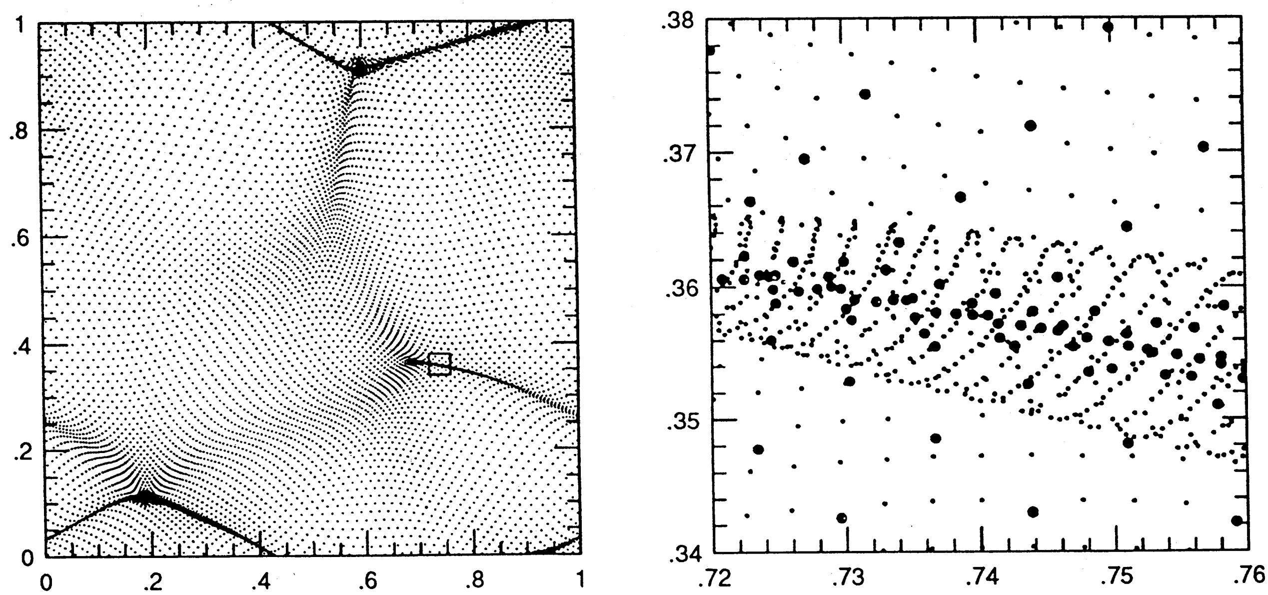

Fig. 2. — Left: positions of gas particles only, at $z = 6$. Right: enlargement of the little box shown in the left panel. Small and large dots represent dark matter and gas particles, respectively.

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Hyesung Kang: University of Princeton, Dept. of Astrophysical Sciences, Peyton Hall, Princeton, NJ 08544-1001, U.S.A.
Paul R. Shapiro and Hugo Martel: University of Texas at Austin, Astronomy Dept., Austin, TX 78712-1083, U.S.A.
Jens V. Villumsen: Ohio State University, Dept. of Astronomy, 174 W. 18th Ave. Columbus, OH 43210, U.S.A.