

THE FU ORIONIS OUTBURST AS A THERMAL ACCRETION EVENT: THEORETICAL AND OBSERVATIONAL IMPLICATIONS

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RESUMEN

Cálculos detallados de discos viscosos de acrecimiento sugieren que las erupciones de tipo FU Ori son indicativas de la existencia de discos protoestelares transportando masa a un ritmo promediado en el tiempo de $(1 - 10) \times 10^{-6} M_{\odot}/\text{año}$ en una fracción ($\approx 10\%$) de objetos estelares jóvenes. El acrecimiento a través de las partes internas de esos discos está autorregulado por la inestabilidad de ionización térmica, de tal manera que períodos largos (≈ 1000 años) de bajo flujo de masa, $(1 - 10) \times 10^{-8} M_{\odot}/\text{año}$, son seguidos por períodos cortos (≈ 100 años) de alto flujo, $(1 - 10) \times 10^{-5} M_{\odot}/\text{año}$. La región del disco inestable se extiende hasta $\approx 1/4 AU$. Más allá de esa región el material es transportado de una forma estable a través del disco con un ritmo \dot{M}_{in} .

En sistemas con $M_* = 1 M_{\odot}$ y radio interior del disco de $3 R_{\odot}$, el ritmo crítico para la erupción es $5 \times 10^{-7} M_{\odot}/\text{año}$, *independientemente de la magnitud de la viscosidad*. La magnitud del parámetro alfa puede restringirse por ajuste de escala de tiempo global hasta 10^{-4} cuando el hidrógeno está neutro, y a 10^{-3} cuando está ionizado. Esta baja viscosidad tiene implicaciones para la auto gravedad como posible mecanismo de transporte de masa a través de casi todo el disco. Se reproducen las curvas de luz de V1515 Cyg, FU Ori y V1057 Cyg, comparando los modelos con las observaciones.

ABSTRACT

Detailed calculations of viscous accretion disks suggest that FU Ori outbursts signify the existence of protostellar disks transporting mass at a time-averaged rate of $(1 - 10) \times 10^{-6} M_{\odot}/\text{yr}$ around some fraction ($\approx 10\%$) of young stellar objects. Accretion through the inner parts of these disks is self regulated by the thermal ionization instability such that long periods ($\approx 1000 \text{ yr}$) of low mass flux: $(1 - 10) \times 10^{-8} M_{\odot}/\text{yr}$, are punctuated by short periods ($\approx 100 \text{ yr}$) of high mass flux: $(1 - 10) \times 10^{-5} M_{\odot}/\text{yr}$. The unstable region of the disk extends only to $\approx 1/4 AU$. Beyond this region matter is transported stably through the disk at the infall rate, \dot{M}_{in} .

In systems for which $M_* = 1 M_{\odot}$ with inner disk edges of $3 R_{\odot}$, the critical rate for outburst is $5 \times 10^{-7} M_{\odot}/\text{yr}$ *independent of the magnitude of the viscosity*. The magnitude of the alpha parameter may be constrained by global timescale fitting to be 10^{-4} where hydrogen is neutral and 10^{-3} where ionized. This low viscosity has implications for self gravity as a possible mass transport mechanism through much of the disk. Light curves of V1515 Cyg, FU Ori, and V1057 Cyg are reproduced and model results are compared to observations.

Key words: ACCRETION, ACCRETION DISKS — STARS: PRE-MAIN-SEQUENCE — STARS: FU ORI, V1515 CYG, V1057 CYG

1. INTRODUCTION

In this contribution are discussed some theoretical and observational consequences of the Bell & Lin (1994) disk accretion model for FU Orionis outbursts. In §2., the importance of these outbursts is briefly discussed. In §3., some useful accretion disk formulae are outlined, and in §4., the Bell & Lin outburst model is summarized. To match global timescales of FU Orionis statistics, a very small viscosity is required. In §5., theoretical implications of the small viscosity for self gravity as a mass transport mechanism are discussed. In §6., model fits to the light curves of FU Ori, V1515 Cyg, and V1057 Cyg are shown. Model features are used to make detailed reply to objections raised for the accretion disk model for outbursts, and predictions of future line-width evolution are made.

2. BACKGROUND

FU Orionis outbursts are temporary large increases in luminosity: $\times(40-250)$ thought to occur repeatedly in all low mass, young stellar systems (Herbig 1977). Time to peak light ranges from one year in the case of prototype FU Ori to 20 years in the case of V1515 Cyg. Outbursts are estimated to last ≈ 100 yr and are thought to recur on a timescale of $10^3 - 10^4$ yr (Herbig 1989). For a discussion of FU Ori timescales see the review by Kenyon (1995) in these Proceedings. Although FU Ori objects are optically revealed, all show long wavelength excesses consistent with slightly flattened envelopes which extend in to several AU (Kenyon & Hartmann 1991).

Broad band photometry of FU Ori objects (hereafter Fuors) show a characteristic broader than black body spectral shape which can be well fitted with the assumption that the system radiates simultaneously at a range of temperatures. Spectra are consistent with the model that the system is a flattened nebula with a temperature distribution: $T_s \sim r^{-3/4}$, characteristic of protostellar disks (Kenyon, Hartmann, & Hewett 1988). Spectral analysis of systems in outburst show absorption features which decrease in line width as a function of wavelength consistent with emission from a Keplerian disk in which rotation rate and surface temperature both decrease with increasing radius (Hartmann & Kenyon 1987). Estimates of accretion rates through FU Orionis disks are in the range of $(1 - 10) \times 10^{-5} M_\odot/yr$ (Kenyon, Hartmann, & Hewett 1988).

From a preoutburst optical spectrum of V1057 Cyg (Herbig 1977) which shows typical T Tauri emission features, it seems that the precursors of FU Ori outbursts are not dramatically different from the normal T Tauri or Class II objects. The suggestion that the preoutburst object was "rather red" together with a surprisingly small estimate of R-I, however, suggest that the precursor may have had a rising spectral energy distribution more typical of the younger Class I objects (Bell et al. 1995).

Accretion disks are thought to play a role in the evolution of most protostellar systems. Observations of broader than black body infrared excesses in young T Tauri objects are widely accepted to be indicative of the presence of flattened solar nebula type disks. Observed correlations of infrared and ultraviolet excesses in visible T Tauri systems suggest that the disks actively transport mass inward onto the central object and do not merely passively reradiate light from the central object (Bertout, Basri, & Bouvier 1988). Estimates of the highest accretion rates in T Tauri systems rarely exceed several times $10^{-7} M_\odot/yr$: two to three orders of magnitude lower than the mass fluxes estimated for Fuors. How, then do these systems make the transition from low to high mass flux state, and why are there so few observations of intermediate mass fluxes?

3. ACCRETION DISK FORMULAE

In this section some useful formulae are briefly derived. Viscosity (responsible for local energy generation and mass transport) is the classic unknown in disk calculations. Molecular viscosity is too small to transport mass through the nebula on reasonable timescales by many orders of magnitude. Shakura & Sunyaev (1973) therefore assume that mass transport occurs due to the turbulent mixing of adjacent annuli over the local pressure scale height: H , at the local sound speed: c_s , with some unknown efficiency: α ; thus arose the standard "alpha law" viscosity: $\nu = \alpha c_s H$. The source of the underlying turbulence remains unknown although proposed processes include convection, magnetic instability, and surface accretion shocks.

Many global properties of accretion disks such as mass flux and surface temperature can be derived independent of the viscosity. While it is convenient that much can be learned about disks without specifying the underlying mass transport mechanism, this necessarily means that the form of the viscosity cannot be derived from observations alone; it must be derived from numerical fits. In the standard disk model (Shakura & Sunyaev 1973; Lynden-Bell & Pringle 1974), optically thick ($\tau \gg 1$), geometrically thin ($H/r \ll 1$) disks are

in vertical thermal balance; all locally generated and absorbed energy (Q^+) is locally radiated (Q^-). In such systems, the vertical and radial evolution is decoupled and can be calculated independently. With \dot{M} and Σ as full plane mass flux and surface density respectively, the vertically averaged equation of motion is given by

$$3\pi\Sigma\nu_c = \dot{M}(r)(1 - \beta\sqrt{R_*/r}). \quad (1)$$

The surface flux, $\sigma T_d^4(r)$, can be shown to depend only on the local mass flux \dot{M} :

$$\sigma T_d^4(r) = \frac{3GM_*\dot{M}(r)}{8\pi r^3} \left(1 - \beta\sqrt{\frac{R_*}{r}}\right), \quad (2)$$

where R_* and M_* are the stellar radius and mass flux respectively. The factor β depends on the inner boundary condition and is generally taken to be one (eg. Lynden-Bell & Pringle 1974).

The radial temperature dependence of the disk can also be derived assuming that the disk has no internal energy sources but merely reradiates light absorbed from the central object. For a strictly flat disk, one quarter of the luminosity from the central object is intercepted by the disk (Adams & Shu 1986). With the projection factor of R_*/r this “reprocessing” flux depends on the stellar flux σT_*^4 :

$$\sigma T_{rp}^4(r) = \frac{\sigma T_*^4}{4} \left(\frac{R_*}{r}\right)^3. \quad (3)$$

For flat disks, at large radius, accretion and reprocessing therefore result in the same radial surface temperature gradient: $T_s \sim r^{-q}$, $q = 0.75$. If there is significant flaring of the disk, q will be smaller. From fitting of T Tauri spectra, estimates of temperature profiles range from 0.5 to 0.75 (Beckwith et al. 1990) consistent with the presence of flat or slightly flared disks.

4. BELL & LIN THERMAL OUTBURST MODELS

Observations confirm the presence of disks and can even provide estimates of disk mass flux, but tell nothing about the physical processes within. Just as in stellar atmospheres where interior properties are derived from model fits to surface observations, so in disk theory intrinsic disk properties such as surface density and central temperature can only be derived from numerical models.

The details of the thermal instability and of the numerical calculations used to investigate this instability are discussed fully in Bell & Lin (1994, hereafter BL94) and Bell et al. (1995, hereafter BLHK95). The essence of the thermal instability is the temperature dependence of the opacity ($\kappa \sim T^{10}$) and the rapid changes in the equation of state variables (C_V , μ , ∇_{ad} etc.) during the ionization of hydrogen. Any system with a temperature range which spans the region of hydrogen ionization is subject to some aspect of the thermal instability. An example of a similar mechanism in a very different context is the κ mechanism responsible for the well known “instability strip” in the HR diagram inhabited by Cepheids and RR Lyrae variables.

Under the assumption of vertical thermal balance ($Q^+ = Q^-$), the vertical structure of the disk may be calculated given a radius and mass flux. Examples of such calculations for four different values of α are shown in Figure 1. In Fig. 1a where the slope is positive the system is stable. If, for example, the disk is perturbed off equilibrium upward, local cooling becomes stronger than local heating, and the disk cools and returns to vertical thermal balance. Where the slope is negative, however, an upward perturbation results in thermal runaway and the disk heats until reaching the stable upper branch.

From the vertical structure models, it can be seen that instability occurs in the range of $(1 - 10) \times 10^{-6} M_\odot/yr$. This mass flux is in agreement with theoretical and observational estimates of molecular cloud core collapse rates. The maximum stable quiescent mass flux: $\dot{M}_{crit} \approx 5 \times 10^{-7} M_\odot/yr$, is consistent with estimates of boundary layer mass flux in T Tauri stars. The critical mass flux can be demonstrated with the simple hydrostatic, vertical structure models shown in Figure 1 and is clearly *independent of the magnitude of the viscous α parameter*. Critical mass fluxes depend to some degree on the assumed opacity law (BL94).

All derivations so far are performed in steady state do not constrain value of α . Changes in the disk, however, occur on the local viscous or thermal timescales and therefore provide a rare opportunity to probe the details of the underlying mass transport mechanism. Time dependent calculations are therefore fit to timescales of FU Orionis outbursts (BL94). To produce systems in which long periods of low mass flux are punctuated by

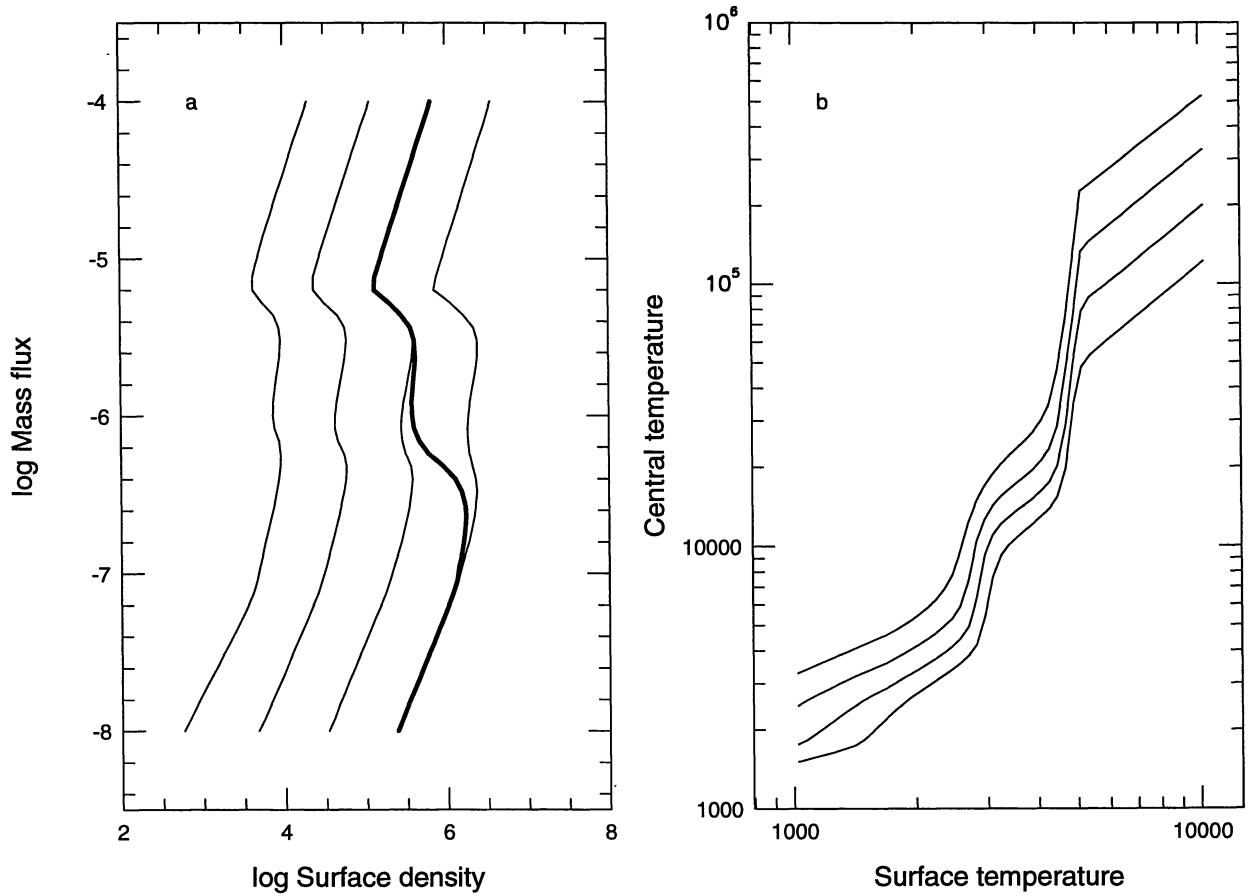


Fig. 1.— Stability at $4.1 R_\odot$ for $\alpha = 10^{-1}$, 10^{-2} , 10^{-3} , and 10^{-4} (a: left to right, b: top to bottom). (a) $\dot{M}(M_\odot/\text{yr})$ vs. $\Sigma(gm/cm^2)$. Regimes with positive slope are thermally stable; regimes with negative slope are unstable. Heavy line is for $\alpha_c = 10^{-4}$ and $\alpha_h = 10^{-3}$. (b) $T_c(K)$ vs. $T_d(K)$.

well-separated episodes of high mass flux, the value of α must be smaller in the quiescent or cold state (α_c) than in the outburst or hot state (α_h). In BL94, FU Ori timescales are reproduced when $\alpha_c = 10^{-4}$ and $\alpha_h = 10^{-3}$ (heavy line Fig. 1a). Larger values of α result in unacceptably short timescales.

These models have very few free parameters. Once the form of the viscosity, opacity, and equation of state are specified, \dot{M}_{in} , α_h , and α_c are the principal variables. In addition, the stellar mass is chosen to be $1 M_\odot$ and the inner disk radius to be $3 R_\odot$. Critical mass flux is dependent on the latter two parameters as follows. Because the initiation of outburst requires potential energy sufficient to ionize hydrogen, \dot{M}_{crit} is inversely related to the central object stellar mass. Larger mass systems show smaller critical mass flux ($\dot{M}_{crit} \approx 10^{-7} M_\odot/\text{yr}$ for $M_* = 3 M_\odot$; Bell 1994). Also there is some stellar mass below which outbursts are not expected to occur; the very youngest protostars are therefore not subject to FU Ori outbursts. The critical mass flux also depends on the radius of the inner edge of the disk which in the standard case is taken to be $3 R_\odot$. A test case in which the inner disk radius is $6 R_\odot$ (BL94) confirms that larger critical mass fluxes are possible for larger inner disk radius. If the disk is truncated at larger radius (for example, matter may be transported along magnetic field lines), larger stable mass fluxes may be possible. An example of such an object may be DR Tau in which an unusually high accretion rate ($10^{-5} M_\odot/\text{yr}$) is coupled with a large estimated stellar radius ($4 R_\odot$) (Hartigan, Edwards, & Ghandour 1995).

Table 1.— Parameters and Results of Outburst Models

Model	\dot{M}_{in} (M_{\odot}/yr)	M_p ($10^{-2} M_{\odot}$)	$\Delta\Sigma/\Sigma$	R_p (R_{\odot})	$T_{max}\dagger$ (K)	$\dot{M}_* M_*\dagger$ (M_{\odot}^2/yr)	$R_t\dagger$ (R_{\odot})
A1	3×10^{-6}	1.0	3	13	7800	5.5×10^{-5}	35
A2	3×10^{-6}	1.0	10	13	7800	5.8×10^{-5}	35
A3	5×10^{-6}	0.7	5	20	8000	6.5×10^{-5}	35
B1	1×10^{-5}	7900	6.0×10^{-5}	26
B2	8×10^{-6}	7700	5.3×10^{-5}	25
B3	5×10^{-6}	0.1	4	34	7900	6.1×10^{-5}	26
C1	1×10^{-6}	0.2	3	10	6700	2.6×10^{-5}	24
C2	1×10^{-6}	0.2	5	10	6700	2.6×10^{-5}	24
C3	8×10^{-7}	0.3	5	10	6900	3.0×10^{-5}	24

†“Current” values at end of light curve fitting.

5. IMPLICATIONS FOR MASS TRANSPORT

The small value of $\alpha = 10^{-4}$ implied by model timescale fitting is consistent with calculations in which turbulent convection provides the local viscosity (Ruden, Papaloizou, & Lin 1988). Quiescent models have typical surface temperatures of 2000K in the inner disk (BL94) and can be shown to be strongly unstable to convection. Viscosity in disk regions expected to go into outburst is therefore consistent with convective turbulence as the physical mechanism for mass transport. Beyond the thermally unstable region, however, as the disk temperature drops, convection is expected to be weak. Such convection as there is has been shown to be incompatible with the presence of local viscous energy generation (Rózyczka, Bodenheimer, & Bell 1994).

In a unified accretion scenario, there may be different sources of mass transport operating at different radii. If α is nevertheless taken to be constant with radius, disk evolution timescales and masses become uncomfortably large (BLHK95). Taking $\tau_{\nu} \sim r^2/\nu \sim (r/H)^2(\alpha\Omega)^{-1}$, with $\nu \approx \alpha H^2\Omega$, $H/r \approx 0.1$, and $\alpha = 10^{-4}$, it takes an estimated 5×10^6 yr for matter to viscously accrete from only 10 AU and 2×10^8 yr from 100 AU. Disk mass is inversely related to viscosity: $\Sigma \approx \dot{M}/(3\pi\nu)$, and low values of α result in uncomfortably large disk masses. By the above estimate, an only 10 AU disk accreting at $10^{-7} M_{\odot}/yr$ is $0.2 M_{\odot}$. These timescales and masses are at odds with evidence which suggests that optically thick disks have lifetimes of less than 10^7 yr (Strom, Edwards, & Skrutskie 1993) and disk masses of no more than $0.1 M_{\odot}$ (Beckwith et al. 1990).

A way out of the dilemma is offered by theoretical derivations which suggest that disks more massive than $0.2 M_{\odot}$ are unstable to mass transport due to a global self gravity instability (Shu et al. 1990). The radius beyond which self gravity is important can be estimated from the radius where the Toomre parameter $Q_T = c_s\Omega/\pi G\Sigma$ (Toomre 1964) drops to one:

$$R(Q_T=1) = 1.5 \text{ AU} \left(\frac{\alpha}{10^{-4}} \right)^{2/3} \left(\frac{H/r}{0.1} \right)^2 \left(\frac{\dot{M}}{10^{-6} M_{\odot}/yr} \right)^{-2/3} \left(\frac{M_*}{1 M_{\odot}} \right) \left(\frac{Q_T}{1} \right)^{-2/3} \quad (4)$$

It can be argued that the process of self gravity acts to maintain the disk at some constant Q_T . By inverting the above equation, the process of self gravity can be expressed as an effective local viscosity for which, in a constant mass flux disk, α increases radially as $r^{3/2}$ (see BLHK95 for further discussion). In the inner disk, self gravity is therefore ineffective and some other form of mass transport (perhaps convective turbulence) must be present to transport mass while at “large” radius ($r > 1/2$ AU), self gravity may dominate.

6. DETAILED LIGHT CURVE FITTING

Using the values of α derived from global timescale fitting in BL94, we then attempt to model specific light curves and to test the predictive power of this outburst model by comparing observations of spectral and line width evolution to model results. Applying standard extinctions taken from the literature, the dereddened

light curves $M_B(t)$ of FU Ori, V1515 Cyg, and V1057 Cyg are fitted with model outbursts (Figure 2); outburst parameters are given in Table 1. V1515 Cyg can be fitted with a self regulated outburst of the BL94 type; the other two objects require application of small perturbations in surface density to produce the observed rapid rise times. Perturbations in surface density are small (M_p less than about $0.01 M_\odot$) and are applied at a radius (R_p) designed to force the outer edge of the thermally unstable region to go into outburst first. In a triggered model, the ionization front propagates in from the outside and has a considerably faster rise time than the self regulated model (see the discussion in BLHK95). In both triggered and self regulated models, outbursts propagate to radii (R_t) less than $1/4 AU$ ($54 R_\odot$). Outside-in outbursts may be triggered by the close passage of low mass binary companions, by the accretion of protoplanetary masses, or perhaps by a viscosity which increases with radius in the quiescent state.

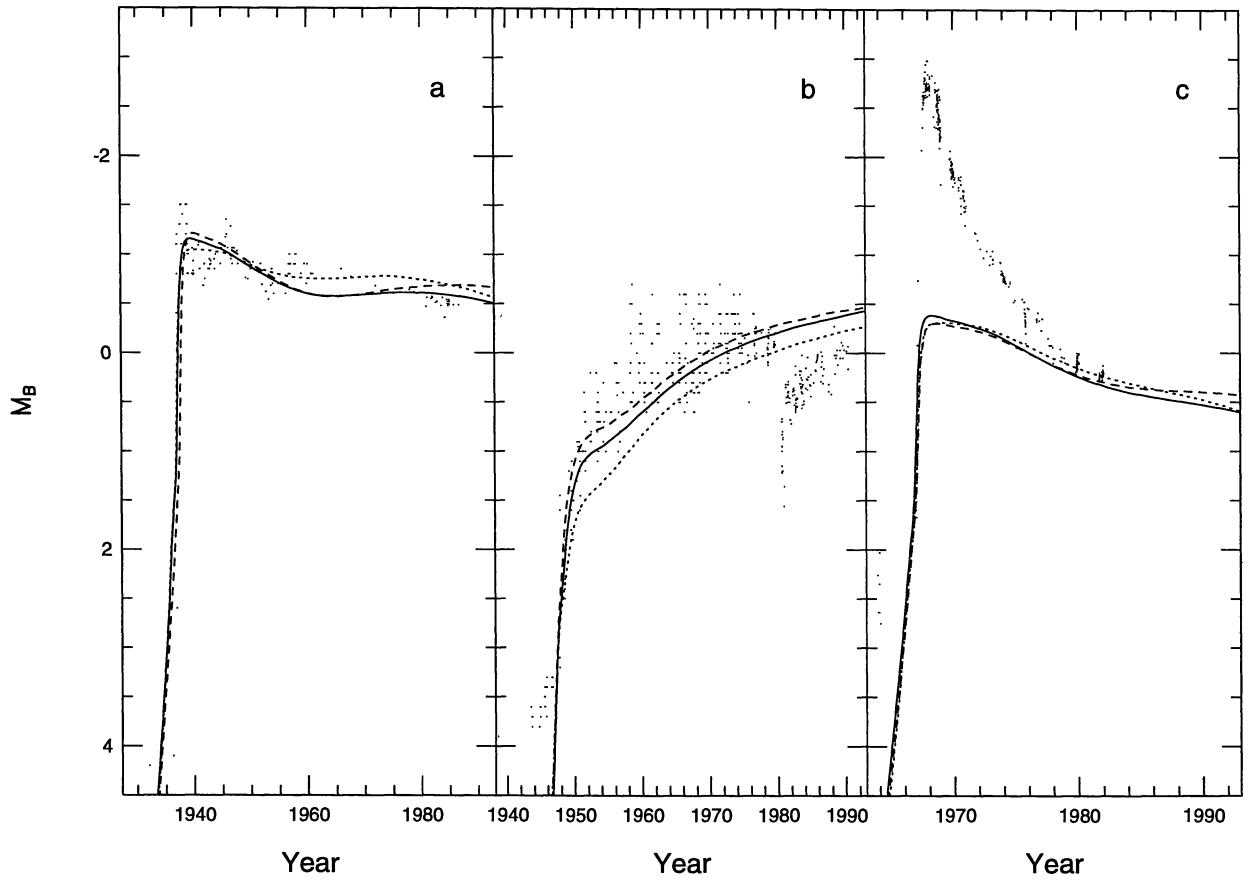


Fig. 2.— Detailed fitting of FU Orionis light curves for (a) FU Ori, (b) V1515 Cyg, and (c) V1057 Cyg. Dots are extinction corrected data points; lines are models with parameters given in Table 1.

The dip in the light curve of V1515 Cyg is thought to be the result of a dust condensation event (Kenyon, Hartmann, & Kolotilov 1991) and is not considered in light curve fitting. The poor fit to the early light curve of V1057 Cyg may be due to numerical limitations during the rapid onset of outburst. Fitting requires a strong triggering of an otherwise only marginally supercritical disk ($\dot{M}_{in} \approx \dot{M}_{crit}$). The ionization front therefore travels only a short distance through the disk ($1/10 AU$), and model evolution is subject to uncertainties and inaccuracies in the inner boundary condition. Because of these uncertainties, fitting of the probably related but

less powerful Exor outbursts (Herbig 1989) has not been attempted. Note that a range of outburst parameters may be used to produce acceptable fits to any given light curve. Note also that fits are made using *absolute* magnitudes as derived from one value of the extinction and may be somewhat altered with different extinctions.

7. MODEL PREDICTIONS

Both detailed light curve fits and the more general self regulated models may be used to make *predictions* about observable Fuor features. Solutions to several of the principal objections to the accretion disk model for FU Ori outbursts may be found in the *radially restricted* nature of model results. It has been argued (Herbig 1989) that decay from peak light in the disk outburst model ought to occur simultaneously at all wavelengths because all disk models, regardless of mass flux, have the same temperature profile (eq. 2) and therefore have the same spectral slope: $\lambda F_\lambda \sim \lambda^{-4/3}$ (Lynden-Bell & Pringle 1974). Observations of V1057 Cyg, however, show that decay since peak light is greater in the optical and ultraviolet then in the near infrared (Kenyon & Hartmann 1991). Further, the few preoutburst observations which exist suggest that changes are also greatest at short wavelength during the rise to outburst. These features find natural explanation in a disk in which only the innermost annuli are subject to outburst.

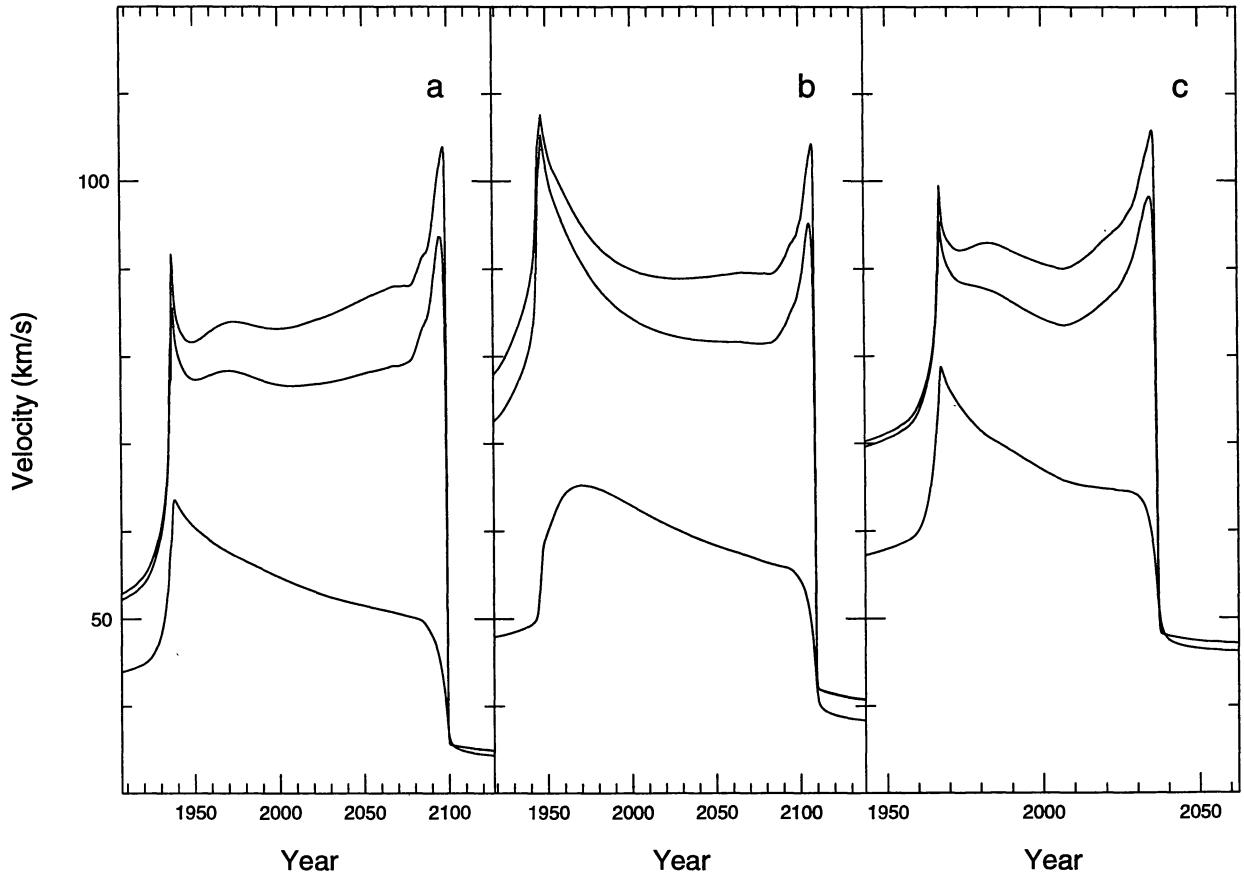


Fig. 3.— Model line-width evolution for models (a) A1, (b) B1, and (c) C1 in km/s. Lines are for 4000 Å, 6000 Å, and 2.3 μ m top to bottom corresponding to temperatures of 7200, 4800, and 1300 K respectively. Note the initial downward trend caused by the outward movement of ionization front. The upturn for the shorter two wavelengths at later times is due to the nearly simultaneous collapse of the entire unstable region of the disk during the decay from outburst.

Another major objection to the disk outburst model for FU Ori events is the observation that since peak light the optical line widths of V1057 Cyg have decreased (Herbig 1989). The observed inverse dependence of line width on wavelength in Fuors is due to the radial temperature structure of the disk. A given wavelength probes the disk at a temperature given by the Wien law. The radius which is at this temperature is indicated by the line width (Hartmann & Kenyon 1987). Observation of the *evolution* of these line widths can therefore reveal important information about the evolving radial temperature structure.

In the standard disk outburst model during the decay from peak light, cooling occurs simultaneously at all radii. Line widths increase at every wavelength as a given temperature is radiated by successively smaller radii. Calculations of model line width evolution is shown in Figure 3 for three wavelengths and shows considerably more complicated behavior. The initial drop in velocity is due to the outward propagation of the ionization front; as successively larger radii go into outburst, lower velocity components are added to the line widths. Once the ionization front has propagated to its maximum radial extent, mass fluxes throughout the inner disk drop essentially simultaneously; velocities at the shorter wavelengths therefore rise as a given temperature is radiated by ever smaller radii. Longer wavelengths, however, ($\lambda > 1.5\mu\text{m}$ corresponding to a surface temperature of 2000K) probe radii which never go into outburst. In this case velocities only increase during outburst because of contributions from the long wavelength tails of the bright inner annuli. As the disk decays after peak light, this component decreases and the longer wavelength velocities decline monotonically (see discussion in BLHK95).

This line width evolution is a strong prediction of the thermal disk model for FU Orionis outbursts. The decline in velocities after the onset of outburst has already been observed in V1057 Cyg. It would be interesting to follow the line width evolution of known Fuors to see whether they follow the pattern shown by the models.

8. DISCUSSION

The ubiquity of FU Orionis outbursts may have important implications for our understanding of the evolution of young stellar objects. Because the largest T Tauri stellar mass fluxes are estimated to be little over $10^{-7}M_{\odot}/\text{yr}$, even integrated over the probable lifetime of active T Tauri disks of a million years, disk mass fluxes would not contribute significantly to the final stellar mass. In this outburst model, however, a system's instantaneous stellar mass flux: M_* , may be very different from its time averaged mass flux: \dot{M}_{in} . If the disk transports the $(1-10) \times 10^{-6}M_{\odot}/\text{yr}$ expected by this thermal outburst model even for only a hundred thousand years, the fraction of the final stellar mass which was accreted through the disk may be large.

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