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A METHOD OF NUMERICAL INTEGRATION FOR CALCULATING ASTRONOMICAL REFRACTION

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RESUMEN

Se calcula la refracción astronómica basada en una integración numérica de ecuaciones diferenciales, las cuales describen la propagación de un rayo de luz a través de una atmósfera estratificada de manera concéntrica. El cálculo hace uso de una atmósfera patrón y de las condiciones meteorológicas locales en el Observatorio Nacional de Llano del Hato en Mérida, Venezuela ubicado a 3600 m de altura. Se deriva una fórmula de interpolación la cual permite un cálculo rápido de la refracción. La comparación de éste, con otros métodos, arroja diferencias mínimas.

ABSTRACT

Astronomical refraction is calculated on the basis of a numerical integration of differential equations which describe the propagation of a beam of light through a concentrically stratified atmosphere. The calculations make use of the standard atmosphere and the local meteorological conditions at the National Observatory Llano del Hato in Mérida, Venezuela located at an elevation of 3600 m. An interpolation formula is derived which permits a rapid calculation of the refraction, and when compared with other methods shows only minor differences.

Key words: **ASTROMETRY — ATMOSPHERIC EFFECTS**

1. INTRODUCTION

'Astronomical refraction', i.e., the correction which has to be applied to a measured zenith distance in order to eliminate the effect of the deviation of a light beam as it passes through the atmosphere, has been the subject of many studies. The exact determination of this correction is only possible through a comparison of the measured angles with an exact and error free system of astronomical coordinates. A number of reasons, mostly of instrumental nature, and uncertainties in the refraction correction, have so far impeded the construction of such a 'reference system'. The Hipparcos catalogue will make such a comparison possible and thus will tell us which of the methods developed so far comes closest to the true refraction.

The refraction of light in the air is strictly a function of the gradient of the refractive index, and the latter is a function of the density of the air. Thus, if we had an exact model of the atmosphere, i.e., an exact description of the density as a function of the three-dimensional space coordinates X, Y, and Z, we can follow a light beam through the air, as proposed for geodetic applications by Stock (1986). This is done with a step by step numerical integration of a set of differential equations and thus differs from the methods used by other authors; for example, Fukawa & Yoshizawa (1985), Yatsenco (1995), or Stone (1996). The latter states that *'atmospheric refraction should be determined ideally by tracing the path of light through the Earth's atmosphere'.* This is exactly what this paper pretends to do.

2. GEOMETRY OF THE LIGHT PATH

We shall make the assumption that the structure of the atmosphere is described by concentric shells; hence the refractive index $\eta(h)$ is only a function of the altitude h above sea level. The geometry of the light path is shown in Figure 1, where the observed zenith distance is z_0; z is the angle of the beam with respect to the Y-axis at the point X,Y; z' is the angle of the beam with respect to the local zenith at X,Y;

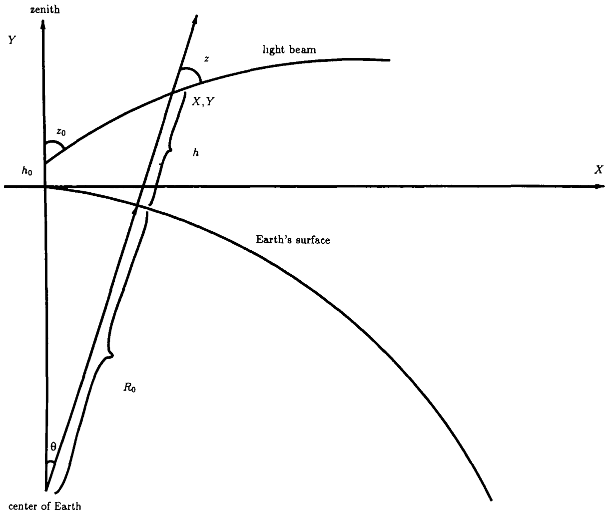

Fig. 1. Geometry of a light beam as it passes through a concentric atmosphere. Symbols are explained in the text.

R_0 is the radius of the Earth at the observing site, and θ is the angle relative to the at the center of the Earth.

Let s be a coordinate along the light path. Then, according to Stock (1986), the variation of z' as the beam passes from s to $s + ds$ is given by

$$\frac{dz'}{ds} = -\frac{1}{\eta(h)}\sin(z')\frac{d\eta(h)}{dh}. \quad (1)$$

This equation follows from differentiating Snell's law of refraction and then applying it to the light beam in Fig. 1. Furthermore, we have the following relations

$$\frac{dx}{ds} = \cos(z), \quad (2)$$

and

$$\frac{dy}{ds} = \sin(z). \quad (3)$$

Evidently, with a knowledge of $d\eta(h)/dh$ this system of differential equations can be solved.

3. THE ATMOSPHERIC MODEL

To construct the function $\eta(h)$ we use the physical data of the Earth's atmosphere of the Standard Atmosphere (1976), which gives the physical parameters of the air at each elevation. These, together with the measured meteorological parameters at the observing site, are interpolated by exponential functions, thus constructing continuous functions which extend to an elevation of 90 km, where the mentioned standard data end. From these functions the density ρ of the air can be calculated for any elevation with the help of known relations. The latter, for instance, may be found in § 3 of the paper by Stone (1996). The refractive index is then calculated from

$$\eta(h, \lambda) = a(\lambda)\rho(h), \quad (4)$$

where λ is the wavelength, and $a(\lambda)$ a well known function.

4. RESULTS

The above integration was carried out for a specific site, namely the Astronomical Observatory in the Venezuelan Andes, located at an elevation of 3600 m. Our results refer to the visual wavelengths, 5500 Å, and calculations range from 0° to 80° zenith distance. For the purpose of interpolation and a more rapid handling of the resuls a formula of the form

$$R = A\tan(z) + B\tan^3(z), \quad (5)$$

was fitted to the data by least squares. The results of the polynomial are given in the Table 1. The coefficients A and B are given at the bottom of the table.

Of the standard atmosphere we need the density as function of the elevation h which can readily be converted into the respective refractive index. The integration is started at an elevation of 3600 m for which the density and hence, the refractive index, is calculated from the average local night-time meteorological parameters. The next point is taken from the table of the Standard Atmosphere at an elevation of 4000 m. We should mention here that the value calculated for the initial point agrees closely with what would be obtained directly from the table. From then on, points of the relation of the refractive index versus the elevation were taken from the table at increasing intervals, from 1000 m in the lower troposphere to 20,000 m at the top of the stratosphere. The refractive index was interpolated from point to point by an exponential function. The gradient of the refractive index, which is needed in equation (1), was calculated numerically from the interpolating functions. The refractive index is in this manner represented by a continuous function. Its derivative is discontinuous at the junction points, but the numerically calculated differences have no discontinuity. The path length of the integration was chosen as 5.0 m up to an elevation of 10,000 m, and then increased up to 50.0 m at the end of the integration at an elevation of 120 km. The results are given in Table 1.

At this point, a comparison with other methods and at different wavelengths may be of interest. Comparisons can be made with the widely used Pulkovo Table (Orlov 1956) and with data by Fukawa & Yoshizawa (1985) for 5753 Å. The differences be-

TABLE 1

ASTRONOMICAL REFRACTION AT THE NATIONAL OBSERVATORY LLANO DEL HATO IN MERIDA, VENEZUELA

Zenithal Distance z_0(°)	Atmospheric Refraction $R_0('')$	$A\tan(z)$ $+B\tan^3(z)$[a] $R_0('')$	Zenithal Distance z_0(°)	Atmospheric Refraction $R_0('')$	$A\tan(z)$ $+B\tan^3(z)$[a] $R_0('')$
1	0.690	0.689	41	34.319	34.306
2	1.380	1.379	42	35.543	35.532
3	2.071	2.070	43	36.811	36.797
4	2.763	2.762	44	38.117	38.103
5	3.457	3.456	45	39.469	39.454
6	4.153	4.152	46	40.867	40.852
7	4.851	4.850	47	42.317	42.302
8	5.553	5.551	48	43.822	43.806
9	6.257	6.256	49	45.387	45.369
10	6.966	6.965	50	47.014	46.996
11	7.679	7.678	51	48.708	48.692
12	8.398	8.395	52	50.478	50.461
13	9.121	9.119	53	52.328	52.311
14	9.851	9.848	54	54.263	54.247
15	10.586	10.583	55	56.295	56.278
16	11.329	11.325	56	58.429	58.412
17	12.079	12.075	57	60.676	60.657
18	12.837	12.833	58	63.041	63.023
19	13.603	13.599	59	65.547	65.528
20	14.378	14.374	60	68.195	68.178
21	15.165	15.160	61	71.007	70.992
22	15.961	15.956	62	74.000	73.985
23	16.768	16.763	63	77.191	77.178
24	17.587	17.582	64	80.606	80.594
25	18.421	18.414	65	84.270	84.258
26	19.267	19.260	66	88.211	88.202
27	20.128	20.120	67	92.465	92.460
28	21.003	20.995	68	97.075	97.075
29	21.894	21.887	69	102.094	102.096
30	22.804	22.796	70	107.575	107.582
31	23.731	23.724	71	113.588	113.603
32	24.680	24.671	72	120.220	120.244
33	25.648	25.639	73	127.578	127.609
34	26.639	26.629	74	135.786	135.827
35	27.652	27.642	75	145.010	145.057
36	28.690	28.680	76	155.445	155.500
37	29.756	29.745	77	167.351	167.410
38	30.848	30.839	78	181.071	181.116
39	31.973	31.962	79	197.039	197.046
40	33.127	33.117	80	215.861	215.763

[a] $A = 39.5006'' \pm 0.0021''$; $B = -0.0450'' \pm 0.0001''$.

tween the data from these two sources and ours are shown in Figure 2. Comparisons at two other wavelengths can also be made with the data from Fukawa & Yoshizawa. The respective results are shown in Figure 3. From the figures, one can conclude that the different methods of calculating the astronomical

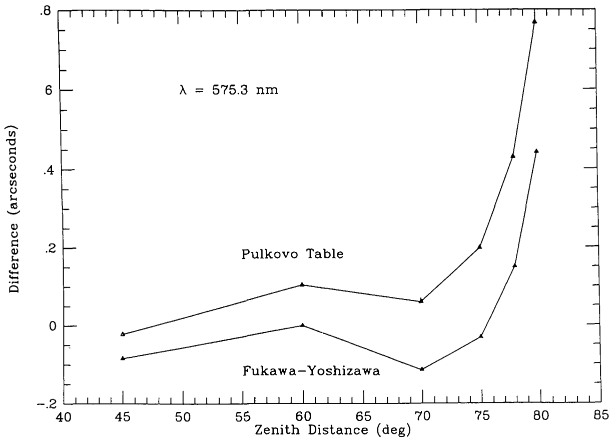

Fig. 2. The refraction of this paper minus the refraction from the Pulkovo Table and the calculationsd by Fukawa & Yoshizawa as function of the zenith distance for 5753 Å.

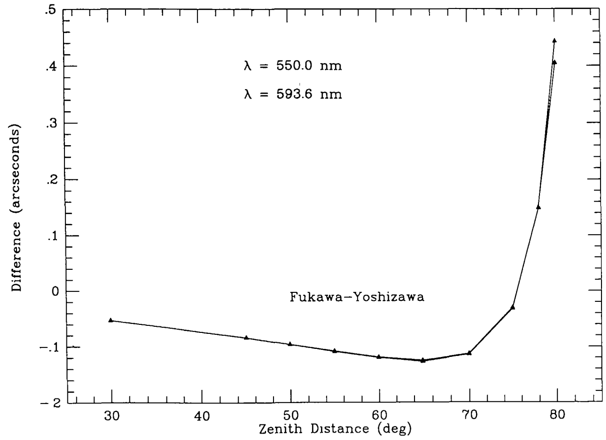

Fig. 3. The refraction of this paper minus the refraction calculated by Fukawa & Yoshizawa for 5500 Å and 5936 Å as function of the zenith distance.

refraction lead to practically identical results down to a zenith distance of 70°. Beyond this distance a major discrepancy builds up.

5. CONCLUSIONS

We have shown that accurate information on astronomical refraction can be obtained by directly integrating a light beam. The results are very close to those obtained by other methods. In fact, for zenith distances $< 78°$ the differences are less than 200 mas. Comparison with other independent data is needed, in order to determine which method comes closest to the true refraction. The Hipparcos catalogue may provide the necessary reference frame.

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