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GRAVITATIONAL COLLAPSE AND BINARY PROTOSTARS

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RESUMEN

La teoría estándar de la formación estelar predice que el colapso del gas se inicia primero en el interior de núcleos moleculares con una gran concentración de materia en el centro (ley de potencias), propagándose después hacia afuera. Si bien es improbable que esos núcleos se fragmenten durante la fase de colapso protoestelar, la detección de compañeras de estrellas de la presecuencia principal requiere que las estrellas binarias se hayan formado durante la fase de colapso por fragmentación. Núcleos moleculares con una concentración moderada en sus interiores (p. ej., distribución gaussiana) pueden fragmentarse siempre y cuando el cociente entre la energía térmica y gravitatoria (α) sea menor que ~ 0.4 (es decir, inferior al que da el equilibrio virial). Las observaciones en longitudes de onda submilimétricas de núcleos moleculares son consistentes con perfiles de densidad gaussiano hacia el centro de ellos. Los modelos teóricos de nubes sostenidas magnéticamente, contrayéndose a través de la difusión ambipolar, predicen también perfiles gaussianos (y $\alpha < 0.4$) cuando el campo magnético se ha debilitado lo suficiente para que el colapso dinámico se inicie. Esta evidencia sugiere que tanto las estrellas binarias como múltiples se forman principalmente en núcleos con una concentración de gas moderada en su centro. Discutimos una serie de modelos hidrodinámicos de fragmentación en tres dimensiones, presentando algunas sugerencias para la investigación futura.

ABSTRACT

The standard theory of star formation is based largely on the 'inside-out' collapse of strongly centrally-condensed (power-law) molecular cloud cores. Such clouds are unlikely to fragment during the protostellar collapse phase, whereas detections of companions to pre-main-sequence stars require that binary stars are formed no later than during the protostellar collapse phase, i.e., by fragmentation. Clouds which are only moderately centrally-condensed (e.g., Gaussian) can fragment into binary protostars, provided their ratio of thermal energy to gravitational energy (α) is less than ~ 0.4 (i.e., less than virial equilibrium). Sub-mm observations of pre-collapse cloud cores are consistent with density profiles that flatten at the center and appear to be Gaussian. Theoretical models of magnetically-supported clouds contracting through ambipolar diffusion also have Gaussian-like profiles (and $\alpha < 0.4$) when the magnetic field has weakened enough for dynamic collapse to be underway. This evidence all suggests that binary and multiple stars form primarily from the collapse and fragmentation of moderately centrally-condensed cloud cores. Recent three dimensional hydrodynamics models of fragmentation are discussed and suggestions are made for future research.

Key words: **BINARIES: GENERAL — INFRARED: STARS — ISM: CLOUDS — STARS: FORMATION — STARS: PRE-MAIN-SEQUENCE**

1. INTRODUCTION

For several decades, gravitational collapse and the formation and evolution of protostars have been relegated primarily to the dark realm of theoretical modeling. Only in relatively recent years have these topics become illuminated and constrained by observations from across the electromagnetic spectrum. Acceptable examples, or at least candidate objects, now exist for essentially the entire sequence of protostellar collapse phases, ranging from (a) pre-collapse, dense cloud cores (Ward-Thompson et al. 1994), (b) "class 0" protostars (André, Ward-Thompson, & Barsony 1993), seen at mm-wavelengths with most of their mass still collapsing, (c) deeply embedded (class I) young stellar objects (YSOs), radiating largely in the infrared, to (d) optically visible, pre-main-sequence (PMS) stars, some of which still show evidence for significant infall (van Langevelde et al. 1994).

The intent of this review is to highlight one particular area in which recent observations affect our theoretical understanding of protostellar evolution – the formation of binary stars. Binary star formation inevitably involves rotating gas, and the gravitational collapse of rotating gas inevitably leads to the formation of a protostellar disk. The most successful models of binary star formation either involve the fragmentation of a rotating protostellar disk, or else involve fragmentation immediately prior to the formation of a circumstellar disk.

2. STANDARD THEORY OF STAR FORMATION

While protostellar disks will be seen to be crucial to the formation of binary stars, nearly all of our understanding of the basic physics of the star formation process derives from more idealized models: the collapse of non-rotating, non-magnetic, spherically symmetric clouds. Larson (1969) pioneered this field, finding that a dense molecular cloud would undergo two distinct phases of collapse. Starting from densities of $\sim 10^{-20}$ g cm^{-3}, clouds which are more massive than the Jeans mass collapse while remaining isothermal at about 10 K, because their low infrared opacity allows dust grains to radiate away the compressional energy generated by the collapse. Once densities on the order of $\sim 10^{-13}$ g cm^{-3} are reached, the infrared optical depth approaches unity, and compressional energy is increasingly trapped within the cloud. The central temperature begins to rise, reinforcing the radially outward force of gas pressure, and halting the collapse by the time the temperature rises to $\sim$ 200 K. A *first*, or outer protostellar core forms at this time, and gains mass as the rest of the cloud continues to infall onto it. Within a few percent of the cloud free fall time, the first core increases in mass to $\sim 0.04 M_{\odot}$, and its central temperature reaches $\sim$ 2000 K, sufficient to dissociate molecular hydrogen. Dissociation serves as an energy sink, softening the equation of state enough that a second collapse phase ensues. The second collapse continues until densities on the order of stellar densities are reached, by which time atomic hydrogen provides the bulk of the gas pressure and this pressure halts the collapse at the center. The *second*, final, inner protostellar core forms at this time, containing only $\sim 0.02 M_{\odot}$. The remainder of the evolution consists of the accretion of the cloud envelope onto this final protostellar core, lasting on the order of a free fall time.

Because the first core disappears soon after the formation of the second core, most attention has focused on events following the initiation of the second collapse phase. In fact, the most successful models of protostellar evolution begin with the formation of the second core, and simply follow the accretion of the cloud envelope onto the second core from that point onward (Stahler, Shu, & Taam 1981). Even an initially uniform density cloud attains a $\rho \propto r^{-2}$ profile by the time the first core forms (Larson 1969), so it is natural to consider the singular isothermal sphere ($\rho = c_s^2/(2\pi G r^2)$, where c_s is the sound speed and G is the gravitational constant) as an initial condition for the collapsing envelope. A similar density profile is also produced during the contraction of magnetically-supported clouds evolving by ambipolar diffusion (Lizano & Shu 1989), and given the degree to which magnetic fields are thought to control star-forming molecular clouds (e.g., Shu, Adams, & Lizano 1987), the singular isothermal sphere has long been an accepted initial condition for the standard theory of star formation.

The singular isothermal sphere is an unstable equilibrium. Shu (1977) derived a similarity solution for the collapse of the singular isothermal sphere, showing that collapse would occur from the 'inside-out', through the outward motion of an expansion wave starting at the core. Matter inside the expansion wave has a profile $\rho \propto r^{-3/2}$ and falls freely onto the core, while the matter exterior to the expansion wave is still at rest. The mass accretion rate onto the core is constant (Shu 1977) at $\dot{M} = 0.975 c_s^3/G$. For 10 K gas, $\dot{M} = 2 \times 10^{-6} M_{\odot}$/yr. The Shu (1977) similarity solution has also been extended to include the effects of small perturbations due to rotation (Terebey, Shu, & Cassen 1984), non-axisymmetry (Tsai & Bertschinger 1989), and magnetic fields (Chiueh & Chou 1994).

With the dynamics of the envelope given by Shu's (1977) similarity solution, Stahler, Shu, & Taam (1981) were able to calculate the evolution of the second protostellar core, and predict total luminosities (L) and effective temperatures (T_e). Stahler (1983) then was able to define the 'birthline' in the $L - T_e$ diagram where young stars first appear optically following the cessation of accretion and the removal of the circumstellar envelope. The birthline agrees quite well with the location of T Tauri stars, and this agreement can be taken as observational evidence that a theory of star formation based on spherically symmetry protostars captures much of the important physics.

One can use this theoretical framework to attempt to place observed and hypothetical protostellar and young stellar objects into a temporal sequence. Working forward in time, the sequence is:

* *"Class -II" ("class minus two") objects* are pre-collapse, dense cloud cores (Ward-Thompson et al. 1994), such as the Myers & Benson (1983) ammonia cloud cores that do not contain infrared sources. Their spectra are like those of grey bodies with temperatures of 5 K to 12 K (Ward-Thompson et al. 1994). "Class -II" objects provide the initial conditions for protostellar collapse.

* *"Class -I" ("class minus one") objects* have not been observed as yet, but would correspond to first protostellar cores with sizes on the order of 10 AU and characterized by central temperatures of several hundred K and distinctive spectral energy distributions (see Fig. 1). These cores disappear soon after the formation of the final, second protostellar core, and presumably predate the occurrence of outflows.

* *Class 0 objects* have been identified by André, Ward-Thompson, & Barsony (1993) and may be second protostellar cores which are still less massive than their cloud envelopes ($M_* < M_{env}$). The presence of outflows suggests that the final stellar core has already formed.

* *Class I objects* are young stellar objects (YSO) composed of second protostellar cores undergoing accretion with $M_* > M_{env}$ and exhibiting robust outflows.

* *Class II objects* are classical T Tauri stars with $M_* >> M_{env}$, still creating outflows.

* *Class III objects* are weak-lined T Tauri stars with very little remaining envelope material ($M_* >>> M_{env}$).

The nomenclature for Classes I, II, and III is defined (e.g., Adams, Lada, & Shu 1987) on the basis of spectral slopes at infrared wavelengths, and Class 0 was thereafter coined by André, Ward-Thompson, & Barsony (1993). There is a certain perverse logic then in advancing the concept of the "negative Roman numeral" to accompany the "Roman zero", and therefore terming the evolutionary phases prior to class 0 as classes "-I" and "-II".

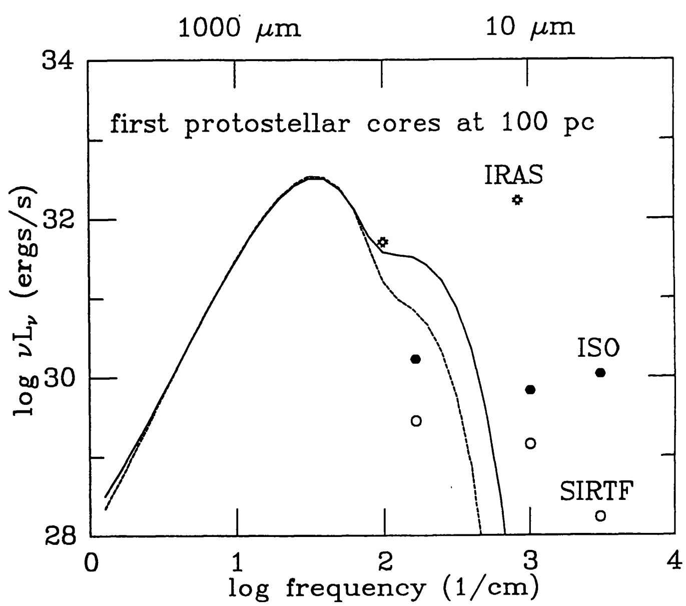

Fig. 1.— Spectral energy distribution predicted for the hypothetical "class -I" objects (first protostellar cores). Shown are pole-on views for both single (solid line) and binary (dashed line) protostars, compared to sensitivities of space IR telescopes. Figure from Boss & Yorke (1995).

3. BINARY PMS STARS

Spherically symmetrical protostar models, combined if necessary with a circumstellar disk to yield infrared emission, and a star-disk boundary layer for emitting ultraviolet radiation, are able to explain many of the observational characteristics of YSOs. The theory of low-mass star formation would be nearly complete (excluding the question of YSO winds and outflows) at this point, if all stars were single like our sun.

However, the sun is well-known to be in the minority among low-mass stars; the monumental survey of all G-dwarf stars within 22 pc of the sun by Duquennoy & Mayor (1991) found that at most one third of all G-dwarf primaries could be true single stars with no companions greater than 0.01 $M_{\odot}$ in mass. Evidently most stars occur in pairs, but even this fact need not concern the theory of star formation so long as stars acquire their companions well after their own formation process is essentially complete, such as during the PMS or later phases. Classical dynamical mechanisms such as orbital capture and rotational fission during contraction to the main-sequence (MS) might then be invoked to explain binary formation as a separate event largely unrelated to the star formation process.

The possibility of decoupling star formation from binary formation has been eliminated by recent surveys of the frequency of companions to PMS stars. If binary stars form during or after the PMS phase, the frequency of binary companions in a large sample of PMS stars must be less than that for MS stars. Surveys of PMS stars have actually shown that the opposite is true – the binary frequency of YSOs in the Ophiuchus and Taurus star-forming regions is *higher* than that of the nearby G-dwarfs (Simon et al. 1995). In fact, the speckle infrared imaging survey by Ghez, Neugebauer, & Matthews (1993) found that for a limited range of separations around 100 AU, the binary star frequency for PMS stars was four times greater than that of MS stars! Other surveys using CCD imaging (Reipurth & Zinnecker 1993), speckle infrared imaging (Leinert et al. 1993), and lunar occultations (Richichi et al. 1994) have also found evidence for an excess of PMS binaries. Evidently binary star formation must occur prior to the PMS phase (Mathieu 1994), i.e., during protostellar collapse.

The requirement that binary star formation occurs during protostellar collapse is a strong argument in favor of the fragmentation mechanism, which is usually defined as break-up during the dynamic collapse phase (e.g., Boss & Bodenheimer 1979). Fragmentation can be quite efficient at producing binary systems with initial separations on the order of 100 AU, so if the excess of binary PMS stars seen by Ghez et al. (1993) around 100 AU should come at the expense of binaries with other separations, such a binary distribution may be a signature revealing that fragmentation is the mechanism responsible for forming the majority of binary stars.

Fragmentation has also been implicated as the cause of binary star formation through the analysis of YSO clustering and binary separation data by Larson (1994). By plotting the surface density of companion stars on the plane of the sky versus the angular separation of the primary and companion, Larson (1994) found that the data was well fit by two different power laws, with a break occurring at a projected separation of 0.04 pc. On scales larger than 0.04 pc, the distribution apparently is caused by self-similar clustering (probably involving magnetic fields and turbulent velocity fields), but on smaller scales the change in slope implies that a different process is operating. Given that the changeover occurs at a length scale associated with the Jeans mass ($\sim 1M_{\odot}$) in dense clouds, Larson (1994) attributed the formation of companions within 0.04 pc to the fragmentation of thermally-supported clouds. This length scale (0.04 pc) is roughly the maximum separation observed for PMS or MS binary stars, and is also comparable to the minimum separations between the clumps in the pre-collapse clouds mapped by Ward-Thompson et al. (1994), implying that only the very widest binaries could be explained by the structure observed in pre-collapse clouds; the bulk of binary PMS and MS stars have separations more on the order of 100 AU and so must have been formed by fragmentation during the subsequent evolution of the pre-collapse clouds.

4. PRE-COLLAPSE CLOUDS

Two extremes exist for the radial density profile of a pre-collapse cloud – uniform density ($\rho = c$) and the singular isothermal sphere ($\rho \propto r^{-2}$). The singular isothermal sphere formally exhibits a singularity at the origin, but has the advantage of being an unstable equilibrium state. The singularity can be considered simply as an approximation for the core of a cloud that begins its collapse from a state with a relatively high degree of central concentration (Shu et al. 1987). Uniform density, isothermal clouds contain no singularities, but are overly idealized in that they have no internal pressure gradients and so are not even close to being in an unstable equilibrium.

Somewhere between the two end members of uniform density and the singular isothermal sphere must lie reality, but until recently observations have not had enough spatial resolution to probe the structure of dense cloud cores on the necessary length scales (i.e., well inside about 0.04 pc). Early estimates of the density structure inside cold cloud cores such as those surveyed by Myers & Benson (1983) showed that the cores are strongly centrally concentrated, and could be fit with power-law density profiles consistent with the singular isothermal sphere or with its innermost collapsing region. However, given that about half of the Myers cores were shown by *IRAS* to contain embedded YSOs (and *ISO* and *SIRTF* will presumably find many more embedded IR sources), the fact that cloud cores containing suspected protostars or YSOs are strongly centrally concentrated says little about the initial conditions prior to formation of the embedded IR source, because even an initially uniform density cloud collapses to produce a $\rho \propto r^{-2}$ envelope. Hence the only way to constrain the initial conditions for protostellar collapse is to map a pre-collapse cloud. Ward-Thompson et al. (1994) have done just that.

Using sub-mm continuum observations, Ward-Thompson et al. (1994) mapped 21 'starless' Myers cores, finding for the first time that the central clumps of the starless cores are less centrally peaked than cores containing *IRAS* sources. The starless cores are more centrally concentrated than a uniform density sphere, however, so reality does indeed appear to lie between the two extremes stated above. When a power-law fit is attempted, at least two different power-laws are needed to try to fit the flux densities, corresponding to mass densities of $\rho \propto r^{-1.25}$ in the inner regions, and $\rho \propto r^{-2}$ in the outer regions. However, Ward-Thompson et al. (1994) found that a Gaussian fit to the flux density profiles (see Fig. 2) provided superior fits to the data, and so they used Gaussian profiles to estimate the total flux densities for the starless Myers cores. For a Gaussian radial density profile of the form $\rho = \rho_o exp(-r^2/r_o^2)$, it can be shown that the column density N as a function of projected separation X has the form $N(X) = \pi^{1/2}\rho_o r_o exp(-X^2/r_o^2)$. For low optical depth, as is appropriate for sub-mm observations, the flux intensity should be proportional to this column density (e.g., Casali 1986). Hence a Gaussian flux density profile implies a Gaussian mass density profile. This is in contrast to the results for power-law envelopes, where if the flux density is given by $S \propto r^{-m}$, and the density is $\rho \propto r^{-p}$, then for an isothermal cloud, one has $p = m + 1$ (Ward-Thompson et al. 1994; Adams 1991; Casali 1986). On the basis of these power-law relations the limiting case of a flat flux density profile ($m = 0$) implies a $1/r$ density profile, i.e., the cloud could still be strongly centrally condensed. However, for a Gaussian flux density, the mass density is also Gaussian, allowing the central regions to flatten out in density and avoid the central singularity associated with power-law profiles.

Density profiles around increasing numbers of class I sources have quite shallow ($\rho \propto r^{-1/2}$) profiles (Barsony & Chandler 1993). Profiles flatter than $\rho \propto r^{-2}$ or $r^{-3/2}$ can be interpreted as a sign of magnetic

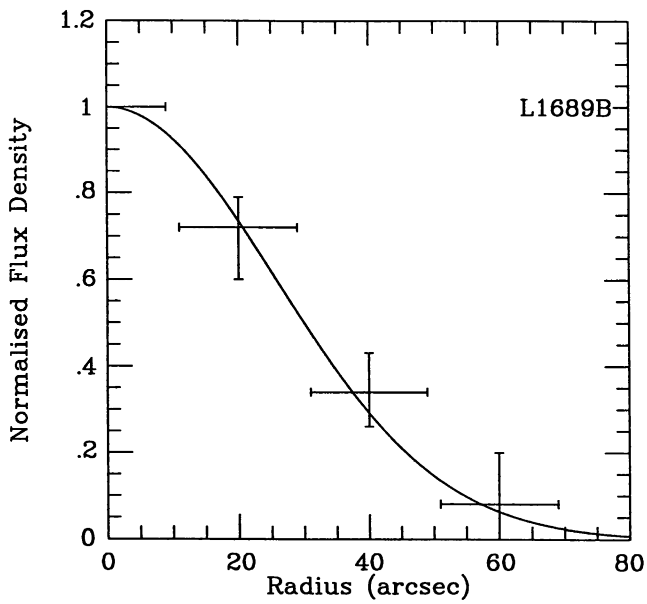

Fig. 2.— Radial flux density profile inferred from sub-mm observations of the L1689B dense cloud exhibiting a Gaussian (solid) profile. Figure from Ward-Thompson et al. (1994).

support modified by ambipolar diffusion, and Ward-Thompson et al. (1994) adopt this explanation for starless cores. Lizano & Shu (1989) modeled the contraction of magnetically-supported, isothermal clouds undergoing ambipolar diffusion, finding that while the cloud envelope tended toward a $\rho \propto r^{-2}$ profile, the central regions remained much flatter (similar to a Bonnor-Ebert sphere, or a Gaussian profile), even at the phase when dynamic collapse is well underway. By modeling a superumbral magnetic cloud with a Gaussian profile, matched to the density distributions in Fig. 4 of Lizano & Shu (1989), the cloud appears to have $\alpha \sim 0.11$ to ~ 0.36, i.e., the thermal support is less than the virial equilibrium value of 1/2, as expected, given that the cloud was partially supported by magnetic fields which are being lost through ambipolar diffusion.

The Bonnor-Ebert sphere is the equilibrium configuration of a pressure-bounded, isothermal sphere, and is the equilibrium expected to be reached by a slowly contracting, isothermal cloud. The collapse of Bonnor-Ebert spheres has been calculated in great detail in spherical symmetry (Foster & Chevalier 1993), and produces line profiles that fit molecular line observations about as well as those produced by the expansion wave solution, which starts from the singular state. Gaussian clouds also collapse to produce radial velocity profiles that are quite close to those in the Shu (1977) similarity solution, which has been shown to reproduce well line profiles in suspected collapsing protostellar clouds (Zhou 1992).

5. FRAGMENTATION OF POWER-LAW DENSITY CLOUDS

We now turn to the question of whether or not fragmentation can occur during the collapse of dense cloud cores with varied initial density profiles. We start with the power-law profiles characteristic of the pre-collapse clouds usually envisioned in the standard theory of star formation.

The collapse of clouds with initial density profiles of the form $\rho \propto r^{-1}$ was investigated by Boss (1987) with a 3D radiative hydrodynamics code, who found that such clouds did not undergo fragmentation during either their isothermal or initial nonisothermal collapse phases, i.e., up to the formation of the first protostellar core. This was true even for clouds with very low initial ratios of thermal to gravitational energy ($\alpha \sim 0.02$), i.e., far from being in virial equilibrium. The implication was that the initial central mass concentration ('singularity') in a power-law profile prejudiced the cloud toward forming only single protostars. This finding was reinforced by the perturbation analysis of the Shu (1977) similarity solution by Tsai & Bertschinger (1989), who found that non-axisymmetry could not grow during the collapse of a singular isothermal sphere.

However, Myhill & Kaula (1992) used Myhill's 3D radiative hydrodynamics code to find that fragmentation could be achieved in a low α (= 0.16), power-law ($\rho \propto r^{-2}$) cloud, provided the initial cloud was in rapid differential rotation ($\beta = 0.17$, where β is the ratio of rotational to gravitational energy), with an angular velocity distribution depending on the density as $\Omega \propto \rho^{2/3}$. In this case, the rapidly rotating inner regions quickly form a centrifugally-supported disk and thereafter fragment (see Fig. 3).

This basic result was confirmed with a different 3D hydrodynamics code by Sigalotti (1994). Sigalotti

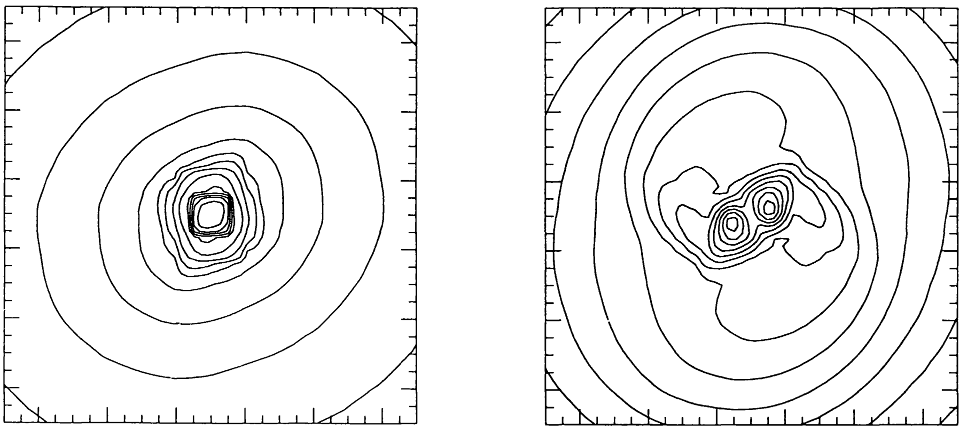

Fig. 3.— Equatorial densities for 3D collapse models with $\rho_i \propto r^{-2}$ and $\Omega_i = c$ (left) or $\Omega_i \propto \rho_i^{2/3}$ (right). Both have $\alpha_i = 0.16$ and $\beta_i \approx 0.17$. Box size is 5300 AU. Figure from Myhill & Kaula (1993).

(1994) first collapsed a uniform density sphere until the central density was 200 times that at the boundary, yielding a radial density profile close to $\rho \propto r^{-2.7}$ and $\alpha \approx 0.15$. He then set the radial components of the velocity to zero, but imposed an angular velocity of the form $\Omega \propto \rho^{2/3}$ (such that $\beta = 0.17$), and let the cloud collapse again from this roughly power-law configuration. The cloud fragmented into a binary system, the same result as found by Myhill & Kaula (1992). Sigalotti (1994) also computed a model with the same assumptions except that the initial radial velocity field was set equal to 0.1 that of the initial collapsing sphere at that time, plus a component of random noise. This time the differentially rotating, power-law cloud fragmented into a binary joined by a bar-like region that later sub-fragmented into four more clumps, yielding a total of six fragments. Myhill & Kaula (1992) also had a power-law density, differentially-rotating model which appeared to form a multiple system, in this case a quadruple system.

The $\Omega \propto \rho^{2/3}$ angular velocity profile adopted by Myhill & Kaula (1992) and Sigalotti (1994) represents the maximum amount of differential rotation that is likely, because it is based on a simple calculation of how the angular velocity of a contracting sphere would increase with the density if it collapsed with conserved angular momentum (Boss 1987). If magnetic fields control the contraction of pre-collapse clouds prior to the onset of dynamic collapse (e.g., Lizano & Shu 1989), then the magnetic field lines will work to maintain solid body rotation in the cloud and so will prevent the development of significant differential rotation. Given that one of the justifications for using a power-law initial condition is that such a profile is expected to result from the contraction of magnetically-supported clouds, it thus does not seem consistent that a power-law pre-collapse

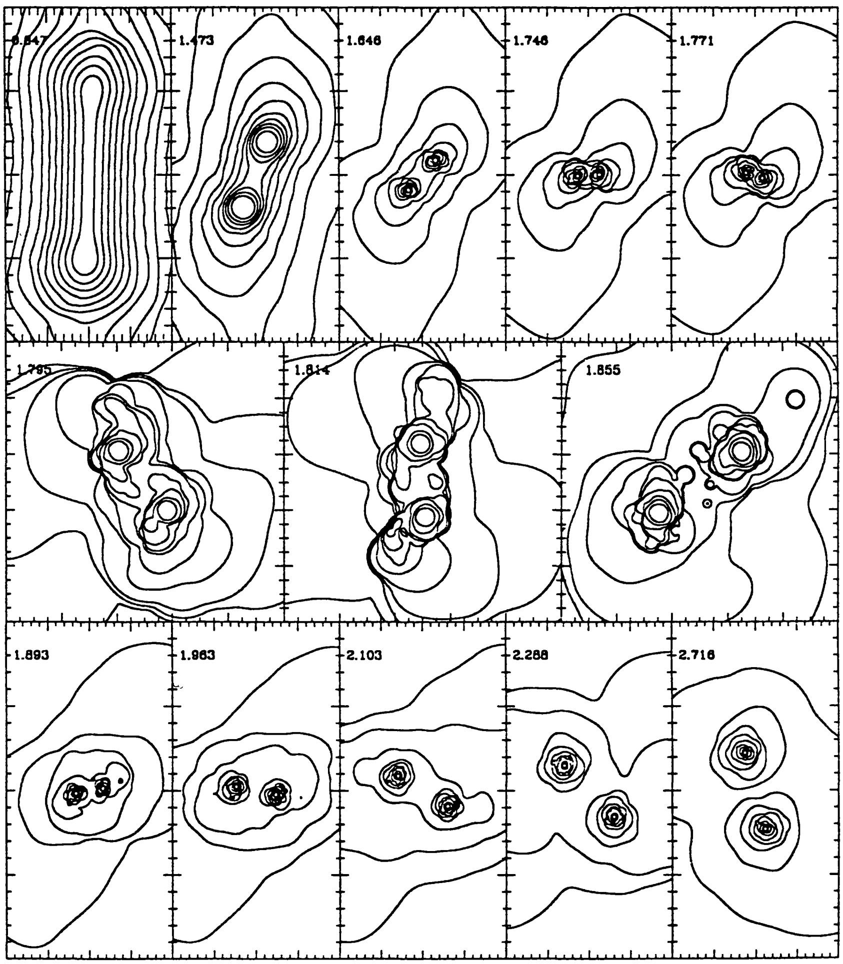

Fig. 4.— Density contours in a rotating reference frame for the 3D collapse of an initially uniform density cylinder. Times are given in units of the free fall time of 4.3×10^5 yrs. Initial cloud length is 0.45 pc. Figure from Bonnell et al. (1991).

cloud will be differentially rotating to the extent required to achieve the fragmentation found by Myhill & Kaula (1992) and Sigalotti (1994). Ultimately the question of the degree of initial differential rotation must be answered by observations of the angular velocity profiles in pre-collapse clouds; solid-body rotation is usually assumed in estimating cloud rotation rates (Goodman et al. 1993), but observations are becoming sufficiently refined to be close to relaxing this assumption (Goodman & Barranco 1994).

Unless pre-collapse clouds are strongly differentially rotating, then, starting collapse from the singular isothermal sphere has the strong disadvantage of eliminating the possibility of fragmentation into binary protostars during the collapse phase, putting the theory in conflict with observations of the frequency of binary PMS stars.

6. FRAGMENTATION OF UNIFORM DENSITY CLOUDS

In contrast to power-law clouds, initially uniform density clouds are considerably more tolerant of fragmentation, and partially for this reason the collapse of initially uniform density clouds has accounted for the bulk of the 3D collapse calculations performed to date. Other reasons include the theoretical desire to start with a structureless cloud, in order to learn how structure develops, and the need to reproduce and investigate previous work before moving on to new problems. The early work has been reviewed by Boss (1990) and Bodenheimer, Ruzmaikina, & Mathieu (1993); here we just describe 3D calculations that have been performed in the last few years.

Several recent studies have been at least partially motivated by the finding that dense molecular cloud cores have elongated shapes on the plane of the sky that are statistically more consistent with prolate than with oblate geometries (Myers et al. 1991). Bonnell and Bastien and their colleagues have used a smoothed particle hydrodynamics (SPH) code to study exhaustively the collapse of uniform density cylinders, an extreme form of a prolate cloud, finding that such clouds readily collapse and fragment into binary systems. For a narrow cylinder with $\alpha = 0.5$ and $\beta = 0.09$, one fragment forms out of each end of a cylinder (see Fig. 4) rotating about an axis perpendicular to its long axis (Bonnell et al. 1991). When rotation about an arbitrary axis is introduced, fragmentation into multiple systems can occur, with the binary or single members of the multiple system being non-coplanar (Bonnell et al. 1992). When one end of the cylinder is given more mass than the other, the binary system that forms is unequal in mass (Bonnell & Bastien 1992), as is most often the case in binary systems.

Nelson & Papaloizou (1993) developed their own SPH code and studied the collapse of initially uniform density clouds with prolate boundaries, closer to the geometry suggested by Myers et al. (1991). Nelson & Papaloizou (1993) find that in the absence of rotation, such clouds do not fragment into two clumps unless they have an prolate axis ratio that is considerably larger ($\sim 6:1$) than that inferred ($\sim 2:1$) for dense cloud cores (Myers et al. 1991). The fragments that do form are initially closer together than in the case of the cylinders studied by Bonnell et al. (1991). Monaghan (1994) used his SPH code to study the collapse of very thin, prolate clouds with differential rotation, showing that during the low density regime where molecular lines cool the cloud, the ellipsoid can fragment along its length into six or more clumps.

Bonnell (1994) has also studied the collapse of initially spherical, uniform density clouds, using a piecewise adiabatic pressure law ($\gamma = 1$ to 7/5) to model the radiative transfer effects that occur for $\rho > 10^{-13}$ g cm^{-3}. The model starts with enough rotation ($\beta_i = 0.054$) to collapse and produce a centrifugally-supported disk with a radius of about 100 AU. The disk flattens and deforms into a bar, from the tips of which grow trailing spiral arms. The ongoing accretion of mass adds sufficient matter to the arms to lead to the formation of a binary (possibly tertiary) protostellar system with a separation on the order of 50 AU.

Uniform density, spherical clouds are also quite tolerant of the formation of multiple systems. Boss (1993a) showed that when such a cloud is given a bar-like initial density perturbation, this perturbation can grow (for low α_i) and fragment the cloud into a quadruple system composed of two identical binaries, each consisting of a 10:1 mass ratio binary. Burkert & Bodenheimer (1993) used their 3D hydrodynamics code to show that a sphere perturbed with a smaller bar-like perturbation can fragment into a binary system, but with a thin bar linking the two binary members. This thin bar then sub-fragmented into 9 distinct clumps, for a total of 11 fragments, all on a scale of about 100 AU (see Fig. 5). The result was reproduced with different numerical grid resolution and so appears to be robust.

7. FRAGMENTATION OF GAUSSIAN DENSITY CLOUDS

The observations of pre-collapse clouds by Ward-Thompson et al. (1994) imply that power-law initial density profiles are too steep, and uniform density profiles are too flat, but that a Gaussian distribution is just

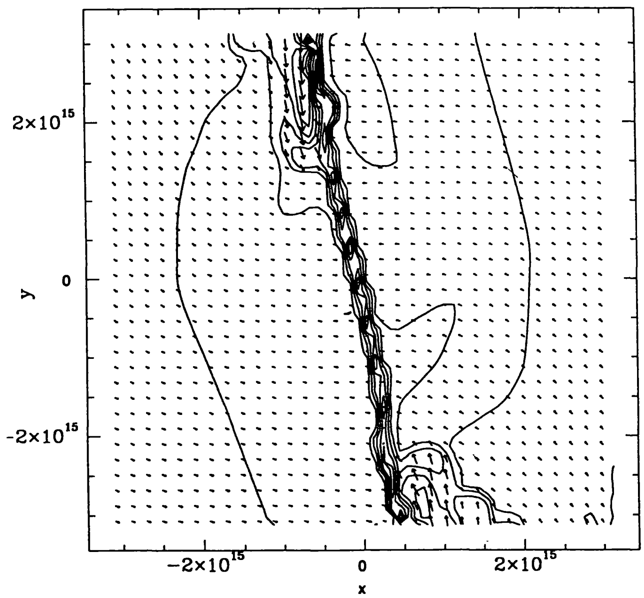

Fig. 5.— Equatorial density contours for inner region of a spherical, uniform density cloud that fragments into a binary linked by a narrow bar that sub-fragments into 9 more clumps. Size of box is 430 AU. Figure from Burkert & Bodenheimer (1993).

right (the Goldilocks dilemma). The best guess for the structure of a typical pre-collapse cloud thus appears to be a prolate cloud with a 2:1 axis ratio (Myers et al. 1991), a Gaussian radial density profile (see section 4), and rotating like a solid-body (Goodman & Barranco 1994) with $\beta \sim 0.02$ (Goodman et al. 1993).

Because of their nearly flat central regions, Gaussian density profile clouds permit binary fragmentation to occur in much the same manner as in uniform density clouds (Boss 1987), though clearly the scale of the fragmentation process is limited by the spatial extent of the initial roughly flat region. Boss (1993b) used a 3D radiative hydrodynamics code to study the collapse of clouds with initial Gaussian radial density profiles (truncated so that the central densities were 20 times the boundary densities) and in solid-body rotation. Boss (1993b) found that these reasonably realistic clouds would collapse and undergo fragmentation (see Fig. 6) provided that $\alpha_i < 0.45 - 0.36\beta_i$ for a prolate axis ratio of 2:1. Given that β_i is typically 0.02 for a dense

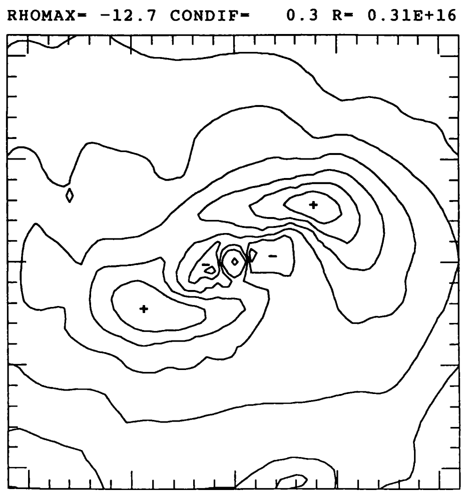

Fig. 6.— Binary formed from collapse of a Gaussian cloud with prolate axis ratio 2:1, $\alpha_i = 0.39$, $\beta_i = 0.12$, and $M = 1.5 M_{\odot}$. Size of box is 410 AU. Figure from Boss (1993b).

cloud core (Goodman et al. 1993), the criterion for fragmentation then becomes roughly $\alpha_i < 0.44$. Note that α for the singular isothermal sphere is 0.75, whereas virial equilibrium for a finite density ellipsoid requires $\alpha + \beta = 0.5$, so pre-collapse Gaussian profile clouds which are only slightly less supported by thermal pressure than they would be in virial equilibrium would be expected to fragment. This critical value for fragmentation (0.44) is larger than the estimated $\alpha \sim 0.3$ for the magnetically contracting model of Lizano & Shu (1989) at the phase where dynamic collapse begins, so fragmentation is expected to occur in this case.

Not only can Gaussian clouds collapse to form binary stars, but multiple systems can result as well. Boss (1991) used a second-order accurate 3D hydrodynamics code to calculate the isothermal collapse of a spherical Gaussian cloud with a small bar-like density perturbation, $\alpha_i = 0.26$, and $\beta_i = 0.16$. This rapidly-rotating cloud collapsed to fragment into a binary with a separation of about 100 AU. Each of the two binary fragments then sub-fragmented into two, yielding a possibly stable, hierarchical quadruple system (see Fig. 7). A very similar model was calculated by Klapp et al. (1993), who used a first-order accurate hydrodynamics code but still obtained a quadruple system (see Fig. 7) that is at least qualitatively very close to that of Boss (1991).

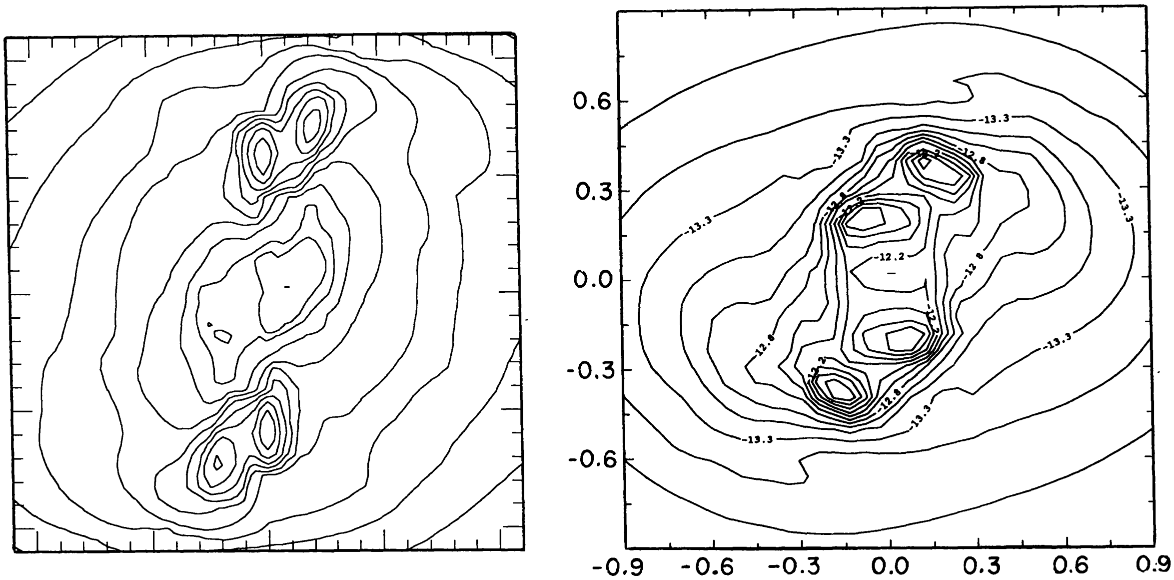

Fig. 7.— Hierarchical quadruple protostellar system formed by two different 3D hydrodynamics codes. Left: Box size is 270 AU. Figure from Boss (1991). Right: Box size is 120 AU. Figure from Klapp et al. (1993).

8. FRAGMENTATION DURING THE SECOND COLLAPSE PHASE

All of the preceding models have been directed toward evaluating the likelihood of fragmentation during the first dynamic collapse phase leading up to the formation of the first protostellar core. Here we consider the possibility of fragmentation during the second collapse phase, which occurs after molecular hydrogen dissociation decreases the gas pressure support of the first protostellar core.

Boss (1989) modeled the second collapse phase by following the collapse of a uniform density cloud with $\alpha = 0.4$ and $\beta = 0.1$, so that the virial equilibrium value of $\alpha + \beta = 0.5$ was achieved. The first core collapsed to densities on the order of 10^{-1} g cm^{-3} and central temperatures of $\sim$ 5000 K (Eddington approximation radiative transfer was used). Once β exceeded the critical value 0.274, significant non-axisymmetry began to grow in the rotationally flattened protostellar disk. A strong bar developed, which fragmented into a binary, but the binary fragments merged together again due to loss of orbital angular momentum caused by the growth of trailing spiral arms. As more mass and angular momentum accreted onto the central object, the bar fragmented once again into a binary, which then merged again due to outward angular momentum transport by gravitational torques. Evidently this model was on the verge of forming a binary, but did not quite succeed.

Bonnell & Bate (1995) have re-investigated the question of fragmentation during second core formation, using their SPH code and a piecewise adiabatic pressure law to represent the thermodynamics ($\gamma = 7/5$ to start, then 1.1 during the second collapse, and 5/3 to form the second core of atomic hydrogen). Bonnell & Bate (1995) computed the collapse of a variety of first core models, ranging from uniform density to Gaussian to power-law profiles, and with variations in α_i ($\sim$ 0.1 to $\sim$ 0.4) and β_i (0.03 to 0.25) such that either $\alpha + \beta = 0.5$ or $\alpha + \beta < 0.5$. All of the models collapsed to form either rotationally flattened disks or rings. The disks contained central cores, and as more matter accreted onto the disks, the trailing spiral arms swept up enough mass to form a companion core; the rings fragmented directly into a number of protostellar objects.

Bonnell & Bate (1994) followed the evolution of one of the binary second cores from Bonnell & Bate (1995) even farther in time, and found that continued mass accretion could lead to the formation of two more companions as the spiral arms gathered infalling mass, possibly leading to a quadruple protostellar system (see Fig. 8). The linear scale of these systems is quite small, on the order of $10R_{\odot}$, so such a process would produce very close binary stars. Thus it appears that fragmentation need not be limited to the first collapse phase, and that fragmentation potentially can account for the formation of binary stars with a very wide range of separations.

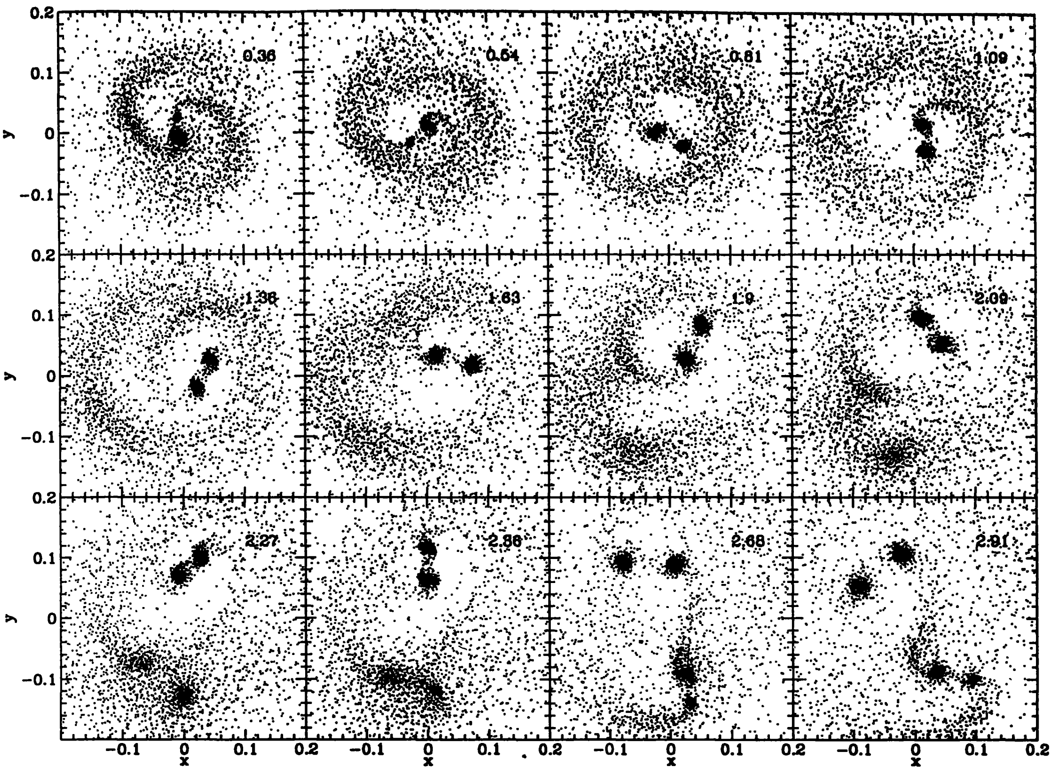

Fig. 8.— Time evolution of a system of second protostellar cores that accrete more mass and form a quadruple system. Box size is 40 $R_{\odot}$. Figure from Bonnell & Bate (1994).

9. SUMMARY

A great deal of effort has gone into studying the 3D collapse of dense molecular cloud cores, and the results appear to have been well worth the effort. There is a remarkable degree of agreement between the results obtained with different 3D hydrodynamics codes of a given type, and even between completely different types of code (i.e., finite-difference and SPH codes), both on realistic cloud models and on more specialized test cases (e.g., Boss & Bodenheimer 1979; Myhill & Boss 1993). The field of 3D protostellar collapse calculations has thus advanced to a point where the numerical results are reliable. These calculations have shown that collapsing molecular cloud cores are likely to undergo fragmentation, with the result being that the fragmentation mechanism is generally considered to be the leading explanation for the formation of binary and multiple star systems (e.g., Bodenheimer, Ruzmaikina, & Mathieu 1993).

Probably the most striking result of recent 3D hydrodynamics calculations is the increasing tendency for protostellar collapse to lead to fragmentation into multiple rather than binary protostellar systems. Several reasons may be noted: the calculations are being performed with higher order accuracy codes with less numerical diffusion, with higher spatial resolution, and starting from initial conditions that lend themselves to multiple fragmentation more than a uniform density sphere does. There is also a striking consistency between recent determinations of pre-collapse cloud density profiles, 3D fragmentation studies starting from Gaussian profiles, and the need to produce copious binary and multiple PMS stars during protostellar formation. Given a Gaussian profile pre-collapse cloud (Ward-Thompson et al. 1994), with $\alpha \sim 0.3$ (e.g., Lizano & Shu 1989), protostellar collapse will lead to the formation of binary and multiple protostars (Boss 1993b).

We are not quite ready to write the textbook summaries, turn off the lights, and go home, however. A number of key points remain to be addressed, of which the "theory gap" of Clarke (1992) perhaps looms largest. The 3D calculations described here demonstrate that binary systems composed of first (or even second) protostellar cores may readily form through fragmentation, but it has not yet been demonstrated that these systems will survive their subsequent orbital evolution and mass accretion without suffering orbital mergers. The development of an implicit technique for 3D hydrodynamics code may be necessary if we are to calculate the long-term evolution of protostellar systems. The neglect of magnetic fields in nearly all 3D fragmentation calculations is also a potentially serious omission – clearly 3D magnetohydrodynamics codes needed to be applied to the fragmentation problem. Theorists must continue to make contact with observations, which provide essential constraints on initial conditions for collapse and on the characteristics of the intermediate and final phases. Hopefully observers will continue to determine the properties of "class -II" objects (pre-collapse cores), perhaps see if "class -I" objects (first protostellar cores) can be identified by *ISO* and *SIRTF* or other telescopes, and accumulate statistical knowledge (e.g., binary frequencies) of the key characteristics of class 0 objects.

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