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CHARACTER OF DYNAMO-GENERATED MAGNETIC FIELDS IN VISCOUS PROTOSTELLAR DISKS

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RESUMEN

En este artículo consideramos campos magnéticos en discos protoestelares modelados mediante un modelo estacionario de viscosidad α. Argumentamos que la condición para que en tales discos exista un campo magnético dinámicamente importante es que éste se mantenga por un proceso de dínamo de orígen turbulento. Concentrándonos en un modelo específico de un disco viscoso, calculamos las propiedades de los campos magnéticos generados por dínamo, discutiendo la importancia de este campo en la estructura y evolución de los discos protoestelares.

ABSTRACT

Magnetic fields are likely to be present in protostellar disks. In this paper we consider magnetic fields in disks that can be modeled by a viscous, steady-state α model. We argue that in such disks a dynamically important magnetic field can be present only if maintained by a turbulent dynamo process. Concentrating on a specific model of a viscous disk we calculate the properties of dynamo-generated magnetic fields and discuss the importance of this field to the structure and evolution of protostellar disks.

Key words: **STARS: CIRCUMSTELLAR MATTER — MAGNETIC FIELDS**

1. INTRODUCTION

Astronomical observations of young low-mass stars have led to wide acceptance of the idea that these stars are surrounded by *accretion* disks (for a recent review see Strom & Edwards 1993). Furthermore, it seems that much of our solar system's gross architecture has arisen from physical processes and conditions characteristic of accretion disks (Levy 1993). Because we believe that the gaseous solar nebula from which the solar system evolved over 4×10^9 years ago exemplifies contemporaneously observable disks around solar-mass protostars, we can make a persuasive case that *all* protostellar disk are in fact accretion disks (hereafter called APDs for accretion protostellar disks). It is often assumed that turbulent viscosity is solely responsible for accretion in such disks.

Are APDs magnetized? Presently, there is no observational evidence for magnetic fields in protostellar disks. It has proven very difficult to detect magnetic activity in "classical" T Tauri stars, objects that we envision to be young pre-main-sequence stars surrounded by an APD. Nonthermal radio activity from a typical protostellar object, T Tau, was reported (Phillips et al. 1993), pointing to the possibility of magnetic field existence *somewhere* in the T Tau system. With the available spatial resolution, one cannot determine whether such a magnetic field is rooted in the protostar itself or, alternatively, originates from a disk. If we accept the notion that the solar nebula was representative of APDs, there is another line of argument for the existence of a magnetic fields in APDs: remanent magnetization of primitive meteorites. The residual magnetization of

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carbonaceous chondrites, which are generally assumed to be relics from the nebular epoch of the solar system and thus owe their magnetization to nebular magnetic fields, has been used to estimate the intensity of such fields. Values of field strengths ranging from 0.1 to 1 Gauss are often quoted (Butler 1972; Brecher 1972).

In principle, we expect APDs to be magnetized. Two different scenarios for magnetization of APDs have been offered in the literature. First, APDs may be threaded by a magnetic field generated in a molecular cloud surrounding a star and its APD. Within a disk the inward dragging of field lines leads to magnetic field amplification. This mechanism has obtained a lot of attention because the resultant structure of the magnetic field seems to be favorable for launching centrifugally driven MHD winds (Pudritz & Norman 1983; Königl 1989) that may be the physical mechanism that drives energetic bipolar outflows – a ubiquitous feature of young stellar objects. Second, conditions in APDs (differential rotation and the presence of turbulence) are favorable for MHD dynamo action. Magnetic fields due to the dynamo mechanism are *internally* generated from an arbitrarily small seed field. In contrast to the first scenario, no externally imposed fields are necessary to maintain a magnetic field within a disk.

In §2 we investigate the feasibility of both the above mentioned scenarios in the context of *viscous* APD. We conclude that as long as the dynamics of a disk is governed by viscous forces, the first scenario is unlikely to provide any significant disk magnetization and couldn't launch centrifugally driven winds. The second scenario may lead to the presence of a significant magnetic field. Thus, in §3 we describe the basic physics of MHD dynamo as applied to APDs, we then give the criteria for existence of dynamo- generated field, discuss the evolution of the field, as well as the strength and topology of an equilibrated magnetic field. Finally, in §4 we remark on the influence of such a field on the structure and evolution of a viscous APD.

2. MAGNETIC FIELDS IN VISCOUS PROTOSTELLAR DISKS: EXTERNALLY MAINTAINED FIELD VERSUS INTERNALLY GENERATED FIELD

The observations suggest that APDs are in the so-called viscous stage of their evolution. They are often modeled by a Keplerian, axisymmetric, geometrically thin, steady-state, turbulent disk, based on the widely adopted α prescription of turbulent viscosity introduced by Shakura & Sunyaev (1973). Our considerations explicitly assume that APDs are described by a viscous model. Table 1 shows the properties of the APD as described by a steady-state model with $\dot{M} = 10^{-7} M_{\odot}/\mathrm{yr}$ and $\alpha = 0.08$. We have assumed a disk surrounding a 1 $M_{\odot}$ star and extending up to 60 AU from the star. These values yield a disk with a mass $M_d < 0.1 M_{\odot}$ and overall properties corresponding to the average T Tauri disk. Physical quantities describing a disk are temperature, T, disk half-thickness, H, surface density, Σ, resistive magnetic diffusivity, η_r, and turbulent magnetic diffusivity η_t. The entries in Table 1 are calculated using formulas given by Stepinski et al. (1993). The quantity β_{BH} will be described in §3.1 and the quantity B_{eq} will be described in §3.3.

Table 1.— Summary of Steady-State APD model with $\dot{M} = 10^{-7} M_{\odot}/\mathrm{yr}$ and $\alpha = 0.08$

Radius (AU)	T (°K)	H (cm)	Σ (g/cm^2)	η_r (cm^2/s)	η_t (cm^2/s)	β_{BH}	B_{eq}(gauss)
1	520	8.4×10^{11}	190	3×10^{16}	1×10^{16}	5.2	2
3	177	2.4×10^{12}	113	1×10^{16}	1.8×10^{16}	27	0.6
5	140	4.7×10^{12}	65	5.3×10^{15}	3.1×10^{16}	88	0.3
10	50	7.9×10^{12}	65	4.1×10^{15}	3.1×10^{16}	115	0.13
20	18	1.3×10^{13}	65	3.2×10^{15}	3.1×10^{16}	150	0.06
40	6.3	2.2×10^{13}	65	2.2×10^{15}	3.1×10^{16}	217	0.03

We are interested in a magnetic field within a disk. We assume that a field is axisymmetric and use cylindrical polar coordinates (r, ϕ, z). For the purpose of the present discussion we use the following definitions:

- An externally maintained field (EMF) is the field that results from inward dragging of externally imposed field lines. The only source of magnetic field amplification is the radial velocity. The possible amplification of magnetic field by turbulent dynamo is ignored.
- An internally generated field (IGF) is the field that results from a *self-excited* dynamo process. The possible existence of an external field is ignored.

Fig. 1 shows schematic plots of poloidal field lines for an EMF and an IGF, respectively. Let's first discuss the features of an EMF configuration. We stress that the prolonged existence of an EMF is possible only if accompanied by the *simultaneous* existence of a magnetic field at "infinity." If the currents supporting a magnetic field at "infinity" vanish, an EMF in a disk vanishes on a time scale of the order of the Kepler time. This is because, in a typical APD, the time scale for ohmic dissipation is of the order of the Keplerian period. An EMF has a dipole symmetry: B_r and B_ϕ are odd with respect to the equator ($z = 0$) and B_z is even. Lubow et al. (1994a) calculated a steady-state configuration of an EMF in a viscous accretion disk. They show that the configuration of the final field depends on the single dimensionless parameter $\mathcal{D} = (r/H)\mathcal{P}$ where $\mathcal{P} = \eta/\nu$ is the magnetic Prandtl number. Here $\eta = \eta_r + \eta_t$ is the total magnetic diffusivity and ν is turbulent kinematic viscosity. Significant amplification of a magnetic field occurs only for $\mathcal{D} < 1$; for $\mathcal{D} > 1$ there is no amplification: the strength of the magnetic field within a disk is equal to the strength of the magnetic field at "infinity." Lubow et al. (1994a) also show that the field emerging from the disk makes an angle i with the vertical so that $i = \arctan(1.52\mathcal{D}^{-1})$. In a turbulent disk, magnetic diffusivity is dominated by turbulent magnetic diffusivity and $\eta \approx \nu$, yielding $\mathcal{P} \approx 1$. Thus $\mathcal{D} \approx r/H$, which for a thin disk is much larger than 1. In addition, i is a very small angle, well short of the 30° required for the operation of a centrifugally driven wind (Blandford & Payne 1982). We conclude that if APDs are indeed viscous disks, the concept of EMF fails to yield any significant magnetization within a disk; it also fails to provide the field topology necessary to launch centrifugally driven winds. One can argue that APDs are not turbulent but instead are powered by centrifugally driven winds themselves. The existence of such disks is often *postulated* (for example Wardle & Königl 1993); however, a self-consistent solution has yet to be found. A recent attempt to find such a solution (Lubow et al. 1994b) shows that it may be unstable.

Now let's discuss the features of an IGF configuration. The existence of an IGF depends on whether or not a self-excited dynamo process works in an APD. In the self-excited dynamo the magnetic field necessary to produce an electromotive force is itself generated with the help of the currents resulting from the very existence of an electromotive force. Only at the very first instant is an arbitrarily small field produced by some other source necessary to initiate the dynamo process. Thus, contrary to the EMF, an IGF may exist without any externally imposed fields. Because the system of electric currents that support an IGF has a finite spatial extent, we expect an IGF configuration to form closed field lines (see Fig. 1). We also expect that an IGF has a quadrupole symmetry: B_r and B_ϕ are even with respect to the equator and B_z is odd. In principle, an IGF may have either dipole of quadrupole symmetry; however, the predominance of the quadrupole is understandable in simple physical terms. A major part of the field generation results from azimuthal stretching of the radial component of the field by the Keplerian shear to produce a toroidal magnetic field. For a dipole field, the height-averaged radial magnetic field vanishes, so that this shearing generation of the toroidal field is weak, whereas for the quadrupole field, the radial field component is strong in the disk, resulting in very effective generation of toroidal magnetic field by the Keplerian shear.

A self-excited dynamo works if there is sufficiently intense motion with a *proper* geometrical structure so the induction effect balances or exceeds the dissipative effect. Can a self-excited dynamo operate in a protoplanetary disk? The answer is positive, providing that the disk is turbulent (to provide the proper geometry of motion) and

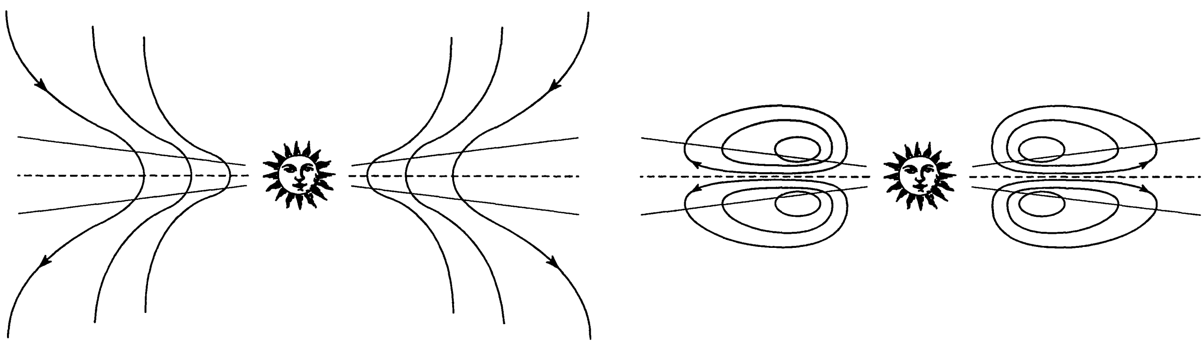

Fig. 1.— Schematic plots of poloidal field lines for EMF (left) and IGF (right). Note that an EMF has a quadrupole symmetry and is characterized by open field lines, whereas an IGF has a dipole symmetry and is characterized by closed field lines.

has high enough conductivity (so the dissipative effect is not overwhelming). A working dynamo will amplify a seed magnetic field until the resulting Lorentz force become great enough to destroy the proper geometry of the motion and halt the growth of the magnetic field. Thus the ultimate strength of an IGF depends on this nonlinear back-reaction called magnetic quenching. In the next section we show that APDs can maintain a relatively strong IGF by means of turbulent dynamo.

3. TURBULENT DYNAMOS IN PROTOSTELLAR DISKS

It is easy to show that the combination of Keplerian rotation with any axisymmetric radial flow cannot support a dynamo. A more general theorem had been proven by Cowling (1934) who has shown that an axisymmetric magnetic field cannot be sustained by a dynamo regardless of how general the motion of the fluid is. This seems to eliminate the possibility of having a self-excited magnetic field with a physically realistic geometry in an APD. Closer examination, however, shows that such a possibility exists. Fluid motions in viscous APDs are not fully axisymmetric. Because turbulence is present, the velocity field includes a random component. Random velocity generates a magnetic field with a random component. Instead of looking for a full magnetic field we can concentrate on solving the dynamo problem for the *mean* magnetic field. This is the turbulent dynamo approach. The important point to make is that in such an approach the mean magnetic field can be axisymmetric and the mean velocity field can be toroidal and the turbulent dynamo can be successful. This does not violate Cowling's theorem because *full* fields are neither axisymmetric nor toroidal.

3.1. Turbulent Dynamo and the Balbus-Hawley Instability

At this point it is fair to remark that one of the main shortcomings of viscous disk theory is the uncertainty as to the nature of turbulence. The fact that we have to rely on turbulence of unknown origin to sustain the disk dynamo may be viewed as a disappointment; one would prefer to have a physical process that would maintain the magnetic field independently from the postulated turbulence. Balbus & Hawley (1991) have shown that Keplerian flow in the presence of some seed magnetic field develops shear instability (later to be known in the literature as Balbus-Hawley instability) that leads to turbulence. This result was welcome news for the astrophysical community because **a)** it pinpointed the source of turbulence in accretion disks for the first time, making it possible to calculate the rate of angular momentum transport from first principles; and **b)** it suggested the possibility that differential rotation alone, without any preexisting turbulence, may amplify magnetic field. Subsequently, a dynamo scheme invoking this instability had been proposed by Tout & Pringle (1992).

Let's examine whether the BH instability can occur in APDs. Recall that only in a perfectly conducting medium the BH instability works for arbitrarily small values of magnetic field. If resistivity is finite there is a *minimum* required field strength for an instability to work. The condition for the minimum magnetic field strength is given by Balbus & Hawley (1991) as

$$\frac{3\Omega\eta_r}{V_a^2} << 1 \tag{1}$$

where Ω is Keplerian angular velocity and V_a is the Alfven velocity. In terms of β, the ratio of gas pressure to magnetic pressure, condition (1) becomes $\beta << \beta_{\rm BH}$ where the quantity $\beta_{\rm BH}$ is given by

$$\beta_{\rm BH} = \frac{3\times 10^{16}}{\eta_r}\ \frac{T}{100^\circ{\rm K}}\ \left(\frac{r}{1{\rm AU}}\right)^{3/2} \tag{2}$$

The values of $\beta_{\rm BH}$ for our model of an APD are given in Table 1. A magnetic field of one and maybe two orders of magnitude smaller than the value of the equipartition with the gas pressure is necessary in a disk as a *prerequisite* for the BH instability to occur in an APD. A magnetic field of such magnitude is not trivial. (In contrast, the value of $\beta_{\rm BH}$ in accretion disks around compact objects is typically of the order of 10^{10} and clearly the minimum required field strength is trivially small.) For example, $\beta_{\rm BH}(r = 3{\rm AU}) = 27$ corresponds to a magnetic field of about 0.1 gauss. Thus, a field at least as strong as the one inferred from primitive meteorites is required to preexist in order to jump-start the BH instability at 3 AU from the star. Notwithstanding that a relatively smaller field is required at larger distances (see Table 1), the BH instability may only work if some other mechanism will preamplify the magnetic field to the required intensity. Turbulent dynamo may be just such a mechanism. Thus, we believe that turbulent dynamo is the primary mechanism for producing IGF in APDs.

3.2. Criteria for Magnetic Field Generation in Protostellar Disks

The behavior of a magnetic field in a disk is determined by an induction equation. We assume that the only component of large-scale velocity is V_ϕ and it depends only on the radial coordinate. Because a magnetic field is axisymmetric we can express it in the form $\mathbf{B} = B_\phi \mathbf{e}_\phi + \nabla \times A_\phi \mathbf{e}_\phi$. The term $\nabla \times A_\phi \mathbf{e}_\phi$ represents a poloidal field with components $B_r = -\partial A_\phi/\partial z$ and $B_z = (1/r)\partial/\partial r(rA_\phi)$. Here $\mathbf{B}$ and $\mathbf{V}$ are large-scale (mean) magnetic and velocity fields. We make use of the fact that the disk is thin; the ratio $H_0/r_0 << 1$, where r_0 is the characteristic radius of the disk and H_0 is the characteristic thickness of the disk. We further assume that a disk dynamo is the so-called $\alpha\Omega$ process. Under these assumptions an induction equation is reduced to the set of two scalar equations

$$\frac{\partial B_\phi}{\partial t} = -r\frac{\partial \Omega}{\partial r}\frac{\partial A_\phi}{\partial z} + \eta\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2}\right) B_\phi \tag{3}$$

$$\frac{\partial A_\phi}{\partial t} = \alpha_h B_\phi + \eta\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2}\right) A_\phi \tag{4}$$

Recall that $\eta = \eta_r + \eta_t$. A function α_h, known in dynamo theory as the so-called α-effect, is responsible for producing a poloidal field out of a toroidal field by means of helical turbulence. In a turbulent disk $\alpha_h \approx \alpha z \Omega$ and $\eta_t \approx \alpha H^2 \Omega$ (Stepinski & Levy 1991). The value of η_r must be calculated from the model of a disk (Stepinski et al. 1993). The criteria for the existence of dynamo IGFs can be obtained from so-called *local* approximation. Because $r_0 >> H_0$, radial derivatives are much smaller than vertical derivatives; the magnetic field diffuses more rapidly to the vertical boundaries of the disk than along the radius. In the local approximation, radial derivatives are neglected. Reduced equations (3) and (4) depend only on t and z, with r appearing only *parametrically*. The solutions describe the vertical distribution of the magnetic field for fixed values of the radial coordinate. Because the reduced system of equations (3)–(4) is linear, normal mode solutions can be obtained.

It can be shown (Stepinski & Levy 1991) that the radial dependence of the coefficients in the reduced system (3)–(4) can be confined to only one radially varying coefficient called an *effective dynamo number* $D_{\rm eff}$

$$D_{\rm eff} = \frac{r\frac{d\Omega}{dr}\alpha_h(r)H^3(r)}{\eta^2(r)} \tag{5}$$

The reduced system (3)–(4) can be solved numerically subject to *vacuum* boundary conditions. The solutions depend on the value of $D_{\rm eff}$; growing modes are only possible if $|D_{\rm eff}| > |D_{\rm crit}|$. Stepinski & Levy (1991) calculated that in a thin disk $|D_{\rm crit}| \approx 10$ and the first excited mode is nonoscillatory and has a quadrupole symmetry. The critical number for a mode with a dipole symmetry has been found to be much larger. Therefore, as expected, IGF due to turbulent dynamo has a quadrupole symmetry. In the first approximation the relationship $|D_{\rm eff}| > 10$ defines radial regions of an APD where a magnetic field can be generated. Fig. 2 shows the variation of $|D_{\rm eff}|$ as a function of the distance from a star for a steady-state model of a viscous disk

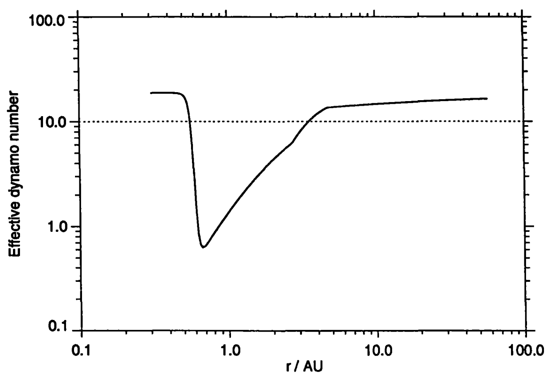

Fig. 2.— Effective dynamo number as a function of radial distance in a disk. The solid line denotes $|D_{\rm eff}|$ as a function of r and the dotted line denotes $|D_{\rm crit}|$. Magnetic field is generated in regions where $|D_{\rm eff}| > |D_{\rm crit}|$.

with $\dot{M} = 10^{-7}M_{\odot}/\text{yr}$ and $\alpha = 0.08$. We found that, indeed, the dynamo works in most regions of a disk with the exception of a radially intermediate portion where the ionization is the lowest. We call this region the "magnetic gap." The location and the width of this gap depend on the particular model, but the case shown in Fig. 2 is typical.

3.3. Radial Distribution of Dynamo-Generated Magnetic Field

From the local analysis one can extract a rather simple criterion for the existence of dynamo-generated magnetic fields in an APD; however, there is no possibility to calculate either the strength of the equilibrium field or the time needed to achieve such an equilibrium. In addition, one has to check whether the inclusion of the radial diffusion (omitted in the local analysis) would not lead to the disappearance of the "magnetic gap" feature. Once radial diffusion is included we can seek the solution to the dynamo equations in the form $\mathbf{B}(t,r,z) = Q(t,r)\mathbf{b}(z;r)$, where $Q(t,r)$ describes the field distribution along the radius. The field distribution across the disk is represented by function $\mathbf{b}$ given by the solution to the local approximation to the set of equations (3)–(4) and normalized such that $\max(|\mathbf{b}(z)|)$ is 1. The time evolution of Q is governed by the following equation

$$\frac{\partial Q}{\partial t} = \eta \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} \right) Q + \gamma(r,Q)Q \tag{6}$$

The function $\gamma(r,Q)$ is local growth rate taken from the local approximation but modified to include the nonlinear stage of the magnetic field evolution. According to kinematic dynamo theory, in the regions of a disk where $|D_{\text{eff}}| > |D_{\text{crit}}|$, a dynamo will amplify the magnetic field without limit. In reality, the Lorentz forces must eventually become great enough so as to modify amplification sources, thus reducing dynamo action and halting the growth of the magnetic field. We describe this effect using the concept of α_h- quenching by prescribing α_h as a function of the field strength $\mathbf{B}$, or in our case, Q. We have chosen $\alpha_h(r,Q) = 1/(1+Q^2/B_{\text{eq}}^2)$, where B_{eq} is the value of magnetic field at which the Alfven velocity is equal to the turbulent velocity, and is called the equipartition value. Note that B_{eq} denotes the value of a magnetic field that is in equipartition with the kinetic energy of *turbulence* and not with thermal pressure. We expect that the equilibrium strength of a dynamo-generated magnetic field scales with B_{eq}. This is based on both theoretical considerations and observations of magnetic fields in galactic disks – the only objects for which a dynamo theory can be observationally tested.

Fig. 3 shows the time evolution of the magnetic field in a protostellar disk described by a steady-state, viscous model with $\dot{M} = 10^{-7}M_{\odot}/\text{yr}$ and $\alpha = 0.08$. At first, the field increases sharply at the inner radii, decays at the middle radii (within a magnetic gap), and remains unchanged at outer radii. By the time $t = 20$ yr, the magnetic field in the innermost portion of the disk achieves equilibrium. As time progresses the magnetic field achieves equilibrium at larger and larger portions of the inner disk. At the same time, the field continues to decay at the region of the magnetic gap and the magnetic field in the outer parts starts to show some growth. By the time $t \approx 200$ yr the whole inner region has reached equilibrium. Radial diffusion from the regions of strong magnetic field stops the further decay of the field within the magnetic gap. The magnetic field in the outer parts of a disk continues to grow. By the time $t \approx 2000$ yr, the magnetic field in almost the entire disk has reached equilibrium. Thus the equilibrium is achieved on a time scale that is very short in comparison with a typical disk lifetime. It is also important to note that magnetic fields in different parts of the disk achieve equilibrium on very different time scales, tenths of years in the innermost parts, hundreds of years at a few AU, and thousands of years at the outermost parts.

The equilibrium configuration of the magnetic field follows closely the distribution of B_{eq}. This may be an artifact of the chosen form of α_h-quenching, but the important result is that the magnetic field equilibrates at about the local value as given by the α_h-quenching function at the regions where the linear theory predicts growth of a magnetic field. In the region where the local theory predicts that the field would decay, the equilibrium is achieved by the balance of this decay and the transport of magnetic field from the neighborhood regions by means of radial diffusion. The magnitude of the equilibrium magnetic field in this region is a few orders of magnitude smaller than the α_h-quenching function would predict. We conclude that 1) the time needed to amplify a seed magnetic field to a significant value varies drastically with radial location, but is everywhere very short relative to the disk lifetime, and 2) the equilibrium value of the magnetic field is given by the α_h-quenching function and is likely to be of the order of B_{eq}, except in the "magnetic gap," where it is insignificantly small.

We assume that in an APD the Rossby number is about 1/2 or $l_0\Omega \approx v_0$, where v_0 is the turbulent speed and l_0 is the turbulent mixing length. Under such an assumption the equipartition magnetic field is given by

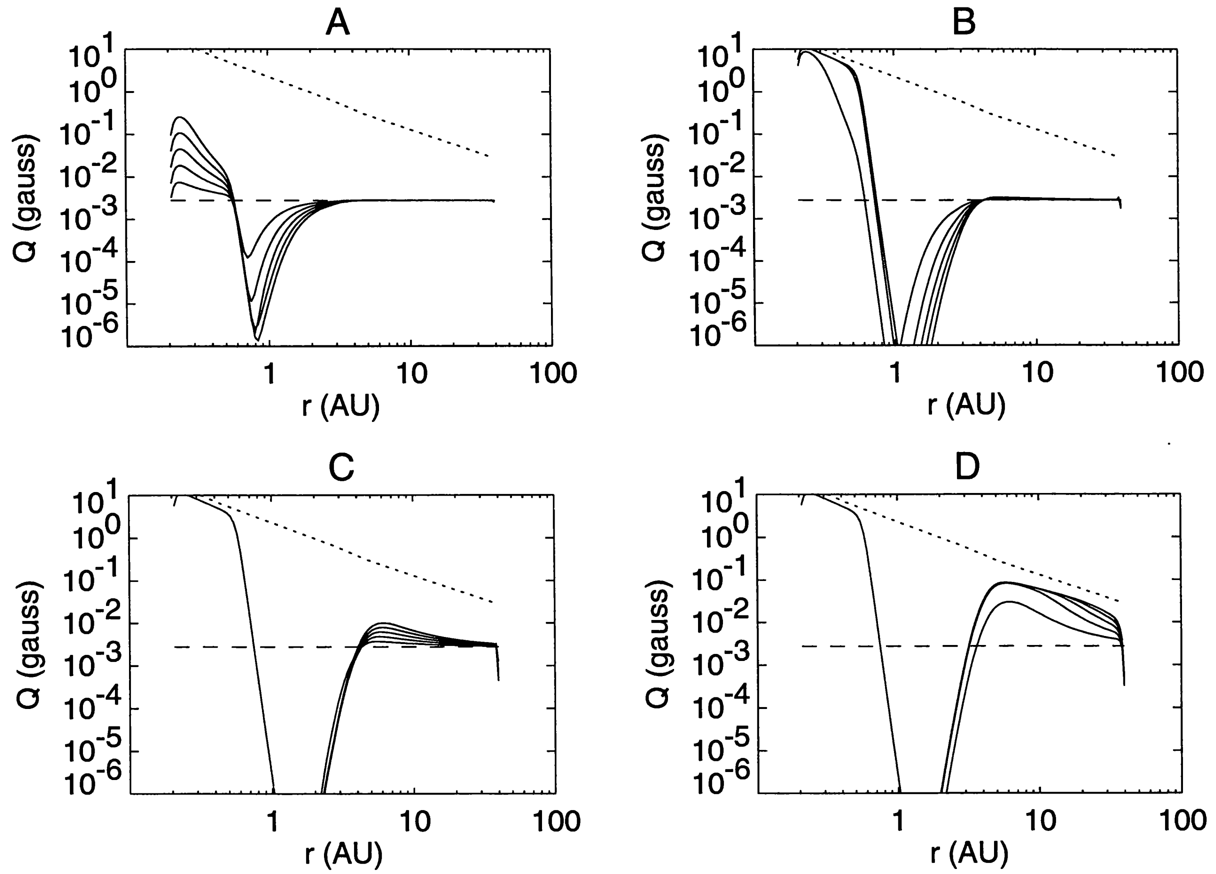

Fig. 3.— Time evolution of function $Q(r)$. The dashed line indicates the initial condition $Q_0 = 0.1B_{\rm eq}(r = 40{\rm AU})$. The dotted line indicates the equipartition strength of magnetic field $B_{\rm eq}$. Solid lines on panel A show the time evolution of the magnetic field in steps of 0.4 yr up to 2 yr. Panel B shows the evolution of magnetic field up to 20 yr in steps of 4 yrs, panel C up to 200 yr in steps of 40 yr, and panel D up to 2000 yr in steps of 400 yr.

$$B_{\rm eq} = \left(\frac{4\pi k}{\mu m_H}\right)^{1/2} \alpha^{1/2}\, \rho^{1/2}\, T^{1/2} \tag{7}$$

The values of $B_{\rm eq}$ for our model of an APD are given in Table 1. It is easy to see that $\beta_{\rm eq}$, the ratio of gas pressure to magnetic pressure exerted by the field having strength $B_{\rm eq}$, is equal to $\sqrt{(2/\alpha)}$ and is independent of the radial location. For our model $\beta_{\rm eq} = 25$. In most parts of the disk $\beta_{\rm eq} > \beta_{\rm BH}$, meaning that turbulent dynamo is able to amplify a magnetic field to the level higher than required for the BH instability to operate. It is thus possible that the final amplitude of the magnetic field is influenced by the BH instability. In the magnetic gap, however, the field is always too small to start the BH instability.

4. CONCLUSIONS: ARE DYNAMO-GENERATED MAGNETIC FIELDS IN VISCOUS PROTOSTELLAR DISKS DYNAMICALLY IMPORTANT?

From the point of view of star formation theory, the existence of dynamo-generated magnetic field in viscous APDs is important only if such a field is able to influence the properties of those disks. The evolution of an APD is primarily governed by the conservation of the angular momentum equation

$$\frac{\partial V_\phi}{\partial t}(2\pi r\Sigma) = \frac{\partial}{\partial r}\left[\dot{M} r V_\phi + (2\pi r^2)(2H)\left(\frac{\langle B_\phi B_r\rangle}{4\pi} + \langle t^{vis}_{\phi r}\rangle\right)\right] + \frac{1}{2} r^2 B_\phi B_z|^{+H}_{-H} \tag{8}$$

The terms in brackets represent the flux of angular momentum due to bulk motion, to the stress of vertically averaged magnetic field, and to viscous stress $\langle t_{\phi r}^{vis} \rangle$. The last term on the right-hand side accounts for the angular momentum leakage from the disk. This is the term responsible for the evolution of a disk in models invoking centrifugally driven winds. Note, however, that a dynamo-generated field subject to vacuum boundary conditions has $B_\phi(H) = 0$; thus the only contribution of a magnetic field to disk evolution comes from the term $t_{\phi r}^{mag} \propto \langle B_\phi B_r \rangle$. Reyes-Ruiz & Stepinski (1995) have shown that for a dynamo-generated field $t_{\phi r}^{vis}/t_{\phi r}^{mag} \approx \alpha^{1/2}\beta/6$. For our model of an APD this ratio is of the order of unity, except in the region of the magnetic gap where it is very large. Thus, an APD with a dynamo-generated magnetic field behaves like a nonmagnetized disk with a radially *variable* α coefficient. Calculations by Reyes-Ruiz & Stepinski (1995) revealed that a magnetized disk develops the surface density bulge located in the magnetic gap. The bulge persists for a time comparable to the lifetime of a disk. The presence and the persistence of this bulge may be more important for the process of planet formation within an APD than to the actual process of central star formation.

Finally, a dynamo-generated field, as presented in this paper, is unlikely to allow centrifugally driven winds to be launched from a disk. However, a dynamo-generated field in the presence of the externally imposed field has to be calculated in order to make a more complete assessment of the situation. In addition, the presence of a magnetic field in a disk may result in a magnetized corona around it, which is heated by a process similar to solar coronal heating (Heyvaerts & Priest 1989). Such a corona may be thermally (as oppose to centrifugally) driven into a wind. These issues require further investigation.

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