RevMexAA (Serie de Conferencias), **8**, 111–114 (1999)

NEW STATISTICAL TECHNIQUES FOR MEASURING THE INITIAL MASS FUNCTION

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RESUMEN

La relación masa–magnitud en un diagrama color–magnitud no es unívoca ni biyectiva. La rotación estelar y la presencia de sistemas binarios no resueltos introducen tales degeneraciones en la relación que hacen imposible la estimación de la función actual de masas, dada una función de luminosidad. Proponemos dos estadísticas no paramétricas que permiten resolver el problema, utilizando técnicas probabilísticas bayesianas, basadas en el conteo en celdillas en los diagramas color–magnitud de cúmulos estelares.

ABSTRACT

The relation linking magnitude to a given mass in a colour–magnitude diagram is neither unique nor bijective. Stellar rotation and unresolved binaries produce a degeneracy that undermines any attempt to use luminosity functions to derive the present-day mass function. We argue that only Bayesian techniques must be used, and propose two non-parametric statistics to derive the initial mass function based on counts in cells of stars in colour-magnitude diagrams.

Key words: **STARS: EVOLUTION — STARS: FORMATION — STARS: HR DIAGRAM — STARS: LUMINOSITY FUNCTION, MASS FUNCTION — METHODS: STATISTICAL**

1. INTRODUCTION: A CRITIQUE OF THE STANDARD METHOD

The problem of going from light to mass is very often found in astrophysics, from the largest structures traced by galaxies on cosmological scales to the measure of the mass distribution of an ensemble of stars through the problem of the 'missing mass' on galactic and cluster scales. In the context of stellar clusters, the standard method uses the determination of the luminosity function $\Phi(M_j)$ in some photometric band j, assumes that bins in magnitude ΔM_j correspond to bins in mass Δm and then derives the present-day mass function $\psi(m)$ through the relation

$$\psi(m)\,\Delta m \;=\; \Phi(M_j)\cdot\left(\frac{dL}{dM_j}\right)\cdot\left(\frac{dm}{dL}\right)\,\Delta M_j \;\; . \qquad (1)$$

Transforming a given magnitude interval into a mass bin depends then on the *slope* of the bolometric correction $L(M_j)$ and on the *slope* of the mass-luminosity relation $m(L)$. The latter can be derived with double-lined eclipsing binaries, although its dependence with metallicity has not been hitherto measured (see Malkov, Piskunov, & Shpil'kina 1997, for a recent summary for low mass stars). In addition, the recent Hipparcos determinations of the positions of the main sequences of clusters with the same or different metallicities (Mermilliod et al. 1997) show that there are other (hidden?) parameters that play a key role, such as perhaps the Helium abundance, in shifting the sequences in a quite unexpected way. Some doubts may be cast, therefore, on the current metallicity scalings. In addition, the very strong dependence of the bolometric correction with colour makes its derivative highly uncertain. Although careful analyses have been attempted (e.g., Tarrab 1982) the uncertainties are simply too large to produce a precise estimate of the present-day mass function in clusters. From a purely numerical point of view, the transmission of errors makes the transformation technique (eq. 1) very unreliable.

But this is the least of the problems, because the hypothesis on which eq. 1 is based, the one-to-one correspondence between mass and magnitude, is quite simply wrong. There are at least two effects that have been neglected in the past and that introduce a wide degeneracy into this relation: stellar rotation and the presence of unresolved binaries. The generic trend produced by stellar rotation in a colour-magnitude diagram (CMD) is to redden the colour (produced by the decrease in the effective temperature) and increase the magnitude, although the effect depends very strongly on the inclination of the rotational axis (see Lastennet & Valls-Gabaud 1999, for further details). This effect causes a systematic overestimation of the ages derived by isochrone fitting, but also shows that a position in a CMD is not univocally linked to a given mass, since the amplitude of the angular velocity and the inclination introduce a degeneracy. Unresolved binary stars are another example of the lack of one-to-one correspondence between mass and luminosity, since depending on the mass of the secondary, the colour and magnitude of the combined system will change dramatically. Hence, deriving mass functions from luminosity functions is quite simply nonsensical and inconsistent with both data and methodological assumptions.

2. NON-PARAMETRIC BAYESIAN TECHNIQUES

The CMDs do contain, however, the information required to derive the present-day mass function in a cluster, provided realistic and careful simulations are carried out including both rotation and unresolved binaries (see Lastennet & Valls-Gabaud 1999, for a brief description). To recover this information, however, a Bayesian approach is required, given the degeneracy of the problem. Basically, one wants to maximise the probability that a prior model M agrees with the observations D: $P(M \mid D) = P(D \mid M)\, P(M)$. A maximum likelihood technique has already been attempted (Tolstoy & Saha 1996) but suffers from over-restrictive assumptions. In this paper we propose the use of two statistics that are generic and allow us to derive reliable confidence intervals on the parameters of the simulated CMDs.

The first statistic, Saha's W, is based on the fact that the probability distribution function for the counts in cells of a CMD is given by the multinomial function. It has been used previously in the context of comparing discrete data with numerical N-body simulations (Valls-Gabaud et al. 1997; Sevenster et al. 1999), and given a formal proof by Saha (1998). Given an observed CMD, and a synthetic one, we ask what is the probability that both diagrams are samples of the same underlying distribution function. Hence, assume that the CMD is divided into B boxes, with S observed stars $\{s_i\}$, and M model stars $\{m_i\}$ (depend on parameters). The unknown distribution function appears via the unknown box weights $\{w_i\}$ which are the values of the distribution function in that particular box, so that the multinomial distribution is

$$P(s_i, m_i \mid w_i) = M!\, S! \prod_{i=1}^{B} \frac{w_i^{m_i+s_i}}{m_i! s_i!}, \qquad \sum_i w_i = 1 \ . \tag{2}$$

Marginalizing, and using Bayes' theorem, one obtains that the likelihood is

$$\text{Prob} \propto W = \prod_{i=1}^{B} \frac{(m_i + s_i)!}{m_i! s_i!} \ . \tag{3}$$

The normalization here is irrelevant, since we are using likelihood ratios when comparing several (different) simulations of different models. The relative likelihood gives the best fitting model, irrespective of its underlying parameters. Parameter fitting (i.e., binary fraction, slope of the IMF, etc.) can be achieved by fixing $\{s_i\}$, varying the model parameters, and hence $\{m_i\}$, and measuring their confidence intervals. This does not tell us, however, if this best fit is a good fit. For this, we produce several realizations of the same model, fixing the parameters (and hence $\{m_i\}$), and vary $\{s_i\}$. By comparing the resulting W distribution from the value obtained with the actual data, we can decide, statistically, whether the fit was good or not.

The second statistic, Peacock's D, is a generalization of the powerful one-dimensional non-parametric Kolmogorov-Smirnov statistics for two dimensions. Although in one dimension the direction of binning is unimportant (in the sense that $P(\geq x) = 1 - P(\leq x)$), in N dimensions one has 2^N-1 possible independent directions. The trick invented by Peacock (1983) is to define D as the maximum absolute difference between the observed and predicted normalized cumulative distributions in the 4 possible ways to cumulate following the directions of the coordinate axes. That is,

$$(x < X_i, y < Y_i) \qquad (x < X_i, y > Y_i) \ ,$$

$$(x > X_i, y < Y_i) \qquad (x > X_i, y > Y_i) \ ,$$

for the $i = 1, N$ data points. The only problem with this is that the distribution of D is not independent of the shape of the distribution function, unlike the 1-D case. Fasano & Franceschini (1987) have carried out extensive numerical simulations for a variety of distribution functions in two dimensions and provide fitting formulae for the rejection probabilities. Yet, in our case, the underlying distribution function is very distorted when different IMFs or binary fractions are used, and the fitting formulae are of little help. The solution is of course to 'calibrate' the distribution of D using sample realizations of the same model distribution, just as we did for the W statistic. We thus make Monte Carlo realizations of CMDs for a given set of parameters and compute the distribution function of D. The comparison of this distribution with the value of D obtained with the actual data gives the probability that the model is statistically compatible with the dataset. This is illustrated in Figure 1.

In summary, the non-parametric statistics we propose provide useful estimates of the IMF from typical CMDs of clusters, without the need for ad hoc fitting criteria, provided realistic simulations can be made. Both W and D are non-parametric, robust statistics, and provide confidence intervals for parameters. The limitation of the W statistic is the number of cells used, but it can provide a high mass resolution. Alternatively, the D statistic gives the global slope, but it is mainly limited by the number of stars (single or otherwise) present in the observed CMD.

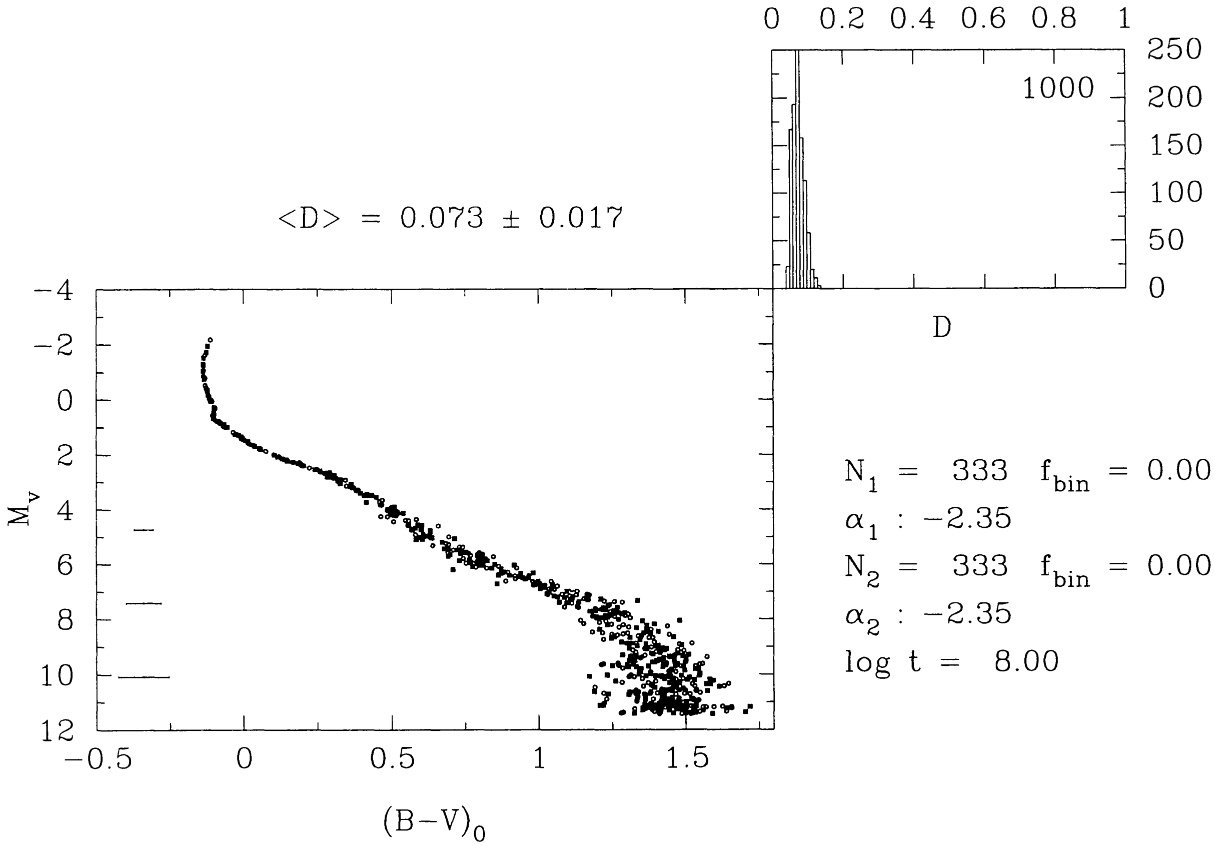

Fig. 1. The upper panel gives the distribution function of D derived from 1000 samples of $N_2 = 333$ stars that follow Salpeter's IMF ($\alpha_2 = -2.35$), contain no binaries ($f_{bin} = 0$) at an age of 10^8 years. The average value is $\langle D \rangle = 0.073 \pm 0.017$. The lower panel shows two realizations extracted from the sample of 1000, with N_1(circles) $= N_2$(squares) $= 333$. The comparison of the D statistic measured from actual data with this D distribution allows us to accept or reject the hypothesis that both the observed CMD and the synthetic one are samples of the same underlying model (in this case $\alpha_2 = -2.35$, $f_{bin} = 0$ and an age of 10^8 yrs).

3. CONCLUSIONS

The non-uniqueness of the mass-magnitude relation for stars in clusters makes it impossible to use the derived luminosity function to infer the mass function. The effects of rotation and of unresolved binaries in colour-magnitude diagrams introduce a degeneracy that can only be tackled using Bayesian techniques. We have introduced two non-parametric estimators to test the statistical agreement between synthetic CM diagrams with a given IMF with the observed data. Bayesian techniques are the only ones that allow us to make further progress in the measure of the IMF.

We thank the Organizing Committee for partial financial support to attend this very enjoyable meeting.

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