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ON THE THERMOMECHANICAL OSCILLATIONS OF X-RAY NOVAE

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RESUMEN

Se estudian las oscilaciones termomecánicas de una cáscara esférica radiante que contiene gas. Con una razonable elección de los principales parámetros se encuentra que el periodo de oscilación es similar al de las novas de rayos X.

ABSTRACT

Thermomechanical oscillations of a radiating spherically symmetric shell containing a gas are studied. With a reasonable choice of the relevant parameters the oscillation period is similar to that of X ray novae.

Key words: **HYDRODYNAMICS — STARS–NOVAE, CATACLYSMIC VARIABLES — X-RAYS–STARS**

1. INTRODUCTION

In this work we study a highly simplified model of X-ray nova burster, with spherical symmetry, introduced in previous work (Aquilano, Castagnino, & Lara 1986,1987) in order to describe the luminosity fluctuations of these astronomical objects.

The model consists on a central spherical nucleus, a white dwarf star, sorrounded by gas, enclosed in a spherical dust shell, in thermal equilibrium with the gas. The shell blackbody radiation and the luminosity fluctuations are caused by the oscillation of the shell radius. Although this model is extremely simple, it predicts quite well the observational data of X-ray novae (see Hoffman, Marshall, & Lewin 1978; Lewin & Clark 1980).

On the other hand we believe that the study of this oscillating system has also a mathematical interest, by itself. In fact, gravity forces, pressure and radiation force induce the shell to oscillate, if the relevant parameter lies between certain bounds (otherwise the shell will collapse to the white dwarf or will be ejected). We shall compute these bounds and show that, in this problem, a primary bifurcation exists when we describe the solutions in terms of the ratio of the shell mass and the gas mass.

2. THE CLASSICAL SOLUTION

To build our simplified model we shall consider that the shell is in thermal equilibrium with the gas which undergoes an adiabatic evolution, as we have said. Besides, we shall suppose that the masses of the shell and the gas are constant of the motion, and that the gas density is uniform and equal to

$$\rho(R) = \frac{\lambda}{R^3} \ , \tag{1}$$

where $R(t)$ is the shell radius and λ a constant

$$\lambda = \frac{3}{4\pi} m_g \ , \tag{2}$$

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with the mass of gas m_g. The pressure is

$$P = \alpha \rho^{5/3} , \qquad (3)$$

where α is a constant to be determined by the initial conditions. Different α gives different evolutions.

From equations (1) and (3) the temperature and the pressure turn out to be

$$T(R) = \frac{\alpha}{K R^2} \lambda^{5/3} , \qquad (4.a)$$

and

$$P(R) = \frac{\alpha}{R^5} \lambda^{5/3} , \qquad (4.b)$$

where K is the constant of the perfect gas law: $P = \rho K\ T$; P is the pressure, ρ is the density and T is the temperature.

Then the shell classical equation of motion reads

$$\ddot{R}(t) = f(R) = A_2 \frac{1}{R^2} + A_1 \frac{1}{R^6} + A_0 \frac{1}{R^3} \ ; \qquad (5)$$

where

$$A_2 = -G(M + \frac{m_s}{2}) , \qquad (6)$$

$$A_1 = - \frac{4\pi b}{3 m_s} (\frac{\alpha}{K})^4 \lambda^{8/3} , \qquad (7)$$

and

$$A_0 = \frac{4\pi\alpha}{m_s} \lambda^{5/3} . \qquad (8)$$

The gravitational constant is G, the mass of the white dwarf star is M, the shell mass is m_s and b is $4\sigma/c$, where σ is the Stefan-Boltzmann constant, and c is the velocity of light.

The first term on the r.h.s. of eq. (5) is caused by the attraction of the central mass M and the self-gravity of the shell, the second one is originated by the emitted radiation, and the last one is the internal gas pressure. Equation (5) shows the balance of two attractive terms (the first and the second) and expansive term (the third).

We obtain a dimensionless version of the equation of motion if we introduce the dimensionless variables

$$x = \frac{R}{R_0} , \quad \tau = \frac{t}{t_0} ; \qquad (9)$$

where R_0 is the singular point of eq. (5) neglecting the radiation term, being

$$R_0 = -\frac{A_0}{A_2} , \quad f(R_0) = 0 , \qquad (10)$$

and thus t_0 is the inverse of the oscillation frequency if there is no radiation around the singular point R_0, i.e.,

$$t_0^2 = \frac{1}{\omega_0^2} = \frac{A_0^3}{A_2^4} . \qquad (11)$$

We can also introduce the dimensionless coefficient

$$\Omega = -\frac{A_1}{A_0} \frac{1}{R^3} = \frac{4\,b}{243\,K^4} \frac{G^3 M^2}{\gamma} \frac{m_s^3}{m_g^3} \qquad (12)$$

where

$$\gamma = \frac{m_g}{M} . \qquad (13)$$

In these equations we have neglected the self-gravity term $(m_s/2)$, in the constant A_2 (eq. 6), with respect to M, because in our system $m_s << M$.

Using the new variables the equation (5) reads

$$x'' = -\frac{1}{x^2} + \frac{1}{x^3} - \frac{\Omega}{x^6} , \qquad (14)$$

where the primes symbolize the τ derivation.

Equation (14) will be integrated with the initial conditions $x'(\tau = 0) = 0$, i.e., the shell will initially be considered at rest. If $\Omega = 0$ a closed analytical solution is available.

In the general case $\Omega \neq 0$ we can make a study of the phase space to see what kind of motion the shell undergoes.

Equation (14) has real positive singular points if $\Omega < \Omega_c = 0.105$ (Ω_c is critical Ω). These points are

$$x^{\pm} = \frac{1-B}{4} \left[1 \pm (1 - f(i)^{1/2} \right] , \qquad (15)$$

where

$$B = -(1 + 8\,i)^{1/2} \quad , f(i) = \frac{16\,i}{B(B-1)} ,$$

and

$$i = \left(\frac{\Omega}{16}\right)^{1/3} \left\{ \left[1 + \left(1 - \frac{\Omega}{\Omega_c}\right)^{1/2}\right]^{1/3} + \right.$$

$$\left. + \left[1 - \left(1 - \frac{\Omega}{\Omega_c}\right)^{1/2}\right]^{1/3} \right\} .$$

If $\Omega = \Omega_c$ there is a unique singular point: $x_c = 0.75$. If $\Omega > \Omega_c$ there are no singular points.

The singular points lay in the interval

$$0 = x^-(\Omega = 0) < x^{\pm}(\Omega) < x^+(\Omega = 0) = 1 \quad ,$$

Thus $\Omega = \Omega_c$ is the bifurcation point for the singular points. The radius $x^+(\Omega)$ corresponds to a center. Around this point the shell undergoes stable oscillations. $x^-(\Omega)$ corresponds to a port, and there are no oscillations around it.

When there is no radiative term $\Omega = 0$, the center is $x^+(\Omega = 0) = 1$ and the port $x^-(\Omega = 0) = 0$.

The phase space, in (x, x') coordinates, has the following characteristics

$$x'' \begin{cases} > 0 & \text{if} \quad x\left[x^-(\Omega) \quad ; x^+(\Omega)\right] \\ < 0 & \text{if} \quad x < x^- \quad \text{or} \quad x^+ < x \quad , \end{cases}$$

and

$$x'' \{ < 0 \; \forall \, x \quad \text{if} \quad \Omega > \Omega_c \quad .$$

The bifurcation point sets a limit for the relation between m_s and m_g,

$$(m_s/m_g)^3 \leq \Omega_c \, \frac{243 \; K^4}{4\pi \, b \, G} \, \frac{\gamma}{M^2} \quad . \qquad (16)$$

When this relation is not satisfied, the shell inevitably collapses onto the white dwarf star.

When $\Omega < \Omega_c$, the shell can oscillate between $x^-(\Omega)$ and $x_{max}(\Omega)$ being these two points defined by the conditions

$$x'(x^- \; ; \; x^-) = 0 \quad \text{and} \quad x'(x_{max} \; ; \; x^-) = 0 \quad .$$

We are interested in the description of oscillations around the center. Linearizing the equation (14) around $x^+(\Omega)$, the oscillation frequency is

$$\omega^2(\Omega) = /\frac{1}{x^7}(3x^3 - 2x^4 - 6\Omega)/_{x=x^+} =$$

$$= \omega^{+2}(\Omega) \; . \qquad (17)$$

It can be easily proven that for any value of x different from x^+

$$\omega(\Omega) \leq \omega^+(\Omega) \quad .$$

Thus $\omega^+(\Omega)$ is a maximum for the frequencies.

The semiperiod can be obtained from

$$S(\Omega) = \frac{\pi}{\omega(\Omega)} \quad .$$

Also we can see the phase diagram for $\Omega = 0$, $\Omega < \Omega_c$ and $\Omega > \Omega_c$ in Figures (1a, 1b, and 1c). This completes our dimensionless analysis.

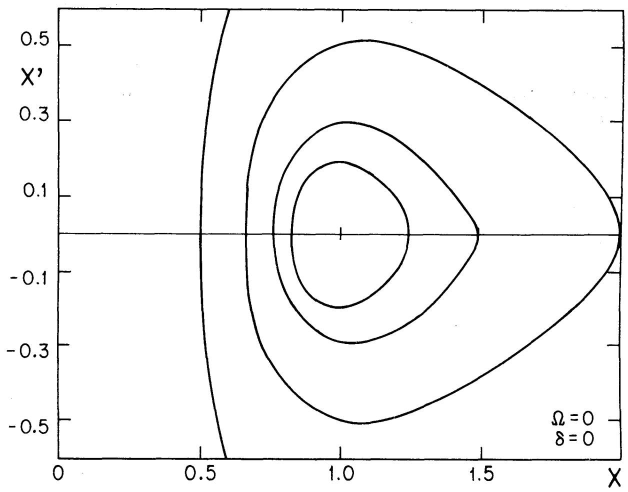

Fig. 1a. Phase diagram for $\Omega = 0$ (i.e., no radiation). All trajectories are stable oscillations.

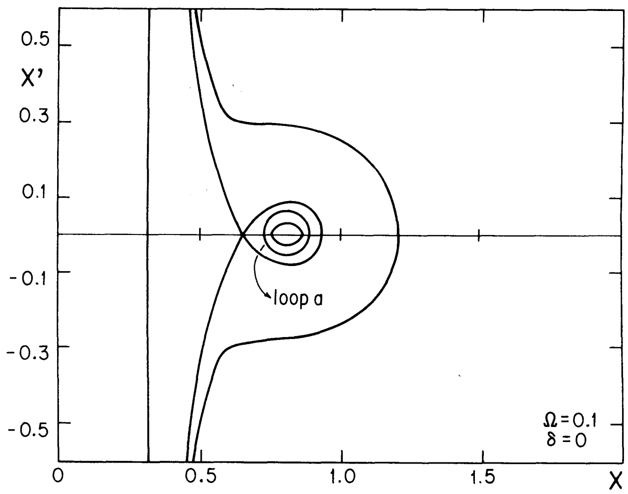

Fig. 1b. Phase diagram for $\Omega = 0.1$ (i.e., $\Omega < 0.1054$). There are two singular points x^+ and x^-. For $0.67 < x < 0.92$ oscillations may occur. For x outside this interval the shell always collapses.

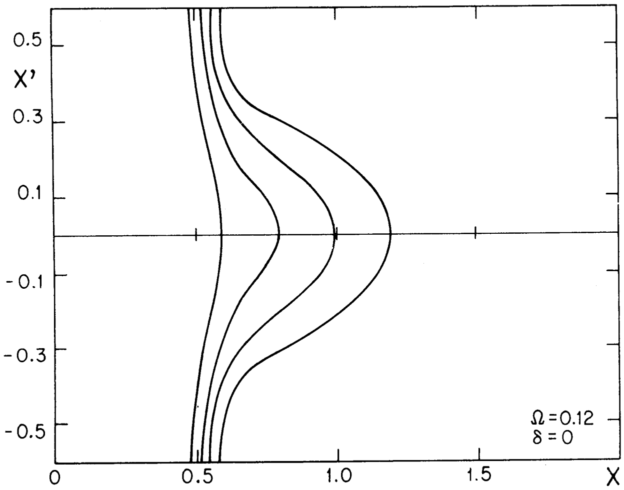

Fig. 1c. Phase diagram for $\Omega = 0.12$ (i.e., $\Omega > 0.1054$). There are no singular points. All trajectories yield as collapse of the shell.

We use eqs. (9), (10) and (13) to restore the dimension and we obtain

$$R_0(M,\gamma,\Omega,\alpha) = \left(\frac{b}{4\,\pi\,K^4}\right)^{1/3} M^{1/3}\left(\frac{\gamma}{\Omega}\right)^{1/3}\alpha \ , \tag{18}$$

and

$$t_0(M,\gamma,\Omega,\alpha) = \left(\frac{1}{2\,\pi^{1/2}\,K^2}\right) \left(\frac{b}{G}\right)^{1/2}\left(\frac{\gamma}{\Omega}\right)^{1/2}\alpha^{3/2} \ . \tag{19}$$

In a similar way we can obtain the temperature, luminosity, gas pressure and density as

$$T(R) = T_0\,T(X) = T_0\,x^{-2}\,,$$

$$T_0(\Omega,\,\alpha) = \left(\frac{3}{b}\right)^{2/3} K^{5/3}\,\Omega^{2/3}/\alpha\,;$$

$$L(R) = L_0\,L(X) = L_0\,x^{-6}\,,$$

$$L_0(M,\gamma,\Omega,\alpha) = \frac{\sigma K^4}{b^2}\,(4\pi)^{1/3}\; 3^{8/3}\,\gamma^{2/3}\,M^{2/3}\,\Omega^2/\alpha^2\,;$$

$$P(R) = P_0\,P(x) = P_0\,x^{-5}\,,$$

$$P_0(\Omega,\alpha) = \left(\frac{3\,k^4}{b}\right)^{5/3}\frac{\Omega^{5/3}}{\alpha^4}\,;$$

$$\rho(R) = \rho_0\,\rho(x) = \rho_0\,x^{-3}\,,$$

$$\rho_0(\Omega,\alpha) = \frac{3\,K^4}{b}\,\frac{\Omega}{\alpha^3}\,;$$

where T_0, L_0, P_0 and ρ_0 are the scale factors determined when the radius of the shell is at the center, defined by eq.(10), for the system without radiative term (see Fig. 2). These factors are given in c.g.s. units. $T(x), L(x), P(x)$ and $\rho(x)$ are shown in Figure 2.

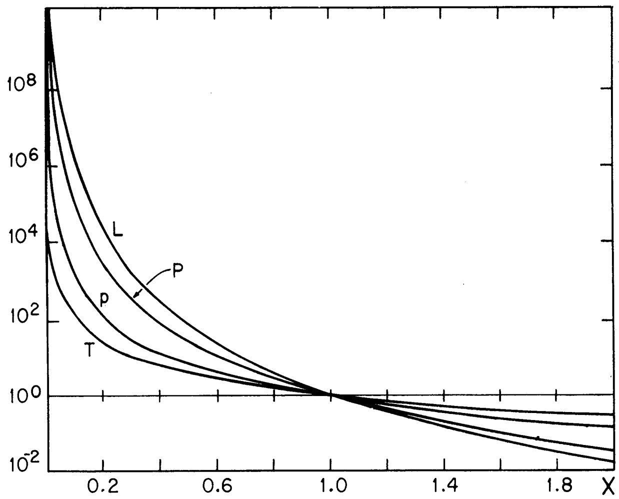

Fig. 2. The dimensionless functions $L(x), P(x), \rho(x)$ and $T(x)$ as function of the dimensionless variable x.

3. POST-NEWTONIAN CORRECTION

As the shell is moving in the strong gravitational field of the white dwarf, the post-newtonian correction is relevant.

If the shell moves with no relativistic velocity the corrected equation of motion obtained using standard techniques (see Weinberg 1972) is

$$x'' = g(x) = -\frac{1}{x^2} + \frac{1+\delta}{x^3} - \frac{\Omega}{x^6}\,, \tag{20}$$

where δ/x^3 is a correction term due to the central mass.

For relativistic velocities we would have the fully corrected equation of motion

$$x'' = h(x,x')\,, \tag{21}$$

where

$$h(x,x') = -(1 - 3/4\,\delta\,x'^2)\,1/x^2 + \\ + \left[\delta + (1 - \delta/4\,x'^2)^{1/2}\right]1/x^3 - \\ - \Omega(1 - \delta/4\,x'^2)\,1/x^6\,, \qquad (22)$$

being

$$\delta(\Omega,\gamma,\alpha) = \tilde{c}^{\,-2} = \left(\frac{c}{R_0/t_0}\right) = G/c^2$$

$$\frac{(4\,\pi\,K)^{4/3})}{b^{1/3}}\,M^{2/3}\left(\frac{\Omega}{\gamma}\right)^{1/3}1/\alpha\,, \qquad (23)$$

and $\tilde{c}$ is the adimensional light velocity.

Equations (14) and (20) are particular cases of eq. (22). The singular points of this equation could be computed as in the case of eq. (14) via the transformation

$$x \rightarrow \tilde{x}\,,\Omega \rightarrow \widetilde{\Omega}\,, g \rightarrow \tilde{g} \quad \text{and} \quad h \rightarrow \tilde{h}\;;$$

where

$$\tilde{x} = x\,\frac{1 - 3/4(\delta\,x'^2)}{\delta + (1 - \delta/4\,x'^2)^{1/2}}\;,$$

$$\widetilde{\Omega} = \Omega\,(1 - \delta/4\,x'^2)^{1/2}\,\frac{(1 - 3/4\,\delta\,x'^2)^3}{[\delta + (1 - \delta/4\,x'^2)^{1/2}]^4}\;,$$

and

$$\tilde{g}(x,x') = \tilde{h}(x,x') = -\frac{1}{\tilde{x}^2} + \frac{1}{\tilde{x}^3} - \frac{\Omega}{\tilde{x}^6}\;.$$

This equation is similar to the classical equation and obviously has the same singular points that eq. (14). Thus anti-transforming it we can obtain $x^+(\Omega,\delta)$.

The equation (20) is a particular case of eq. (22), when the velocity post-newtonian correction is neglected. The singular points in eq. (20) are the same as those of eq. (22).

We show the oscillation semiperiod (S) around a singular point $x^+(\Omega,\delta)$ in Figure 3 which is obtained using the classical computation with the time transformation

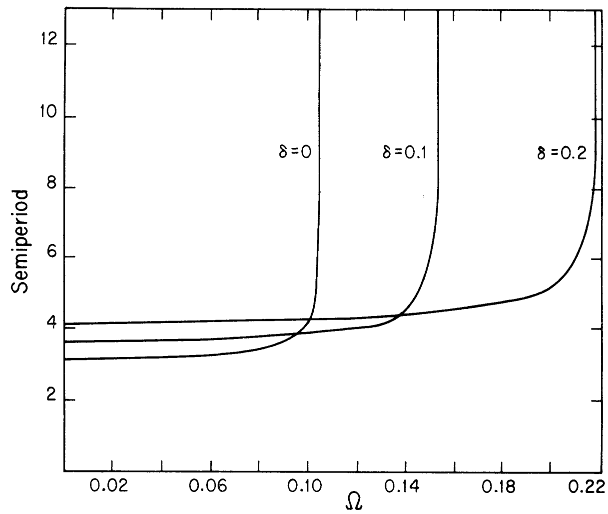

Fig. 3. The semiperiod as a function of Ω for different δ.

$$t \longrightarrow \tilde{t} = t\,\frac{(1 - 3/4\,\delta\,x'^2)^{3/2}}{[\delta + (1 - \delta/4\,x'^2)^{1/2}]^{3/2}}\;,$$

because with this transformation the relativistic equation for small oscillations becomes the classical one.

Also, in Figure 4 the singular points $x^\pm$ are represented as a function of Ω and δ.

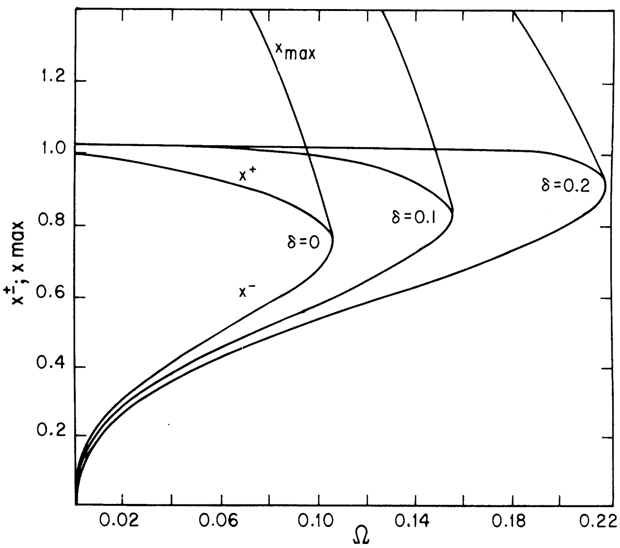

Fig. 4. Dependence on Ω of singular points, for different δ.

TABLE 1

COMPARISON WITH OBSERVATIONAL DATA FROM NOVAE[a]

	O_p	L	L_{obs}	R	R_{obs}	T	T_{obs}	m_s/M	δ
U Sco	1.1×10^8	200	300	3000	4000	6	7	2×10^{-4}	8×10^{-8}
T CrB	2.5×10^8	200	300	5000	6000	4	5	2×10^{-4}	7×10^{-8}
RS Oph	1.1×10^8	200	300	3000	4000	6	7	2×10^{-4}	8×10^{-8}
WZ Sge	1.1×10^8	200	300	3000	4000	6	7	2×10^{-4}	8×10^{-8}
A 0620–00	1.0×10^8	200	300	3000	4000	6	7	2×10^{-4}	8×10^{-8}

[a] O_p in s, L and L_{obs} in 10^{36} erg s^{-1}, R and R_{obs} in 10^{10} cm, T and T_{obs} in 10^3 °K.

4. COMPARISON WITH OBSERVATIONS

We are interested in comparing the observational data with the results of our model, for small oscillations of the shell. If the observational period is O_p and we fix the mechanical parameters Ω and γ; α is determined by means of the transcendental equation

$$O_p = 2t_0(\Omega,\gamma,\alpha)S\Big[\Omega,\delta(\Omega,\gamma,\alpha)\Big] \quad .$$

From equations (10) and (11) we determine the scale factors. The relativistic correction $\delta(\Omega,\gamma,\alpha)$ is obtained for eq. (23) when the density, luminosity and the other parameters can be compared with the observational data.

In our case, we choose $\Omega = 0.1054$ because around this point, we have the maximum density of the observed periods (Figure 3), and we choose $\gamma = 10^{-3}$ for novae, because for these objects a dependence exists between γ and the ratio m_s/M, and $\gamma = 10^{-3}$ correspond to $m_s/M = 10^{-4}$, as is observed in novae (Hoffman et al. 1978; Lewin & Clark 1980). We select 1 $M_\odot$ (solar mass) for the white dwarf star mass.

With this choice of the parameters the luminosity (the emitted energy) turns out to be within reasonable bounds, it never reaches a supernovae luminosity, and it never becomes too small for the kind of objects we are dealing with. Also the temperature is of the order of the observed ones. In fact, for novae it turns out to be of the order of 10^3 °K as observed (Lewin & Clark 1980).

For $\delta = 0$, the dependence of $T(t)$ and $L(t)$ corresponds to "loop a" in Figure 1b.

The computed data for the observed objects are shown in Table 1.

5. CONCLUSIONS

It is interesting to remark that the coincidence of Table 1 with the observed values disappear if we use the classical theory with no relativistic post-newtonian corrections. In fact, if we take $\delta = 0$, i.e., in the classical limit we would obtain bigger α and smaller luminosities, outside the observational bounds. The temperatures would also become extremely small with respect to the real temperatures. It is true that we could solve the problem by changing the value of γ, but in this case we would use a value of m_s/M totally different from 10^{-4} for novae. Thus, the relativistic correction is essential to the model.

Then, a physical object, similar to the one we describe in this paper, will undergo thermomechanical oscillations with the frequency we have calculated.

Our model can be improved in several ways. The one-shell hypothesis is reasonable enough for novae, and it works quite well as we showed, but it is somehow artificial for bursters. Therefore we shall develop a many-shell model in a forthcoming paper. Also we use uniform density, because, in order to simplify the model, we have really focussed all the problems in the shell dynamics. Even so the coincidence with observational data is suggestive. The model can thus be improved with a more realistic density law. We also hope to take into account the thermonuclear explosion and the axial symmetry of the system in future models.

The main motivation of the present work is to provide some easy and dimensionless model to test numerical codes.

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