

# WINDS AND FUNNEL FLOWS FROM YOUNG STARS AND DISKS

Joan Najita <sup>1</sup>

Astronomy Department, University of California  
Berkeley, CA 94720, USA

## RESUMEN

Describimos cómo la interacción entre estrellas magnetizadas y sus discos puede generar vientos energéticos impulsados magnetocentrífugamente. Estos vientos pueden formarse ya sea en estrellas magnetizadas con rotación de ruptura impulsada por acrecimiento a través de un disco, o bien por estrellas magnetizadas rotando más lentamente que se truncan y acoplan con discos keplerianos circunestelares conductores. En este último caso, la pérdida de masa por un viento acelerado de tipo X acompaña el acrecimiento a lo largo de las líneas del campo magnético. La transferencia de masa y momento angular en el viento y los flujos de acrecimiento puede permitir a la estrella permanecer en corrotación con la región interior del disco, a pesar del continuo acrecimiento de la masa estelar. Discutimos también posibles diagnósticos observacionales de la región de interacción estrella-disco, incluyendo la emisión vibracional armónica del CO y las líneas Balmer e infrarrojas del hidrógeno.

## ABSTRACT

We describe how the interaction between magnetized stars and their surrounding disks can generate energetic magnetocentrifugally driven winds which appear to explain many of the observed properties of winds from young stars. These winds can arise both from magnetized stars spun up to breakup by accretion through a surrounding disk as well as from more slowly rotating magnetized stars which truncate and couple to circumstellar conducting Keplerian disks. In the latter case, mass loss in an X-celerator wind naturally accompanies accretion along magnetic field lines. The transfer of mass and angular momentum in the wind and accretion flows may allow the star to remain in corotation with the inner disk region despite ongoing stellar mass accretion. We discuss possible observational probes of the star-disk interaction region; these include CO vibrational overtone emission and the Balmer and infrared lines of hydrogen.

*Key words:* ACCRETION, ACCRETION DISKS — STARS:  
FORMATION — STARS: MAGNETIC FIELDS — STARS:  
ROTATION — MHD

## 1. INTRODUCTION

Observational evidence assembled over the past few decades has led to the realization that circumstellar disks are commonly associated with low-luminosity young stellar objects (YSOs), as is the presence of energetic inflows and outflows of gas within several stellar radii of these stars. This discovery leads us to wonder how these disks and gas flows fit together—what roles do they play?—in the process of making stars. The answer to this question is very relevant to one of the central goals of star formation: to describe how stars acquire their main sequence properties, *i.e.* an understanding of the origin of the IMF and the angular momentum

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<sup>1</sup>Presently at the Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

distribution of stars on the ZAMS. For this purpose, an understanding of the origin and interrelation of these observed phenomena is naturally of great interest.

In this contribution, I describe a generalized model that binds together these observed elements (circumstellar disks and energetic gas flows) into a coherent dynamical picture of the processes of accretion and loss of mass and angular momentum in both revealed and embedded YSOs. Although originally developed in the context of stars already rotating at breakup, the present model is more general in that it describes how these processes may also occur in the case of stars rotating *below* breakup. I also describe how the generalization of our original picture to this latter case also encompasses an understanding of the dynamical basis of accretion along magnetic field lines (funnel flows) and the angular momentum regulation of young stars. Finally, I discuss observational diagnostics that can be used to test the generalized model and the implications of several recent observational results.

## 2. WHY A GENERALIZED MODEL?

Our interest in constructing a generalized model is motivated by the observed properties of winds from both embedded and revealed sources (*c.f.* Edwards et al. 1993a). In both types of sources, measures of accretion activity (e.g., near-infrared continuum excesses) are found to correlate with measures of outflow activity, suggesting that the processes of accretion and outflow are intimately related. The characteristic velocities of winds from both types of sources (hundreds of km/s) locate the origin of the wind deep in the potential well, close to the star. When compared with the system luminosity, the wind velocities and mass loss rates ( $> 10^{-6} M_{\odot}/\text{yr}$  for embedded sources and  $\sim 10^{-8} M_{\odot}/\text{yr}$  for revealed sources) indicate an efficient wind acceleration mechanism with little radiative loss. Most importantly, the physical properties of winds from both embedded and revealed sources form a continuum, indicating that the same wind mechanism operates in both circumstances.

## 3. WINDS FROM RAPIDLY ROTATING STARS

We initially focused on the physical origin of winds from embedded sources (Shu et al. 1988). Understanding the origin of winds from these sources is extremely challenging because the high mass loss rates of these winds severely restrict possible physical explanations for their existence! We imagined such stars to be accreting directly from infalling envelopes or surrounding disks; these stars would quickly be spun up to breakup speed by the accretion of the accompanying angular momentum and develop strong stellar magnetic fields as a consequence of their rapid rotation.

Addressing both the observed correlation between outflow and accretion activity and the indication that the wind originates close to the star, we were able to explain the properties of winds from embedded sources as a consequence of accretion-driven mass loss in which the wind is directly from the stellar surface. The ability to drive such a wind is very important to the mass evolution of the star: since the star is already rotating at breakup, as it is fed more matter from the surrounding disk, it must redistribute the incoming angular momentum in order to continue to accrete and grow in mass. It accomplishes this by driving a powerful wind which carries away a larger amount of specific angular momentum than is brought in by the disk.

As discussed by Shu et al. (1988), cold gas outflows can be driven at high mass loss rates from magnetized stars rotating at breakup along open stellar equatorial magnetic field lines that emerge in the downhill direction of the effective (gravitational+centrifugal) potential in the frame rotating with the star (Fig. 1a). The equatorial region of a star rotating at breakup is a convenient point from which to launch the flow because matter in this region is in centrifugal balance with gravity. Consequently, mass loss can be driven quite easily from this region given only relatively weak photospheric pressure gradients. (The effective potential at the equator is of X-type so we refer to the equator as the “X-point” and the flow that emerges from this region as an “X-wind”.) As the star rotates, the open field lines transfer angular momentum to the wind, accelerating it away from the star, and converting stellar rotational energy into wind kinetic energy. The inertia of the outflowing matter drags the stellar field lines into a trailing spiral pattern, generating toroidal magnetic field from initially poloidal field.

The efficiency of this wind mechanism is confirmed by the results of detailed self-consistent, two-dimensional, axisymmetric MHD calculations (Najita 1992, Najita & Shu 1994) which show that X-winds are characterized by strong toroidal velocities near the star and rapid acceleration to escape speeds on the scale of a stellar radius! The strength of the toroidal magnetic field becomes comparable to that of the poloidal field over the same distance. Scaling the results of these nondimensional calculations to the parameters characteristic of low-luminosity embedded sources demonstrates that measured pre-main-sequence stellar field strengths of  $\sim 1\text{ kG}$

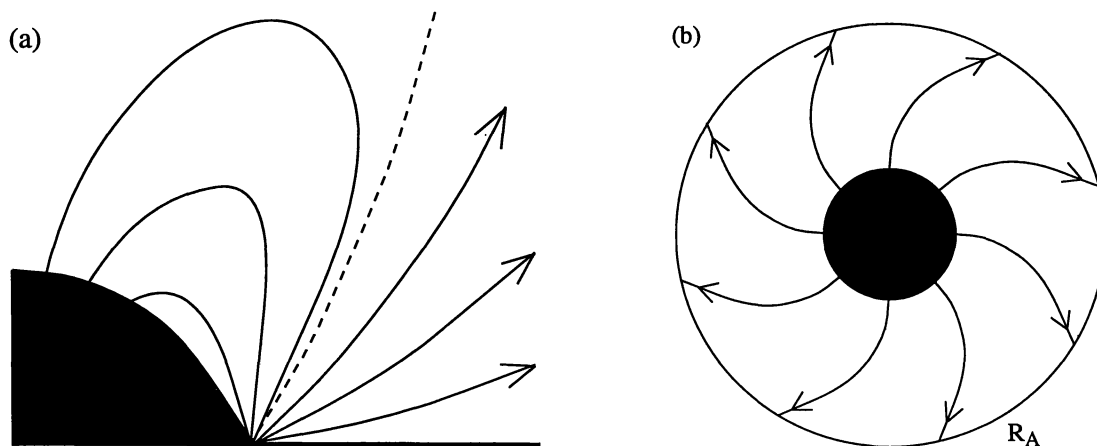


Fig. 1.— Schematic views of the (a) meridional plane and (b) equatorial plane of a magnetized star spun up to breakup by rapid disk accretion. Energetic cold gas outflows are driven along equatorial stellar field lines that emerge into the downhill region of the effective potential, the region bounded by the disk and the critical equipotential (*dashed line*). The inertia of the outflowing matter drags the stellar field into a trailing spiral pattern; the toroidal component of the resulting magnetic field is comparable in strength to the poloidal component at the Alfvén surface  $R_A$ .

(*c.f.* Basri et al. 1992) can drive X-winds with terminal velocities and mass loss rates comparable to those that characterize winds from embedded sources. Since X-winds with these properties carry off  $\sim 3$  times as much specific angular momentum as is accreted through the disk, the steady-state wind mass loss rate is a definite fraction of the disk mass accretion rate,  $\dot{M}_w \simeq 1/4\dot{M}_D$ .

#### 4. THE PUZZLE OF SLOWLY ROTATING T TAURI STARS

While this picture provides a promising description of mass loss from embedded sources, this picture clearly cannot apply without modification to revealed T Tauri stars (TTs) which are known to rotate much below breakup (e.g. Vogel & Kuhi 1981). The naive expectation is that TTs *would* be rotating at breakup since the accretion of matter through a disk would rapidly spin it up to breakup speed. Thus, despite the remarkable success of disk accretion in explaining many properties of optically visible TTs (Adams et al. 1987; Kenyon & Hartmann 1987; Bertout et al. 1988), the observed slow stellar rotation of TTs has been a difficulty in our understanding of both the stellar accretion and mass loss processes of these stars.

Two recent observational results provide clues to the resolution of this difficulty. On the one hand, the slow rotation of TTs has recently been shown to correlate with the presence of inner circumstellar disks (Bouvier et al. 1993; Edwards et al. 1993b). The speculation here is that the observed rotation statistics are the result of magnetic coupling between magnetized young stars and their inner disks. On the other hand, it now appears that TTs do not accrete matter solely through a disk but also by accretion along stellar magnetic field lines (Calvet & Hartmann 1992; Edwards et al. 1994; Hartmann, Hewett, & Calvet 1994). In this way, the star-disk magnetic interaction speculated upon on the basis of TTs rotation statistics also appears to be connected to the final stages of accretion onto young stars! So in addition to the original question of how stars rotating below breakup drive winds, these recent observational results raise an additional fundamental question: if TTs accrete along stellar magnetic field lines, how do they manage to do so without spinning up?

In response to these questions, it turns out that the X-wind model originally proposed for stars rotating at breakup can be applied with only a small modification to the case of rotation below breakup. In fact, the wind calculations originally performed for the case of rotation at breakup can be directly applied to this case! In addition, in making the generalization to the case of rotation below breakup, we also find a physical explanation for the slow stellar rotation of TTs and the possibility of magnetic accretion flows which emerge as integral parts of the whole picture.

## 5. WINDS, FUNNEL FLOWS, AND THE ANGULAR MOMENTUM REGULATION OF T TAURI STARS

The modification required to generalize the original X-wind model to the case of rotation below breakup is apparent from a comparison of Figures 1a and Figure 2a. As discussed by Shu et al. (1994), the magnetic configuration shown in the latter Figure can be obtained by imagining the interaction between the dipole field of a magnetized star and a conducting, accreting, Keplerian disk. For typical TTS parameters, kilogauss strength stellar fields can truncate the circumstellar disk, creating a magnetosphere (*c.f.* Bertout et al. 1988, Königl 1991). A steady-state configuration is possible if the star and its magnetic field corotate with the disk at the truncation radius. Note that because the star rotates below breakup in this case, there is an additional downhill region of the effective potential: the region formerly filled by the rapidly rotating star. For the moment, assume that rest of the interaction between the stellar field and the truncated disk results in magnetic flux trapped at the truncation radius; the magnetic pressure of the trapped flux will then cause the trapped field lines to fan out in the manner shown.

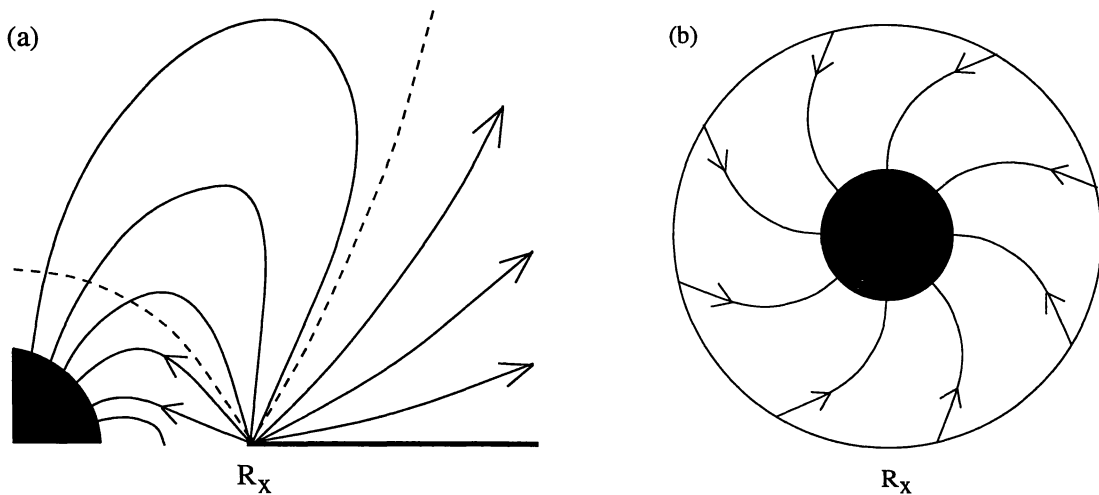


Fig. 2.— Schematic views of the (a) meridional plane and (b) equatorial plane of a more slowly rotating magnetized star that truncates and couples to its circumstellar disk. Both energetic outflows and funnel flows emerge from the disk truncation region. Gas accreting from the disk onto the star in a funnel flow drags the stellar field into a trailing spiral pattern.

Field lines that emerge away from the star direct matter down equipotentials, launching an X-wind in a way that is completely analogous to the case of rotation at breakup. Rotating field lines again transfer angular momentum to the wind, accelerating it outward. As matter flows downhill out of the X-point, the inertia of the wind drags the field into a trailing spiral pattern. These field lines tug backward on their footpoints in the disk, removing angular momentum from the disk which is carried away by the wind.

With sufficient trapped flux at the X-point, some fraction of the field lines connecting the star and disk point in the *other* downhill direction toward the star (Fig. 2a). Thus, disk material can accrete along these field lines onto the star if it can lose angular momentum. How is this accomplished? Because gas leaving the disk typically has too much angular momentum to accrete radially, it spins up as it falls inward, again dragging the field into a trailing spiral pattern (Fig. 2b). Looking at the torques in this part of the problem, we find that the magnetic field tugs backward on the footpoint on the star, spinning it down; and forward on the footpoint in the disk, spinning it up. In this way, magnetic torques can (1) remove angular momentum from the gas, allowing it to accrete, and (2) transfer angular momentum to the disk where it can be removed in the wind. The accretion of the resulting low angular momentum material allows the star to accrete without spinning up!

The transfer of angular momentum that drives these flows also acts to maintain the flux trapped at the disk truncation radius. On the far side of the disk truncation radius, the removal of angular momentum from disk by the X-wind and disk accretion torques drags fields toward the X-point. On the near side of the disk truncation radius, the transfer of angular momentum to the disk by the funnel flow again drags fields toward the X-point, maintaining disk truncation and the trapped flux. Scaling the results of the original X-wind calculations in the



context of this modified picture to the parameters characteristic of revealed sources, we find good agreement with observed quantities such as wind velocities and mass loss rates and stellar rotation periods (Najita & Shu 1994).

## 6. FUNNEL FLOW CALCULATIONS

Calculations of the magnetic structure of the region external to the wind show the way in which the stellar magnetic field geometry is modified by the presence of the X-wind and trapped flux (Fig. 3; Ostriker & Shu 1995). Since the X-wind field lines are *originally closed* stellar field lines that have been opened by the X-wind, these must connect back to the star at infinity. Thus, there are additional open (possibly lightly-loaded) field lines rooted in the star that are equal in number to those in the X-wind.

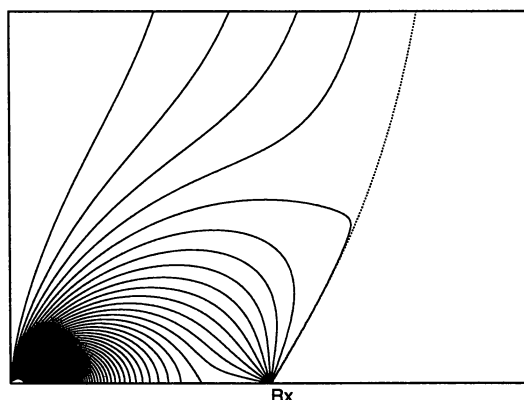


Fig. 3.— The stellar magnetic field geometry as modified by the presence of an X-wind (not shown) and the magnetic flux trapped at the disk truncation radius  $R_x$ .

Although both the field lines in the X-wind and these additional open field lines rooted in the star have footpoints that corotate with the star, there is a fundamental difference between these two sets of open field lines because the former is heavily-loaded whereas the latter is not. While the heavily-loaded X-wind fields rapidly develop a large toroidal magnetic field component, the lightly-loaded fields are able to rapidly untwist so that they remain primarily longitudinal. The X-wind and funnel flow calculations show that the situation of longitudinal field wrapped by toroidal X-wind field is rapidly approached: there is a rapid transition from poloidal to toroidal field in the X-wind solutions, and a rapid decrease of the trapped flux component in the funnel flow solutions.

The probable configuration of longitudinal field wrapped by toroidal field has important implications for both the stability and collimation of the flow. X-winds with large toroidal fields that would otherwise be subject to sausage and kink instabilities (Eichler 1993) may be stabilized by the longitudinal field. The collimation of the X-wind will occur when the X-wind comes into pressure equilibrium with the longitudinal field. This is likely to be achieved at a *finite* cylindrical radius because of the difference between the asymptotic behavior of longitudinal ( $\varpi^{-2}$ ) and toroidal fields ( $\varpi^{-1}$ ).

## 7. OBSERVATIONAL DIAGNOSTICS

In the generalized model, the region in which winds and funnel flows are launched and accelerated extends over several tens of solar radii. Although regions of this size cannot be imaged directly with present observational techniques, they can be probed indirectly using high-resolution spectroscopy if suitable spectral line diagnostics can be identified. It is fortunate that we have recently been able to identify kinematic diagnostics suitable for the study of this region in both revealed and embedded objects.

### 7.1. Kinematic Diagnostics of Inner Disks: CO Overtone Emission

The stability of the CO molecule and the excitation temperatures and critical densities of the  $2.3\mu\text{m}$  CO vibrational overtone bands make these transitions particularly well-suited to the study of the warm ( $10^3 - 10^4\text{K}$ ),

dense ( $10^{10} - 10^{15} \text{ cm}^{-3}$ ) conditions that characterize the inner regions of circumstellar disks. The utility of these transitions as a probe of the star-disk interaction region is supported by recent high resolution spectroscopy of the CO bandhead emission from a handful of embedded and revealed YSOs which strongly indicates a disk origin for the emission (Carr et al. 1993, Najita et al. 1995).

In comparison with near-infrared continuum excesses which have been used to argue for the presence of inner disks but which leave the location and kinematics of the emitting material uncertain, the CO overtone emission often displays the characteristic spectral shape of gas in Keplerian motion, almost certainly identifying it with a circumstellar disk (Fig. 4). Indeed, these observations provide some of the most compelling evidence to date for the existence of inner disks around young stars!

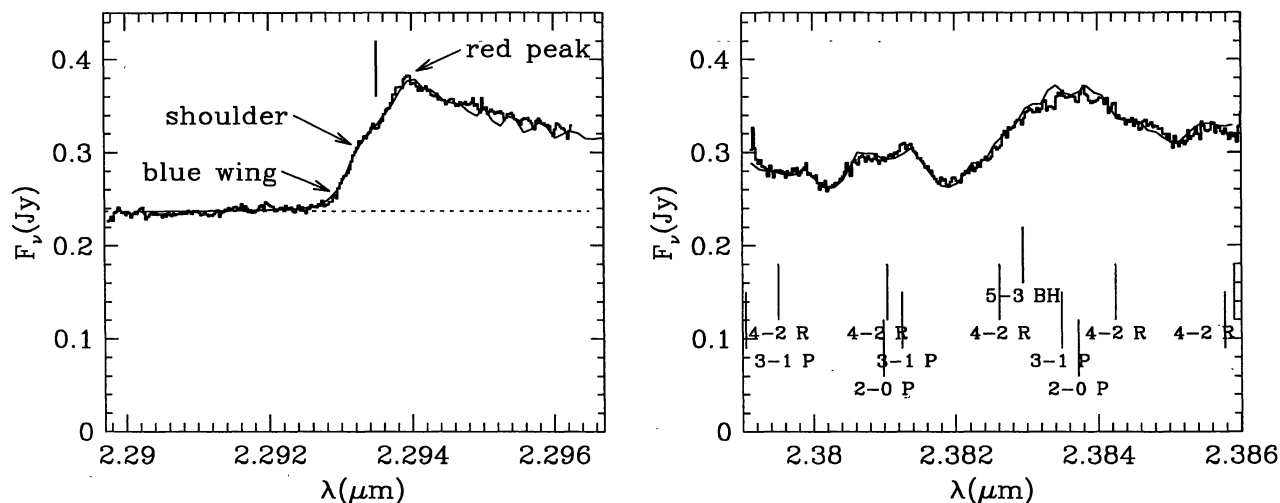


Fig. 4.— The CO( $v=2-0$ ) bandhead (*left*) and CO( $v=5-3$ ) bandhead (*right*) regions of the spectrum of 1548C27 (*histogram*). The 2-0 bandhead shows the characteristic shape of emission from a rotating disk. The overtone lines that fall in the 5-3 bandhead region probe a range of temperatures. Non-LTE modeling of the emission in both bandhead regions as arising from a Keplerian disk (*solid line*) constrains the stellar and disk properties.

We have shown that detailed analysis and modeling of these data can place stringent constraints on stellar properties such as masses and radii as well as on disk properties such as inner disk radii and radial column density, temperature, and intrinsic linewidth distributions (Najita et al. 1995). As a test of the generalized model, we can compare the inner disk radii determined from these data with measured stellar rotation periods to determine whether stars truncate and corotate with their inner disks. Other derived properties such as the radial disk temperature and linewidth distributions can be used to study the heating and turbulence of the inner disk region generated by the interaction of the disk with the X-wind.

Equally exciting is the prospect of CO overtone emission as a probe of X-winds with large mass loss rates. Calculations of the thermal and chemical structure of winds from low-luminosity YSOs show that not only is CO a robust chemical product in these winds, but the inner regions of these winds are also likely to be warm ( $\sim 2000 \text{ K}$ ) and dense ( $> 10^{10} \text{ cm}^{-3}$ ), conditions which are ideally suited to the use of CO overtone emission as a spectral line diagnostic (Ruden, Glassgold, & Shu 1990).

## 7.2. Br $\gamma$ As a Probe Of Infalling Gas

As in the case of the CO overtone bands, the physical origin of the near-infrared hydrogen emission lines in low-luminosity YSOs is of great interest since they are among the few prominent spectral features available for the study of some of the youngest (*i.e.* deeply embedded) stars. As in the case of the hydrogen Balmer lines in the spectra of TTSs, these lines have generally been assumed to originate in a wind.

Recent high resolution spectroscopy of Br  $\gamma$  emission in a small sample of low-luminosity embedded YSOs and active TTSs reveals line profiles with broad wings but which conspicuously *lack* the blue-shifted absorption

components generally expected for emission from a wind (Najita, Carr, & Tokunaga 1995). Even TTS that show strong wind absorption features in the Balmer and NaD lines show no evidence of a wind in the Br  $\gamma$  line (Fig. 5). Instead, we find that the Br  $\gamma$  emission centroids tend to be blue-shifted with respect to the stellar velocity, as is observed in the higher Balmer lines of TTS.

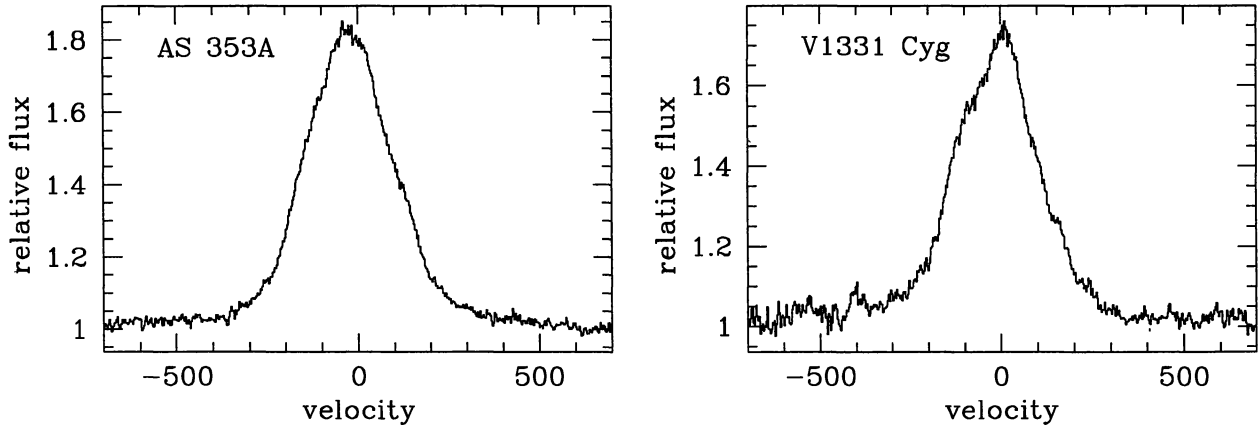


Fig. 5.— Br  $\gamma$  line profiles of two very active T Tauri stars. Although these stars show strong wind absorption features in the Balmer and NaD lines, there is no evidence of a wind in the Br  $\gamma$  line.

These results imply that the Br  $\gamma$  emission in these sources does not arise in a wind, presumably because the actual wind temperature is lower than previous estimates based on spectrally unresolved Br  $\gamma$  observations (*e.g.* Natta & Giovanardi 1990). This suggests that the acceleration regions of winds from active low-luminosity YSOs are better probed by spectral lines that sample cooler gas such as the CO overtone transitions. Instead, the blue-shifted Br  $\gamma$  emission centroids indicate that the bulk of the Br  $\gamma$  emission probably arises from infalling gas, possibly in a magnetosphere (*c.f.* Hartmann, Hewett, & Calvet 1994). The Br  $\gamma$  line appears to be an ideal probe of this infalling gas since it apparently has very little contamination from a wind. Using Br  $\gamma$  as a spectral line diagnostic, studies of magnetospheres and funnel flows carried out for revealed stars (Edwards et al. 1994) can now be extended to the youngest stars!

### 7.3. Time Variability Studies

Finally, time variability studies offer another intriguing way to test the generalized model. The presence of non-axisymmetric structures may give rise to periodic phenomena such as rotating disk hot spots, winds, and funnel flows which vary with the rotation period of the star.

The TTS SU Aur is of particular interest in this regard. Johns & Basri (1995) have reported periodic variability in discrete components of the Balmer line profiles of this star. Periodic behavior is found in both a blue-shifted (wind) absorption feature in the H $\alpha$  profile and a red-shifted (accretion) absorption feature in the H $\beta$  profile. Both features are found to vary at the photometric (rotation) period of the star but are 180° out of phase. In the context of the generalized model, these data can be explained by a tilted stellar magnetic dipole (Fig. 6). In this case, field lines in one part of the magnetosphere naturally bend inward allowing accretion; while on the other side of the magnetosphere, they naturally bend outward allowing mass loss. With the lower hemisphere occulted by the disk, the rotation of the magnetosphere at the rotation period of the star produces the observed periodicity and 180° phase difference in the red- and blue-shifted features.

## 8. SUMMARY

We have proposed a generalized dynamical model of flows from magnetized stars rotating both at and below breakup. In the former case, the star is spun up to breakup by the rapid accretion of mass and angular momentum. The star responds by launching a powerful X-wind which removes the excess fraction of the incoming angular momentum, allowing the star to continue accrete and grow in mass. The latter case can

be viewed as the low mass accretion limit of the rapid rotation case. At a low enough disk accretion rate, the dipole component of the stellar magnetic field is strong enough to truncate the circumstellar disk. The interaction between the star and disk results in trapped magnetic flux near the truncation radius from which both X-winds and funnel flows emerge. Angular momentum transfer in these flows may result in the removal of sufficient angular momentum such that a steady-state configuration is possible: the star rotates below breakup, in corotation with the inner disk. In both instances, disk accretion drives winds (and sometimes funnel flows) through angular momentum redistribution mediated by the stellar magnetic field.

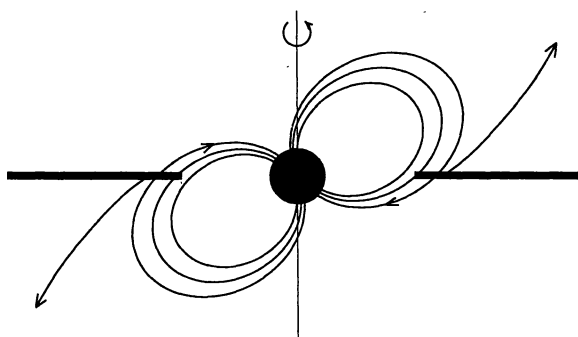


Fig. 6.— Funnel flows and winds from the interaction of a tilted stellar dipole with an accretion disk.

While the case of rotation below breakup may apply to the case of revealed TTSs, the case of rotation at breakup may be more relevant to embedded sources and FU Oris. This picture appears to explain (1) the observed relation between mass accretion and mass loss in embedded and revealed YSOs, (2) observed properties such as the velocity and mass loss rate of winds from embedded and revealed YSOs, (3) how TTSs can rotate slowly despite ongoing disk accretion, and (4) the dynamical basis of funnel flows and their role in the mass and angular momentum evolution of young stars.

The validity of the generalized model can be further investigated using diagnostics that probe the properties of disks, X-winds, and funnel flows in the youngest stars and their more revealed counterparts. The exciting possibilities include diagnostics that have only recently received attention, such as the near-infrared CO overtone and Br  $\gamma$  lines, as well as time variability studies of more traditional diagnostics, such as the hydrogen Balmer lines.

It is a pleasure to thank my colleagues John Carr, Al Glassgold, Eve Ostriker, Frank Shu, and Alan Tokunaga, and especially Eve for her help in preparing Figure 3.

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