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ATTENUATION OF ULTRAVIOLET RADIATION BY DUST IN INTERSTELLAR CLOUDS

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RESUMEN

Se han obtenido soluciones de la ecuación de transporte para la dispersión coherente, no conservativa y anisotrópica para estimar la precisión de métodos aproximados, usados en modelos de nubes en que la luz es atenuada principalmente por el polvo. En los cálculos se ha aplicado el método de armónicos esféricos para distintos parámetros del polvo. Se ha explorado la posibilidad de descubrir cambios en las caracterísiticas del polvo mediante observaciones de regiones fotodisociadas. Se muestra que para altos valores del albedo de dispersión simple y del parámetro de asimetría de la función de fase que son adecuados para el polvo galáctico, no es posible determinar variaciones de más de un factor de 2 en el cociente de gas a polvo.

ABSTRACT

Solutions to the transfer equation for coherent, non-conservative, anisotropic scattering have been obtained in order to estimate the accuracy of approximate methods used in models of clouds where light is attenuated mostly by dust. In the calculations the spherical harmonic method has been applied for different grain parameters. The possibility of discovering changes of dust characteristics through observations of photodissociation regions has been considered. It is shown that for the high values of the single scattering albedo and the asymmetry parameter of the phase function for redistribution that appear to be appropriate for galactic dust, it is not possible to determine variations of more than a factor of 2 in the gas to dust ratio.

Key words: **DUST, EXTINCTION — RADIATIVE TRANSFER — SCATTERING**

1. INTRODUCTION

Chemical and physical processes that take place in interstellar clouds are controlled in many cases by th radiation that penetrates the cloud. To understand those processes it is necessary to understand how radiation i attenuated as it enters a cloud of dust and gas. The UV radiation field of young stars photodissociates molecule and creates atomic hydrogen regions. Thus the observation of molecular clouds at the 21 cm hydrogen line offer an opportunity to study the absorption and scattering properties of dust inside molecular clouds. Althoug H_2 molecules produce line absorption of radiation around 1100 A, the size of a photodissociation region i largely determined by the attenuation of the radiation by dust. Detailed models to interpret the observation have been constructed (e.g., Tielens & Hollenbach 1985; Sternberg & Dalgarno 1989). However, the optica and physical properties of dust grains are largely unknown, and there has been much speculation on how the vary among different sites. Models show that the size of a photodissociation region can be estimated from th optical depth of penetration of the UV radiation, which is $\tau_{\rm UV} \sim 1$ around 1100 A (Escalante 1991). Thi relation can be used to estimate how much the unknown parameters can be varied in theoretical models fc a given region, where the column density of atomic hydrogen $N_{\rm H}$, is known, and $\tau_{\rm UV}$ is fixed. We can writ

$$\tau_{\rm UV} = 1.086kA_{\rm UV} = 1.086k_m\left(\frac{A_{\rm UV}}{A_{\rm V}}\right)R_{\rm V}\,\xi\,N_{\rm H}\ , \qquad (1)$$

where

$$R_{\rm V} = \frac{A_{\rm V}}{E(B-V)}\ ,\ \text{and}\ \xi = \frac{E(B-V)}{N_{\rm H}}\ .$$

The canonical value of the gas to dust ratio, ξ^{-1}, is $(6.0 \pm 1.5) \times 10^{21}$ *atoms* cm^2 mag (Bohlin 1975). The factor k_m is usually introduced to account for radiative transfer effects, but this is only correct in the asymptotic regime as shown below.

Eugenio Mendoza (Johnson & Mendoza 1964) was among the first to point out that the Orion region showed anomalously large values of $R_{\rm V}$, and there is a lot of evidence that the extinction curve $(A_\lambda/A_{\rm V})$ varies strongly in the UV in different sites. Nevertheless, Cardelli, Clayton, & Mathis (1989) have shown a remarkably constant value for $(A_\lambda/A_{\rm V})\,R_{\rm V}$ at several wavelengths. On the basis of observations and detailed models, Rogers and Pedlar have suggested variations in ξ (e.g., Dewdney & Roger 1982; Roger & Pedlar 1981). It is apparent, therefore, that in in a detailed model, the radiative transfer problem, represented by the factor k_m in equation (1), must be solved accurately before conclusions about the variation of dust physical parameters are reached. Below will be shown the amount of variation that can be introduced by the radiative transfer in model calculations of photodissociation regions.

2. THE RADIATIVE TRANSFER PROBLEM

2.1. General Solution for a Slab

Flannery, Roberge, & Rybicki (1980, hereafter FRR) showed that problems with anisotropic coherent nonconservative scattering could be solved with considerable accuracy by using the method of spherical harmonics. Previously this method had been used to solve problems in radiative transfer and neutron transport theory (Chandrasekhar 1960; Davison & Skyes 1957). Here we are interested in the case of clouds illuminated by a point source that represents a star, and we concentrate in the plane-parallel case. The problem can be simplified by assuming a point source very far from the boundary, which is located at $\tau = 0$. The formulation by FRR can then be easily extended by separating the diffuse and direct fields. Once photons from the direct field, of an intensity I^* in the direction $\mu = -1$, are scattered once, they become part of the diffuse field, and contribute to the transfer equation a source term of the form

$$S^*(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1} R(\mu,\mu')I(\tau,\mu')\,d\mu' = \omega I^* R(\mu,-1)e^{-\tau}\ ,$$

where $R(\mu,\mu') = \frac{1}{2}\int p(\cos\Theta)d\phi'$ is the scattering phase function averaged over the azimuthal angle ϕ, and ω is the single scattering albedo. The transfer equation for the diffuse field is then

$$\mu\frac{\partial I^d(\tau,\mu)}{\partial\tau} = I^d(\tau,\mu) - S^d(\tau,\mu) - S^*(\tau,\mu)\ , \qquad (2)$$

with

$$S^d(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1} R(\mu,\mu')I^d(\tau,\mu')\,d\mu'\ .$$

In the spherical harmonics method, the intensity and phase function are approximated as a truncated series of Legendre polynomials (the P_L approximation):

$$I^d(\tau,\mu) = \sum_{\ell=0}^{L}(2\ell+1)F_\ell(\tau)P_\ell(\mu)\ , \qquad (3)$$

$$R(\mu,\mu') = \sum_{\ell=0}^{L}(2\ell+1)\sigma_\ell P_\ell(\mu)P_\ell(\mu')\ . \qquad (4)$$

The coefficients σ_ℓ depend on the phase function of the dust grains. In the case of the Henyey–Greenstein phase function (Henyey & Greenstein 1941), $\sigma_\ell = g^\ell$, where g is the asymmetry parameter. Substituting

Eqs. (3) and (4) in Eq. (2) and using the recurrence relation for the Legendre polynomials $(2\ell+1)\mu P_\ell(\mu) = (\ell+1)P_{\ell+1}(\mu) + \ell P_{\ell-1}(\mu)$, we obtain $L+1$ equations for the unknown functions $X_\ell(\tau)$

$$b_\ell X'_{\ell-1} + b_{\ell+1}X'_{\ell+1} = X_\ell - w_\ell I^* e^{-\tau} \ , \quad X_{L+1} = 0 \ , \qquad (5$$

where the primes denote differentiation with respect to τ, $h_\ell = (2\ell+1)(1-\omega\sigma_\ell)$, $b_\ell = \ell(h_{\ell-1}/h_\ell)^{1/2}$, an $w_\ell = (-1)^{-1}(2\ell+1)\omega\sigma_\ell/h_\ell^{1/2}$. The system of equations (5) differs from Eq. (15) of FRR in the last term, an the reader is referred to their paper for details of the calculations. The $L+1$ equations (5) can be written as

$$\mathbf{B}\ \ \mathbf{X}'(\tau) = \mathbf{X}' - I^*\mathbf{W}e^{-\tau} \ , \qquad (6$$

with

$$\mathbf{X} = \begin{pmatrix} X_0(\tau) \\ X_1(\tau) \\ \vdots \\ X_L(\tau) \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 & b_1 & & & \\ b_1 & 0 & b_2 & \bigcirc & \\ & b_2 & 0 & \ddots & \\ & & \ddots & \ddots & b_L \\ & \bigcirc & & b_L & 0 \end{pmatrix} \text{ and } \mathbf{W} = \begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_L \end{pmatrix}$$

A particular solution of the inhomogeneous Eq. (6) is $\mathbf{Q}I^*e^{-\tau}$, and the matrix $\mathbf{Q}$ can be found by solvin $(\mathbf{B}+\mathbf{I})\mathbf{Q} = \mathbf{W}$. The solution of the homogenous equation is $\mathbf{A}\exp(\mathbf{B}^{-1}\tau)$, where $\mathbf{A}$ depends on boundar conditions. We have assumed for simplicity no external radiation on the cloud, except for that of the star, an a semi–infinite medium. Thus, $I^d(0,\mu) = I^d(\infty,\mu) = 0$.

2.2. Characteristic Roots and Asymptotic Solutions

In the calculations it is necessary to solve the eigenvalue problem $\mathbf{Av} = \lambda\mathbf{v}$. For odd values of L there ar $(L+1)$ eigenvalues $\pm\lambda_1, \pm\lambda_2, \ldots \pm \lambda_{(L+1)/2}$ (see FRR, Davison & Skyes 1957). One problem with the spherica harmonics method is that cases with high ω and g converge slowly and a high L approximation is needed, eve when only the intensity moments are needed. FRR have discussed limiting cases. The main problem in thos cases is the near singularity of the boundary condition problem. Some authors have noted that finding th roots of the characteristic equation for the eigenvalues can be difficult for some values of g and ω. This is a important problem because for some applications the asymptotic solutions for $\tau \gg 1$ are of the form

$$I^d(\tau,\mu) \sim \sum_{\ell=0}^{L}(2\ell+1)P_\ell(\mu)A_m \exp(-1/\lambda_m) + h^{-1/2}Q_\ell I^* e^{-\tau} \ , \qquad (7$$

where λ_m is the highest eigenvalue; it may be enough to find the eigenvalues rather than to solve the probler completely. Eq. (7) shows that Eq. (1) is actually an effective optical depth with $k_m = \lambda_m^{-1}$. Furthermor problems with spherical symmetry also need the determination of eigenvalues for the same matrix $\mathbf{B}$, and ofte approximations for large L (~ 20) are needed (e.g., Tiné et al. 1992). An efficient method that is based on th mathematical properties of tridiagonal symmetric matrices and achieves full machine precision for most valu of interest of g and ω has been implemented.

2.3. Spherical Geometry

For spherical clouds with a central point source, a similar method as the one outlined above can b carried out. However, it is not as efficient and simple. The eigenfunctions for the spherical case are modifie spherical Bessel functions, which have strong divergences at the origin and at infinity. This fact makes th solution of the system of equations more difficult. Nevertheless, solutions have a similar form as in the cas of plane-parallel geometry, and it is possible to find simple approximate solutions. For example, the exa solution of Siewert & Grandjean (1979) for the mean intensity in an infinite sphere with isotropic scatterin can be reproduced within 3% by combining the direct field with the asymptotic expansion of the diffuse fiel $J(\tau) = A(r^{-2}e^{-\tau} + \beta r^{-1}e^{-1/\lambda_m})$. For $\omega = 0.3$, $\beta = 0.54$. An exact formulation of the problem will be publishe elsewhere.

3. RESULTS

We have calculated the mean intensity for characteristic values of g and ω, and plotted the total (diffus plus direct) mean intensity for the plane-parallel case in Figure 1. Because of the scattering, the intensit increases near the boundary due to the scattered light from other parts of the cloud, but in all cases it decrease

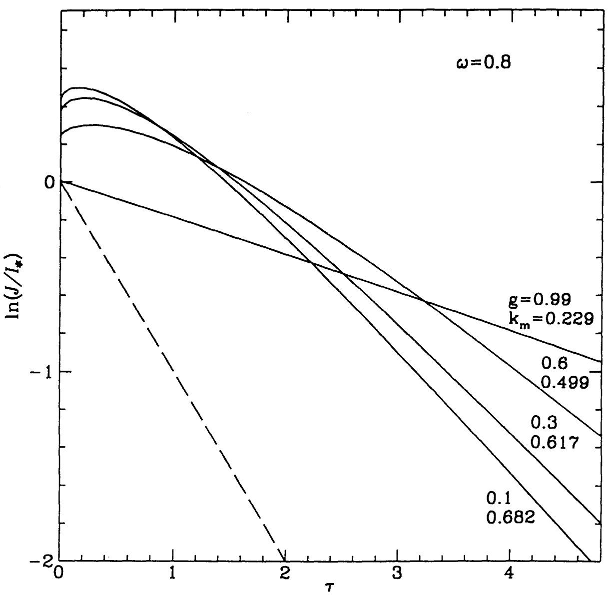

Fig. 1. Mean total intensity in units of the external radiation intensity I^* as a function of optical depth r $\omega = 0.8$ for different values of g. Calculated values of $k_m = \lambda_m^{-1}$ are also shown. The cloud boundary is at = 0. The curve $J(\tau) = I^* e^{-\tau}$ is plotted with a dashed line for reference.

ıd takes the asymptotic behaviour given by Eq. (7) quickly. The increase near the boundary is not observed r low values of ω.

The variation of k_m in Figure 1 show that changes in the apparent effective optical depth of Eq. (1) can : accounted for by radiative transfer effects, up to factors of 2 or 3 for high g, rather than by variations in the ıs to dust ratio ξ^{-1} required by some observations. It must be noted that ξ enters model calculations in the rmation of H_2 as well as in the dust optical depth; thus, more constraints can be imposed on dust parameters ₹oger & Pedlar 1981). Nevertheless, the calculations presented here show that there is more dependence on ,diation transfer than has been commonly accepted.

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